PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 122, Number 1, September 1994

MINIMAL TORI IN $\mathbb{S}^2 \times \mathbb{S}^1$

CHUN-CHUNG HSIEH AND AI NUNG WANG

(Communicated by Christopher Croke)

ABSTRACT. Choi and Schoen (1985) have shown that for a positively Ricci curved three-fold M, the family of embedded minimal surfaces of a fixed genus is compact. It is of interest to know how much the positivity condition can be relaxed.

For three-folds with zero or negative Ricci curvature, Tian [3] observed that the product spaces $\Sigma_g \times \mathbb{S}^1$ have totally geodesic embedded tori, winding around as many times as one wishes. Here Σ_g is a Riemann surface of genus g endowed with metric of constant curvature -1 if $g > 1$ or 0 if $g = 1$. We would like to indicate that the embedded minimal tori in $\mathbb{S}^2 \times \mathbb{S}^1$ do not form a compact family either.

Let

$$ds^2 = dr^2 + \sin^2 r d\theta^2 + dz^2$$

be the riemannian metric on $\mathbb{S}^2 \times \mathbb{S}^1$. The rotationally symmetric minimal tori can be described by $r = r(z)$ and independent of θ. They satisfy

$$r''/(1 + r'^2) = \cot r$$

and the periodic condition

$$r(z + 2\pi) = r(z).$$

The general solutions are elliptic functions and for a sequence of initial values $r(z = 0) = c_1 > c_2 > c_3 > \cdots \to 0$, 2π is a period. Details can be found in any classical treaties, e.g., [1]. The corresponding sequence of embedded minimal tori certainly cannot have a convergent subsequence.

ACKNOWLEDGMENT

This work was completed when both authors were visiting Harvard University. We would like to thank the university for its hospitality. Our gratitude also extends to Mlle. Der Chyi Wu, who brought this problem to our attention.

Received by the editors September 28, 1993 and, in revised form, October 28, 1993.
1991 *Mathematics Subject Classification*. Primary 53A10.
Key words and phrases. Minimal surface, Ricci curvature.
Partially supported by NSC 83-0208-M-002-029.

References

1. F. Bowman, *Introduction to elliptic functions with applications*, Wiley, New York, 1953.
2. Hyeong In Choi and Richard Schoen, *The space of minimal embeddings of a surface into a three-dimensional manifold of positive Ricci curvature*, Invent. Math. **81** (1985), 387–394.
3. Gang Tian, private communication.

Mathematics Institute, Academia Sinica, Taipei, Taiwan

Department of Mathematics, National Taiwan University, Taipei, Taiwan
E-mail address: wang@math.ntu.edu.tw