SPECIAL POINTS IN COMPACT SPACES

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Abstract. Given a collection $\mathcal{C}$, of cardinality $\kappa$, of subsets of a compact space $X$, we prove the existence of a point $x$ such that whenever $C \in \mathcal{C}$ and $x \in C$, there exists a $G_{\delta}$-set $Z$ with $\lambda < \kappa$ and $x \in Z \subset \overline{C}$. We investigate the case when $\mathcal{C}$ is the collection of all cozerosets of $X$ and also when $X$ is a dyadic space. We apply this result to homogeneous compact spaces. Another application is a characterization of $2^{\omega_1}$ among dyadic spaces.

1. Introduction

A space $X$ is homogeneous if for every $x, y \in X$, there exists an automorphism $h$ of $X$ such that $h(x) = y$. This paper is concerned with homogeneous compact spaces. For a cardinal $\kappa$ and a collection $\mathcal{C}$ of subsets of a space $X$, we define what we mean by a $\mathcal{C}_\kappa$-point of $X$ (see Definition 2.1). Our main results are the Point Theorem and the Dyadic Cozero Point Theorem of §2 which prove the existence of $\mathcal{C}_\kappa$-points in compact spaces. In §3 we apply the results of §2 to homogeneous spaces and get the Homogeneous Theorem and the Dyadic Cozero Homogeneous Theorem. This paper contains a number of examples illustrating the sharpness of the theorems and also answering two questions previously posed by the author. Finally, we characterize $2^{\omega_1}$, among dyadic spaces, as the unique zero-dimensional, homogeneous compact space of weight $\omega_1$. This answers a question posed by B. Efimov for the case $\kappa = \omega_1$ of $2^{\kappa}$. We first proved this result assuming the continuum hypothesis $CH$. Subsequently, L. Shapiro removed this extra assumption. We show how it follows from our general theory.

Cardinals are initial ordinals, and $\omega$ and $\omega_1$ are the first two infinite cardinals. The cofinality of the cardinal $\kappa$ is denoted by $\text{cf}(\kappa)$. The set of all subsets of $X$ is denoted by $\mathcal{P}(X)$. We use standard set-theory notions as in Kunen's Set Theory [Ku].

All spaces in this paper are assumed to be $T_2$, i.e., Hausdorff. We use standard topological notions as in Engleking's General Topology [En2] with one exception. We use the term cozeroset instead of the term functionally open. We emphasize two cardinal functions. The weight $w(X)$ of a space $X$ is the...

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least cardinal of a base for $X$, and the character $\chi(x, X)$ of $x$ in $X$ is the least cardinal of a local base at $x$ in $X$. We emphasize one topological property. A space $X$ is dyadic if $X$ is the continuous image of some Cantor Cube $2^\kappa$.

2. The Point Results

We begin with the central definitions of our paper. Let $\kappa$ be an infinite cardinal. A subset $Z$ of a space $X$ is a $G_\kappa$-set (resp. $G_{<\kappa}$-set) if $Z = \bigcap_{\alpha < \lambda} U_\alpha$ where $\lambda \leq \kappa$ (resp. $\lambda < \kappa$) and the $U_\alpha$'s are open subsets of $X$. For $A \subseteq X$, we put $G_\kappa\text{-int}(A) = \bigcup\{Z \subseteq A : Z$ is a $G_\kappa$-set of $X\}$. We say that $A$ is $G_\kappa$-open in $X$ if $A = G_\kappa\text{-int}(A)$. The preceding two notions yield two more if we uniformly replace $\kappa$ by $< \kappa$.

Definition 2.1. Let $\kappa$ be an infinite cardinal, and let $\mathcal{C} \subseteq \mathcal{P}(X)$. A point $x \in X$ is a $\mathcal{G}_\kappa$-point of $X$ if whenever $C \in \mathcal{C}$ and $x \in C$, then $x \in G_{<\kappa}\text{-int}(C)$.

Definition 2.2. Let $\kappa$ be an infinite cardinal. A point $x \in X$ is a $\mathcal{W}_\kappa$-point of $X$ if there exists a disjoint clopen family $\mathcal{B}$ in $X$ of cardinality $\kappa$ such that every neighbourhood $N$ of $x$ intersects all but finitely many members of $\mathcal{B}$.

When the compact space $X$ is understood, we put $\mathcal{C}_X = \{C \subseteq X : C$ is a cozeroset of $X\}$. Note that in a compact space $X$ a cozeroset is the same as an open $F_\sigma$ set.

Proposition 2.3. Let $X$ be a compact space, and let $\kappa$ be an infinite cardinal. Then, a $\mathcal{W}_\kappa$-point is not a $\mathcal{G}_\kappa$-point.

Proof. Let $\mathcal{B}$ be a disjoint, clopen family of cardinality $\kappa$ witnessing the fact that $x$ is a $\mathcal{W}_\kappa$-point of $X$. Let $\mathcal{C}$ be a countably infinite subfamily of $\mathcal{B}$. Then $C = \bigcup\mathcal{C}$ is a cozeroset of $X$ and $x \in C$.

Let $\lambda < \kappa$, and let $Z$ be any $G_\lambda$-set with $x \in Z$. Using regularity, for each $\alpha < \lambda$, get a closed neighbourhood $N_\alpha$ of $x$ with $\bigcap_{\alpha < \lambda} N_\alpha \subseteq Z$. Using the $W_\kappa$-point property of $x$, choose $B \in \mathcal{B}\setminus\mathcal{C}$ such that for every finite $F \subseteq \lambda$, $B \cap \bigcap_{\alpha \in F} N_\alpha \neq \emptyset$. Using compactness of $X$, choose $p \in B \cap \bigcap_{\alpha < \lambda} N_\alpha$. Note that $B \cap C = \emptyset$. Thus, $p \in Z$ and $p \notin C$. Hence $x$ cannot be a $\mathcal{G}_\kappa$-point of $X$. □

Theorem 2.4 (Point Theorem). If $X$ is a compact space and $\mathcal{C}$ is a collection of subsets of $X$ with $|\mathcal{C}| = \kappa$, then there exists a $\mathcal{G}_\kappa$-point in $X$.

Proof. Enumerate $\mathcal{C}$ as $\{C_\alpha : \alpha < \kappa\}$. We will define, by induction on $\alpha < \kappa$, a sequence of zerosets $Z_\alpha$ of $X$. Put $Z_0 = X$. At stage $\alpha$ if there exists a zeroset $Z$ such that $Z \cap \bigcap_{\beta < \alpha} Z_\beta \neq \emptyset$ and $Z \cap C_\alpha = \emptyset$, then let $Z_\alpha$ be one such zeroset $Z$. If there does not exist such a zeroset $Z$, then put $Z_\alpha = X$.

At the end of this induction, by compactness, choose $x \in \bigcap_{\alpha < \kappa} Z_\alpha$. Then $x$ is a $\mathcal{C}_\kappa$-point of $X$. To see this, let $C \subseteq X$ and assume that $x \in C$. Choose $\alpha < \kappa$ with $C = C_\alpha$. We claim that $\bigcap_{\beta \leq \alpha} Z_\beta \subseteq C_\alpha$. If not, then choose $y \in \bigcap_{\beta \leq \alpha} Z_\beta \setminus C_\alpha$. Get a zeroset $Z$ with $y \in Z$ and $Z \cap C_\alpha = \emptyset$. So at stage $\alpha$ we have that $Z_\alpha \cap C_\alpha = \emptyset$. But this is a contradiction. □

The Point Theorem only has content if each $x \in X$ has $\chi(x, X) \geq \kappa$, because a point of character $< \kappa$ is obviously a $\mathcal{C}_\kappa$-point.
We are mostly interested when $f = \mathcal{W}$. Since $|\mathcal{W}| = w(X)^\omega$ for compact spaces $X$, to avoid no content, we restate this special case as:

**Theorem 2.5 (Cozero Point Theorem).** If $X$ is compact and $w(X) = \kappa = \kappa^\omega$, then there exists a $\mathcal{W}_\kappa$-point in $X$.

We now give some examples to illustrate the sharpness, in general, of $\mathcal{W}_\kappa$ in the Cozero Point Theorem. Obviously, if $\lambda < \kappa$, then having a $\mathcal{W}_\lambda$-point is a stronger result than having a $\mathcal{W}_\kappa$-point. In certain spaces there are better results. Every point is a $\mathcal{W}_{\omega_1}$-point in any Cantor cube $2^\kappa$. Every point is a $\mathcal{W}_\omega$-point in any compact basically disconnected space. In the following we will use the basic fact that in a compact space $X$ a $\mathcal{W}_\kappa$-point is not a $\mathcal{W}_{\omega_1}$-point.

**Example 2.6.** For every $\kappa > \omega$ there exists a compact $X_\kappa$ such that
- $w(X) = \kappa$;
- $X$ has no $\mathcal{W}_\omega$-points, for $\lambda < \kappa$.

*Proof.* Case 1. $\kappa$ is a successor; $\kappa = \lambda^+$ where $\lambda \geq \omega$. Put $\alpha \lambda = \lambda \cup \{\infty\}$, the Alexandroff one-point compactification of the discrete space $\lambda$. For $f \in (\alpha \lambda)^\kappa$, define $\text{supp}(f) = \{\beta < \kappa : f(\beta) \neq \infty\}$. Set $X_\kappa = \{f \in (\alpha \lambda)^\kappa : f|\text{supp}(f) \text{ is a one-to-one function}\}$. As a subspace of $(\alpha \lambda)^\kappa$, $X_\kappa$ is closed and hence is compact. Let $f \in X_\kappa$. Choose $\beta \in X_\kappa \setminus \text{supp}(f)$. For $\gamma < \lambda$, define $B_\gamma = \{g \in X_\kappa : g(\beta) = \gamma\}$. Then $\{B_\gamma : \gamma < \lambda\}$ witnesses the fact that $f$ is a $W_1$-point.

Hence, all points of $X_\kappa$ are $W_1$-points, and therefore no point is a $\mathcal{W}_\omega$-point.

We mention that $X_\omega_2$ answers a question of the author raised in [Bel] about whether there exists a compact space in which every point is a $W_{\omega_1}$-point.

Case 2. $\kappa$ is a limit; $\kappa = \sum_{\alpha < \text{cf}(\kappa)} \kappa_\alpha$ where for each $\alpha < \text{cf}(\kappa)$, $\kappa_\alpha$ is a successor cardinal $< \kappa$. Set $X_\kappa = \prod_{\alpha < \text{cf}(\kappa)} X_\kappa_\alpha$ where the $X_\kappa_\alpha$'s are as in Case 1. Fix $\lambda < \kappa$, and assume that $f \in X_\kappa$ is a $\mathcal{W}_\kappa$-point. Choose $\alpha < \text{cf}(\kappa)$ such that $\lambda < \kappa_\alpha < \kappa$. Then $f(\alpha)$ must be a $\mathcal{W}_1$-point of $X_\kappa_\alpha$ — a contradiction. $\square$

The next example shows that the condition $\kappa = \kappa^\omega$ cannot be removed, in general, from the Cozero Point Theorem.

**Example 2.7.** There exists a compact space $Y$ of weight $\omega_\omega$ which has no $\mathcal{W}_{\omega_1}$-points.

*Proof.* For each $n < \omega$, define $Y_n = X_{\omega+n}$ as defined in Example 2.5 and put $Y = \prod_{n < \omega} Y_n$. For each $n < \omega$, let $\pi_n$ be the projection map onto $Y_n$.

Let $f \in Y$. For every $n < \omega$, $f(n)$ is not a $\mathcal{W}_{\omega_1}$-point of $Y_n$ (by the previous example), so let $C_n$ be a cozeroset of $Y_n$ such that $f(n) \in \overline{C_n \setminus \text{int}(C_n)}$. For every $n < \omega$, let $(C^k_n : k < \omega)$ be an increasing sequence of cozerosets of $Y_n$ such that $C_n = \bigcup_{k < \omega} C^k_n$ and such that for every $k < \omega$, $\overline{C^k_n} \subset C_n$. For every $k < \omega$, define $D_k = \bigcap_{i \leq k} \pi^{-1}_k (C^i_k)$. Put $D = \bigcup_{k < \omega} D_k$. Then $D$ is a cozeroset of $Y$ and $f \in \overline{D}$.

Striving for a contradiction, assume that $f$ is a $\mathcal{W}_{\omega_1}$-point and choose $n < \omega$ and a closed $G_{\omega_1}$-set $Z$ with $f \in Z \subset \overline{D}$.

For each $k < \omega$, choose a closed $G_{\omega_1}$-set $Z_k$ of $Y_k$ such that $f \in \prod_{k < \omega} Z_k \subset Z$. $Z_n$ cannot be
contained in $\overline{C_k}$, so let $y \in Z_n \setminus \overline{C_n}$. Define $g \in Y$ by
\[
g(k) = \begin{cases} f(k) & \text{if } k \neq n, \\ y & \text{if } k = n.\end{cases}
\]
Then $g \in \prod_{k \in \omega} Z_k$ and so $g \in \overline{D}$. But this is a contradiction as $\pi_k^{-1}(Y_n \setminus \overline{C_n}) \cap \bigcap_{i < n} \pi_i^{-1}(Y_i \setminus \overline{C_i})$ is a neighbourhood of $g$ which is disjoint from $D = \bigcup_{k \in \omega} \left( \bigcap_{i \leq k} \pi_k^{-1}(C_k^i) \right)$. \square

It might be possible to replace the condition $k = k_0$ in the Cozero Point Theorem by the weaker condition $\text{cf}(k) > \omega$, but we do not know.

**Question 2.8.** If $X$ is a compact space with $w(X) = \kappa$ and $\text{cf}(\kappa) > \omega$, then does $X$ have a $\mathcal{C}_\kappa\mathcal{Z}_\kappa$-point? (Of course, this is true if one assumes GCH; this question is particularly interesting for $\kappa = \omega_1$.)

We do have a positive answer to this question provided $X$ is a dyadic space.

**Theorem 2.9** (Dyadic Cozero Point Theorem). If $X$ is a dyadic space and $w(X) = \kappa$ and $\text{cf}(\kappa) > \omega$, then there exists a $\mathcal{C}_\kappa\mathcal{Z}_\kappa$-point in $X$.

**Proof.** Let $\phi: 2^\kappa \to X$. For each $\alpha < \kappa$, let $\pi_\alpha: 2^\kappa \to 2^\alpha$ be the projection map. We say that a subset $A$ of $2^\kappa$ depends on $\alpha$ if $A = \pi_\alpha^{-1}\pi_\alpha(A)$. We will define, by induction on $\alpha$, where $\omega \leq \alpha < \kappa$, a sequence of closed $G_{|\alpha|}$-sets $Z_\alpha$ of $X$. Put $Z_\omega = X$. At stage $\alpha$ put $W = \bigcap_{\beta < \alpha} Z_\beta$. Then $W$ is a closed $G_{|\alpha|}$-set. Put $Z_\alpha = \text{some nonempty closed } G_{|\alpha|}$-set $Z \subset W$ such that if $Z'$ is any nonempty closed $G_{|\alpha|}$-set with $Z' \subset Z$, then $\pi_\alpha(\phi^{-1}(Z')) = \pi_\alpha(\phi^{-1}(Z))$. Such a $Z$ exists since $2^\alpha$ has no strictly decreasing $|\alpha|$ sequence of closed sets.

At the end of this induction, by compactness, choose $x \in \bigcap_{\alpha < \kappa} Z_\alpha$. Then $x$ is a $\mathcal{C}_\kappa\mathcal{Z}_\kappa$-point in $X$. To see this, assume that $B \in \mathcal{C}_\kappa\mathcal{Z}$ and that $x \in \overline{B}$. Then $\phi^{-1}(B)$ is a cozeroset in $2^\kappa$ and since $\text{cf}(\kappa) > \omega$, we can choose an $\alpha < \kappa$ such that $\phi^{-1}(B)$ depends on $\alpha$. From $x \in Z_\alpha \cap \overline{B}$, it follows that $\phi^{-1}(Z_\alpha) \cap \phi^{-1}(B) \neq \emptyset$.

We claim that $Z_\alpha \subset \overline{B}$. If not, then choose $y \in Z_\alpha \setminus \overline{B}$. Choose a zero set $A \ni y$ such that $A \cap \overline{B} = \emptyset$. Put $Z' = A \cap Z_\alpha$. $Z'$ is a nonempty closed $G_{|\alpha|}$-set with $Z' \subset Z_\alpha$. So at stage $\alpha$ we have that $\pi_\alpha(\phi^{-1}(Z')) = \pi_\alpha(\phi^{-1}(Z_\alpha))$. $Z' \cap \overline{B} = \emptyset$ implies that $\phi^{-1}(Z') \cap \phi^{-1}(B) = \emptyset$. Since $\phi^{-1}(B)$ depends on $\alpha$, we get that $\phi^{-1}(B)$ also depends on $\alpha$. So, $\pi_\alpha(\phi^{-1}(Z')) \cap \pi_\alpha(\phi^{-1}(B)) = \emptyset$. Hence, $\pi_\alpha(\phi^{-1}(Z_\alpha)) \cap \pi_\alpha(\phi^{-1}(B)) = \emptyset$. Thus, $\phi^{-1}(Z_\alpha) \cap \phi^{-1}(B) = \emptyset$—a contradiction. \square

### 3. Applications to Homogeneity

The first application is to resolve a question posed by the author in [Be2]. Let $X$ be a zero-dimensional compact space, and let $\mathcal{C}$ be the family of clopen subsets of $X$. A family of sets $\mathcal{A}$ is *centered* if for every finite $\mathcal{F} \subset \mathcal{A}$ we have that $\bigcap \mathcal{F} \neq \emptyset$. Define $\text{Cen}(X) = \{ f \in 2^\mathcal{C} : \{ B \in \mathcal{C} : f(B) = 1 \} \text{ is a centered family} \}$. $\text{Cen}(X)$ is closed in $2^\mathcal{C}$, hence it is compact. For each $B \in \mathcal{C}$, put $B^* = \{ f \in \text{Cen}(X) : f(B) = 1 \}$. Then $B^*$ is a clopen subset of $\text{Cen}(X)$. We consider $\text{Cen}(\omega^*)$, where $\omega^*$ is the Stone-Cech remainder.
βω\(\omega\). In the above paper we investigated \(\text{Cen}(\omega^*)\), showed that it is in some sense the largest "Combinatorial" remainder of \(\omega\), and mentioned that every point has \(\pi\)-character \(c\). We asked whether \(\text{Cen}(\omega^*)\) was homogeneous.

**Proposition 3.1.** \(\text{Cen}(\omega^*)\) is not homogeneous.

*Proof.* Let \(\bar{0}\) be the constant function \(0\) in \(\text{Cen}(\omega^*)\). Let \(\mathscr{B}\) be a disjoint family of clopen sets in \(\omega^*\) of cardinality \(c\). The disjoint clopen family \(\{B^* : B \in \mathscr{B}\}\) witnesses the fact that \(\bar{0}\) is a \(W^\sigma\)-point; thus, by Proposition 2.3, \(\bar{0}\) is not a \(\mathcal{G}^{\omega}_c\)-point. By the Cozero Point Theorem, since \(w(\omega^*) = c = c^\omega\), \(\text{Cen}(\omega^*)\) must have a \(\mathcal{G}^{\omega}_c\)-point. Since having a \(\mathcal{G}^{\omega}_c\)-point is a topological invariant, \(\text{Cen}(\omega^*)\) is not homogeneous and our proposition is proved. □

It follows from the Point Theorem that \(\omega^*\) has a \(\mathcal{G}^{\omega}_c\)-point. An interesting question which we have not been able to resolve without assuming extra set-theoretic assumptions is the following:

**Question 3.2.** Does \(\omega^*\) have a point which is NOT a \(\mathcal{G}^{\omega}_c\)-point?

Now, let us restate the Point Theorems for a homogeneous compact space. A family \(\mathscr{C} \subset \mathscr{P}(X)\) is called invariant if for all \(C \in \mathscr{C}\) and for all automorphisms \(h\) of \(X\) we have that \(h(C) \in \mathscr{C}\). In this case the existence of one point gives us that all points are \(\mathscr{C}_k\)-points.

**Theorem 3.3 (Homogeneous Theorem).** If \(X\) is a homogeneous compact space and \(\mathscr{C}\) is an invariant family with \(|\mathscr{C}| = \kappa\), then for each \(C \in \mathscr{C}\), \(C\) is \(G_{\kappa}\)-open in \(X\).

**Theorem 3.4 (Cozero Homogeneous Theorem).** If \(X\) is a homogeneous compact space and \(w(X) = \kappa = \kappa^\omega\) and \(C \in \mathcal{G}^{\omega}_c\), then \(\overline{C}\) is \(G_{\kappa}\)-open in \(X\).

The Cozero Homogeneous Theorem cannot be improved by replacing \(G_{\kappa}\)-open by a \(G_{\kappa}\)-set.

**Example 3.5.** There exists a homogeneous compact space \(X\) such that

- \(w(X) = c\);
- \(X\) has a cozeroset \(C\) such that \(\overline{C}\) is not a \(G_{<\kappa}\)-set.

*Proof.* Let \(Y\) be the Alexandroff Double of the Cantor set \(2^\omega\). Then \(Y\) is a first countable, zero-dimensional, compact space with a dense set of isolated points. Put \(X = Y^\omega\). By a result of Motorov [Mo], \(X\) is homogeneous. Let \(D\) be a countable, dense subset of \(2^\omega\), and let \(E\) be the isolated points of \(Y\) which correspond to \(D\). Then \(E\) is a cozeroset of \(Y\) and it is seen that \(\overline{E}\) is not a \(G_{<\kappa}\)-set in \(Y\). Let \(\pi\) be the projection map of \(X\) onto the first factor space \(Y\). It follows that if we put \(C = \pi^{-1}(E)\), then \(C\) is a cozeroset of \(X\) and \(\overline{C}\) is not a \(G_{<\kappa}\)-set in \(X\). □

However, in a dyadic space, the Cozero Homogeneous Theorem can be improved to

**Theorem 3.6 (Dyadic Cozero Homogeneous Theorem).** If \(X\) is a homogeneous dyadic space and \(w(X) = \kappa^+\) with \(\kappa \geq \omega\) and \(C \in \mathcal{G}^{\omega}_c\), then \(\overline{C}\) is a \(G_\kappa\)-set.

*Proof.* Applying homogeneity to the Dyadic Cozero Point Theorem we immediately get that if \(C \in \mathcal{G}^{\omega}_c\), then \(\overline{C}\) is \(G_\kappa\)-open in \(X\). In dyadic spaces, closed \(G_\kappa\)-open sets are actually \(G_\kappa\)-sets, so our result follows. A justification of this
just-quoted dyadic fact goes as follows: Theorem 6 of Engelking [En1, p. 293] implies that in a Cantor cube, a closed $G_{\kappa}$-open set is actually a $G_{\kappa}$-set. The general dyadic case follows from an easy argument showing that this property is preserved under closed continuous images. □

Efimov [Ef] asked whether there existed a zero-dimensional, homogeneous dyadic space not homeomorphic to $2^\kappa$. This was answered by Pashenkov [Pa], who showed that they do exist. However, the smallest weight example that he produced was of weight $2^{\omega_1}$. Using the Cozero Homogeneous Theorem plus the additional assumption of the Continuum Hypothesis, we showed that there could be no such example of weight $\omega_1$. Subsequently, L. Shapiro [Sh] removed the CH assumption completely from this result. After being informed of this, we proved the Dyadic Cozero Homogeneous Theorem. The reader is encouraged to read [Sh] for an inverse limit approach to this problem.

**Corollary 3.7.** The Cantor cube $2^{\omega_1}$ is topologically the only zero-dimensional, homogeneous dyadic space of weight $\omega_1$.

**Proof.** Let $X$ have all the properties in the corollary. Since $\chi(X) = w(X)$ in dyadic spaces and $X$ is homogeneous, we get that $X$ has uniform character $\omega_1$. Since, in a dyadic space, a regular closed set is the closure of some cozeroset, by applying the preceding theorem, we get that every regular closed set is a $G_{\omega}$-set.

To complete the proof, we now use the following result of Schepin [Sc]: A zero-dimensional dyadic space of uniform character $\omega_1$ in which every regular closed set is a $G_{\omega}$-set is homeomorphic to $2^{\omega_1}$. As this result does not explicitly appear in this reference, to help our readers, we explain it using notions and theorems from that paper. Theorem 18 on p. 10 implies that $X$ is open-generated. Since $w(X) = \omega_1$, Theorem 3 on p. 2 implies that $X$ is $AE(0)$. Finally, Theorem 1 on page 1 implies that $X$ is a Cantor cube. □

**References**


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