MINIMAL TORI IN $S^2 \times S^1$

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Abstract. Choi and Schoen (1985) have shown that for a positively Ricci curved three-fold $M$, the family of embedded minimal surfaces of a fixed genus is compact. It is of interest to know how much the positivity condition can be relaxed.

For three-folds with zero or negative Ricci curvature, Tian [3] observed that the product spaces $\Sigma_g \times S^1$ have totally geodesic embedded tori, winding around as many times as one wishes. Here $\Sigma_g$ is a Riemann surface of genus $g$ endowed with metric of constant curvature $-1$ if $g > 1$ or $0$ if $g = 1$. We would like to indicate that the embedded minimal tori in $S^2 \times S^1$ do not form a compact family either.

Let

$$ds^2 = dr^2 + \sin^2 r d\theta^2 + dz^2$$

be the riemannian metric on $S^2 \times S^1$. The rotationally symmetric minimal tori can be described by $r = r(z)$ and independent of $\theta$. They satisfy

$$r''/(1 + r'^2) = \cot r$$

and the periodic condition

$$r(z + 2\pi) = r(z).$$

The general solutions are elliptic functions and for a sequence of initial values $r(z = 0) = c_1 > c_2 > c_3 > \cdots \rightarrow 0$, $2\pi$ is a period. Details can be found in any classical treaties, e.g., [1]. The corresponding sequence of embedded minimal tori certainly cannot have a convergent subsequence.

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