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ON THE HOLOMORPHY CONJECTURE FOR IGUSA'S LOCAL ZETA FUNCTION

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ABSTRACT. To a polynomial f over a p-adic field K and a character χ of the group of units of the valuation ring of K one associates Igusa's local zeta function $Z(s, f, \chi)$, which is a meromorphic function on $\mathbb{C}$. Several theorems and conjectures relate the poles of $Z(s, f, \chi)$ to the monodromy of f; the so-called holomorphy conjecture states roughly that if the order of χ does not divide the order of any eigenvalue of monodromy of f, then $Z(s, f, \chi)$ is holomorphic on $\mathbb{C}$. We prove mainly that if the holomorphy conjecture is true for $f(x_1, \dots, x_{n-1})$, then it is true for $f(x_1, \dots, x_{n-1}) + x_n^k$ with $k \geq 3$, and we give some applications.

INTRODUCTION

0.1. Let K be a finite extension of the field $\mathbb{Q}_p$ of p-adic numbers, R the valuation ring of K, P the maximal ideal of R, and $\bar{K} = R/P$ the residue field with cardinality q. For $z \in K$ we denote by ord $z \in \mathbb{Z} \cup \{+\infty\}$ its valuation, $|z| = q^{-\operatorname{ord} z}$ its absolute value, and $ac(z) = z\pi^{-\operatorname{ord} z}$ its angular component, where π is a fixed uniformizing parameter for R.

Let $f(x) \in K[x]$, $x = (x_1, \dots, x_n)$, be a nonconstant polynomial over K, and $\chi : R^\times \to \mathbb{C}^\times$ a character of $R^\times$, the group of units of R. (We formally put $\chi(0) = 0$.) To these data one associates *Igusa's local zeta function*, which is the meromorphic continuation to $\mathbb{C}$ of

$$Z(s, \chi, f) = \int_{R^n} \chi(\operatorname{ac} f(x))|f(x)|^s |dx|$$

for $\operatorname{Re} s > 0$, where $|dx|$ denotes the Haar measure on K^n, normalized such that R^n has measure 1. Igusa [I1] showed that it is a rational function of q^{-s}. For more information and recent work on Igusa's local zeta function, see for example [D], [DM], [I], [KSZ], [L], and [V]; [D2] presents an overview of the subject.

0.2. The following conjecture [D2, Conjecture 4.4.2] is based on a formula for $Z(s, \chi, f)$ in terms of an embedded resolution of $f^{-1}\{0\}$. See [V1, 0.2–0.3]

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for a short motivation. By the order of an eigenvalue of monodromy we mean its order as root of unity.

Holomorphy Conjecture. *If $f(x)$ is defined over a number field $F \subset \mathbb{C}$, then for almost all completions K of F (i.e. for all except a finite number) we have the following: if the order of χ does not divide the order of any eigenvalue of the (complex) local monodromy of f at any complex point of $f^{-1}\{0\}$, then $Z(s, \chi, f)$ is holomorphic on $\mathbb{C}$.*

0.3. One easily shows that the conjecture is true for $n = 1$ and also when $\{f = 0\}$ is nonsingular (in these cases even for *all* completions K). Denef verified it for the relative invariants f of a few prehomogeneous vector spaces. In [V1] Veys proved the conjecture in general for curves, i.e. for $n = 2$ and any $f \in F[x_1, x_2]$. Also [V3, Theorem 5.1] yields some confirmation in higher dimensions.

0.4. The following statements are the main results of this paper.

Theorem. *If the holomorphy conjecture is true for $f(x_1, \dots, x_{n-1})$, then it is true for $f(x_1, \dots, x_{n-1}) + x_n^k$, where $k \geq 3$.*

Corollary. *The holomorphy conjecture is true for $f(x, y) + z^k$, where $f(x, y)$ is a polynomial in two variables and $k \geq 3$.*

Corollary. *The holomorphy conjecture is true for diagonal forms, i.e. for $f(x) = \sum_{i=1}^{n} a_i x_i^{m_i}$ where all a_i belong to some number field and all $m_i \geq 1$.*

0.5. To prove this we need an elementary number theoretical lemma in order to use the Sebastiani–Thom result which relates the monodromy of $f(x) + g(y)$ to the monodromies of $f(x)$ and $g(y)$; see §1. In §2 we extend a relation between the poles of $Z(s, \chi, f(x) + g(y))$ and the poles of $Z(s, \chi_1, f(x))$ and $Z(s, \chi_2, g(y))$ for $\chi = \chi_1 \cdot \chi_2$, which was known under certain restrictions. Then in §3 we prove the theorem and its corollaries and give some generalizations.

1. Eigenvalues of monodromy of $f(x_1, \dots, x_{n-1}) + x_n^k$

1.1. We first fix some terminology. Let $f \in \mathbb{C}[x_1, \dots, x_n]$. For $P \in f^{-1}\{0\}$ an *eigenvalue of monodromy of f at P* is any eigenvalue of the monodromy operator on any (complex) cohomology group of the local Milnor fiber of f around P. (See [M] for the concept of monodromy.) An *eigenvalue of monodromy of f* is an eigenvalue of monodromy of f at some $P \in f^{-1}\{0\}$. It is well known that all such eigenvalues are roots of unity [SGA7, Exposé I], so we call the *order* of an eigenvalue of monodromy of f its order as root of unity.

1.2. **Theorem** [ST], [S]. *Let $f \in \mathbb{C}[x_1, \dots, x_n]$ and $g \in \mathbb{C}[y_1, \dots, y_m]$. Suppose that $f(0) = g(0) = 0$, and denote by F_f, F_g and F_{f+g} the Milnor fibers of respectively f, g and $f + g$ around 0. Then for all $r \geq 0$ there is an isomorphism*

$$\tilde{H}^{r+1}(F_{f+g}) \cong \oplus_{i+j=r}(\tilde{H}^i(F_f) \otimes \tilde{H}^j(F_g))$$

which is moreover compatible with the corresponding monodromy operators. (Here $\tilde{H}^*$ denotes reduced complex cohomology.)

So in particular if α and β are eigenvalues of monodromy on $\tilde{H}^i(F_f)$ and $\tilde{H}^j(F_g)$, respectively, then $\alpha \cdot \beta$ is an eigenvalue of monodromy on $\tilde{H}^{i+j+1}(F_{f+g})$.

Remark. Remember that $H^0(F_f)$ is the direct sum of $\tilde{H}^0(F_f)$ and the (one-dimensional) eigenspace of the eigenvalue 1 of the monodromy operator on $H^0(F_f)$. Further we will call this 1 the trivial eigenvalue of f (around 0).

1.3. Now to study the holomorphy conjecture we are interested in the *orders* of the eigenvalues of monodromy. In general those orders for $f(x)+g(y)$ are quite unpredictable in terms of the orders for $f(x)$ and $g(y)$. (If α and β are roots of unity with order respectively r and s, we only know that the order of $\alpha \cdot \beta$ divides $\operatorname{lcm}(r, s)$.) We have the following result.

1.4. **Proposition.** *Let $f \in \mathbb{C}[x_1, \ldots, x_{n-1}]$ and $k \in \mathbb{N}$, $k \geq 3$. If there exists a non-trivial eigenvalue of monodromy of f with order r, then there exists an eigenvalue of monodromy of $f + x_n^k$ with order $\operatorname{lcm}(r, k)$.*

Proof. This is an immediate consequence of the number theoretical Lemma 1.5 below. If some eigenvalue of monodromy of f has order r, then $e^{2\pi i \cdot \frac{1}{r}}$ is an eigenvalue of monodromy of f (since all primitive rth roots of unity are simultaneously eigenvalues or not). Also the $e^{2\pi i \cdot \frac{\ell}{k}}$, $\ell \not\equiv 0 \mod k$, are precisely the eigenvalues of monodromy on $\tilde{H}^0(F_{x_n^k})$. So by Theorem 1.2 some eigenvalue of monodromy of $f + x_n^k$ has the desired order. □

1.5. **Lemma.** *Let $r, k \in \mathbb{N} \setminus \{0\}$ and $k \geq 3$. Then there exists $\ell \in \mathbb{N}$ such that*

(1) $\frac{1}{r} + \frac{\ell}{k}$ *has order* $\operatorname{lcm}(r, k)$ *in* $\mathbb{Q}/\mathbb{Z}$, *and*
(2) $\ell \not\equiv 0 \mod k$.

Remark. The order of $w \in \mathbb{Q} \setminus \{0\}$ in $\mathbb{Q}/\mathbb{Z}$ is precisely the order of $e^{2\pi i w}$ as root of unity, and is also the number $b \in \mathbb{N}$ such that $w = \frac{a}{b}$ with $a \in \mathbb{Z}$, $b \in \mathbb{N}$ and $\gcd(a, b) = 1$.

Proof. Denote $d = \gcd(r, k)$, $r = ad$, $k = bd$. So $\gcd(a, b) = 1$ and

$$\frac{1}{r} + \frac{\ell}{k} = \frac{b + \ell a}{abd}.$$

We first suppose that $b \geq 2$. Choose $x \equiv b \mod a$ such that x is prime and big enough, say $x > abd$. (Use Dirichlet's theorem on arithmetic progressions.) Then the ℓ for which $x = b + \ell a$ satisfies (1) and (2). The remaining possibility is $b = 1$ (and so $d \geq 3$). We must find some $\ell \in \mathbb{N}$ for which

(1′) $\frac{1}{ad} + \frac{\ell}{d} = \frac{1+a\ell}{ad}$ has order ad in $\mathbb{Q}/\mathbb{Z}$, and
(2′) $\ell \not\equiv 0 \mod d$.

For this we consider three subcases.

Case 1. For all prime numbers p we have $p|d \Rightarrow p|a$. We can choose $\ell = 1$.

Case 2. The prime 2 is the only prime number p such that $p|d$ and $p \nmid a$. We can choose ℓ to be any even number which is not divisible by d. (Here we need that $d \geq 3$!)

Case 3. There exists a prime number $p_0 \neq 2$ such that $p_0|d$ and $p_0 \nmid a$. Choose $x \in \mathbb{N}$ such that

$$x \equiv 1 \mod a, \quad x \equiv 2 \mod p_0,$$

and moreover such that x is prime and big enough, say $x > ad$. (Use the Chinese remainder theorem and Dirichlet's theorem.) The ℓ for which $x = 1 + \ell a$ satisfies $(1')$. And since $x \not\equiv 1 \bmod p_0$ by the second required congruence, this ℓ also satisfies $(2')$. □

1.6. *Remark.* (i) The requirement that $k \geq 3$ in the above proposition and lemma is used only in the proof of Case 2.

(ii) For $k = 2$ we have the following : $\frac{1}{r} + \frac{1}{2}$ has order $\mathrm{lcm}(r, 2) \Leftrightarrow r \not\equiv 2 \bmod 4$.

2. Poles of Igusa's local zeta function for $f(x) + g(y)$

2.1. In what follows the trivial character behaves slightly differently with respect to the other characters. We therefore introduce the notation $A(s, \chi) := q^{s+1} - 1$ if χ is the trivial character and $A(s, \chi) := 1$ otherwise. Also for $f \in K[x_1, \ldots, x_n]$ we denote the critical set of $f : \mathbb{A}^n_K \to \mathbb{A}^1_K$ by C_f.

2.2. We recall the following. Let $f(x) \in K[x]$, $x = (x_1, \ldots, x_n)$, and $g(y) \in K[y]$, $y = (y_1, \ldots, y_m)$. If $C_f \cap R^n \subset f^{-1}\{0\}$ and $C_g \cap R^m \subset g^{-1}\{0\}$, then the poles of $A(s, \chi)Z(s, \chi, f(x) + g(y))$ are of the form $s_1 + s_2$, with s_1 and s_2 a pole of $A(s, \chi_1)Z(s, \chi_1, f(x))$ and $A(s, \chi_2)Z(s, \chi_2, g(y))$, respectively, for some $\chi_1 \cdot \chi_2 = \chi$ [D2, (5.1)] (cf. the erratum to [D2] at the end of the present paper). We will prove an analogous statement without assumptions on the critical sets of f and g.

First we generalize the concept of Igusa's local zeta function, including Schwartz-Bruhat functions, and we introduce certain exponential sums.

2.3. **Definition.** Let $\Phi : K^n \to \mathbb{C}$ be a Schwarz-Bruhat function, i.e. a locally constant function with compact support.

(i) Let f and χ be as in (0.1). To f, χ and Φ is associated Igusa's local zeta function

$$Z_\Phi(s, \chi, f) = \int_{K^n} \Phi(x)\chi(\mathrm{ac}\, f(x))|f(x)|^s |dx|.$$

(So when Φ is the characteristic function of R^n we get definition 0.1.)

(ii) Let ψ be the standard additive character on K; thus $\psi(z) = e^{2\pi i \,\mathrm{Tr}_{K/\mathbb{Q}_p}(z)}$ for $z \in K$, where Tr denotes the trace. To f and Φ is associated

$$E_\Phi(z, f) = \int_{K^n} \Phi(x)\psi(z \cdot f(x))|dx|$$

for $z \in K$. This function is locally constant and bounded on K. When Φ is the characteristic function of R^n we put $E = E_\Phi$.

2.4. **Proposition** [I1, I2] (see also [D2, Corollary 1.4.5]). *Let $f \in K[x_1, \ldots, x_n]$ and let $\Phi : K^n \to \mathbb{C}$ be a Schwarz-Bruhat function. Suppose that $C_f \cap \operatorname{Supp} \Phi \subset f^{-1}\{0\}$. Then for $|z|$ big enough $E_\Phi(z, f)$ is a finite $\mathbb{C}$-linear combination of functions of the form*

$$\chi(ac(z))|z|^\lambda(\log_q |z|)^\beta$$

with coefficients independent of z. Here χ is a character of $R^\times$, $\lambda \in \mathbb{C}$ is a pole of $A(s, \chi)Z_\Phi(s, \chi, f)$, and $\beta \in \mathbb{N}$ satisfying $\beta \leq$ (multiplicity of pole λ) -1. Moreover all poles λ appear effectively in this linear combination.

2.5 *Remark.* The statement in 2.2 follows from the proposition above using the obvious fact that $E(z, f(x)+g(y)) = E(z, f(x)) \cdot E(z, g(y))$.

2.6. Let $f \in K[x]$, $x = (x_1, \dots, x_n)$. In order to make reductions mod P^ℓ we choose k such that $\pi^k f \in R[x]$ and such that $\pi^k c_i \in R$ for all critical values c_i of f in $f(K^n)$.

We then choose ℓ big enough such that all $\pi^k c_i$ mod P^ℓ are mutually different and denote by Φ_c the characteristic function of $\{x \in R^n | \pi^k f(x) \equiv \pi^k c \bmod P^\ell\}$ for any $c \in R[\frac{1}{\pi^k}]$. Then

$$(*) \qquad C_{f-c_i} \cap \operatorname{Supp} \Phi_{c_i} \subset (f - c_i)^{-1}\{0\}$$

for any critical value c_i of f.

2.7. **Proposition.** *Let $f \in K[x_1, \dots, x_n]$. For $|z|$ big enough $E(z, f)$ is a finite $\mathbb{C}$-linear combination of the form*

$$\chi(\operatorname{ac}(z))|z|^\lambda(\log_q(z))^\beta e^{2\pi i \operatorname{Tr}(c\cdot z)}$$

with coefficients independent of z. Here c is a critical value of f, χ is a character of $R^\times$, λ is a pole of $A(s, \chi)Z(s, \chi, f-c)$, and $\beta \in \mathbb{N}$ satisfying $\beta \le$ (multiplicity of pole λ) -1. Moreover all poles λ appear effectively in this linear combination.

Proof. We choose a set of representatives $\mathscr{R}'$ of R mod P^ℓ such that $\mathscr{R} = \{\frac{c'}{\pi^k} | c' \in \mathscr{R}'\}$ contains all critical values of f. Then

$$\begin{aligned} E(z, f) &= \sum_{c\in\mathscr{R}} E_{\Phi_c}(z, f) \\ &= \sum_{c\in\mathscr{R}} \int_{K^n} \Phi_c(x)\psi(z\cdot(f-c)(x) + z\cdot c)|dx| \\ &= \sum_{c\in\mathscr{R}} \int_{K^n} \Phi_c(x)\psi(z\cdot(f-c)(x))e^{2\pi i \operatorname{Tr}(z\cdot c)}|dx| \\ &= \sum_{c\in\mathscr{R}} e^{2\pi i \operatorname{Tr}(z\cdot c)} E_{\Phi_c}(z, f-c). \end{aligned}$$

It is known that when c is not a critical value of f, then $E_{\Phi_c}(z, f-c) = 0$ for $|z| >> 0$; in fact, this is a particular case of Proposition 2.4.

Then $(*)$ and again Proposition 2.4 yield the stated result, but with λ a pole of $A(s, \chi)Z_{\Phi_c}(s, \chi, f-c)$. Now $Z_{\Phi_c}(s, \chi, f-c)$ and $Z(s, \chi, f-c)$ differ only by a constant factor and have thus the same poles. □

2.8. **Corollary.** *Let $f(x) \in K[x]$, $x = (x_1, \dots, x_n)$, and $g(y) \in K[y]$, $y = (y_1, \dots, y_m)$. All poles of $A(s, \chi)Z(s, \chi, f(x)+g(y))$ are of the form $s_1 + s_2$, with s_1 and s_2 a pole of respectively $A(s, \chi_1)Z(s, \chi_1, f(x) - c)$ and $A(s, \chi_2)Z(s, \chi_2, g(y) + c)$ for some $\chi_1 \cdot \chi_2 = \chi$ and $c \in K$. (In that case c is a critical value of f and $-c$ is a critical value of g.)*

Proof. We apply Proposition 2.7 to f, g, and $f+g$ and identify then the expansions of $E(z, f(x)+g(y))$ and $E(z, f(x)) \cdot E(z, g(y))$. The fact that all poles must appear yields that a pole of $A(s, \chi)Z(s, \chi, f(x)+g(y))$ must be the sum of a pole of $A(s, \chi_1)Z(s, \chi_1, f(x) - c_1)$ and a pole of $A(s, \chi_2)\cdot Z(s, \chi_2, g(y) - c_2)$ with $\chi_1 \cdot \chi_2 = \chi$ and $c_1 + c_2 = 0$. □

3. On the holomorphy conjecture for $f(x_1, \dots, x_{n-1}) + x_n^k$

3.1. **Theorem.** *Let $f \in F[x_1, \dots, x_{n-1}]$ for some number field F. If the holomorphy conjecture is true for $f(x_1, \dots, x_{n-1})$, then it is true for $f(x_1, \dots, x_{n-1}) + x_n^k$, where $k \geq 3$.*

Proof. We take a character χ of order d such that d does not divide the order of any eigenvalue of monodromy of $f + x_n^k$; then we have to prove that $Z(s, \chi, f + x_n^k)$ is holomorphic on $\mathbb{C}$ (for almost all completions K of F). We may suppose that $d \geq 2$ and that 0 is a critical value of f; otherwise the statement is trivial.

For $d_1, d_2 \in \mathbb{N} \setminus \{0\}$ Proposition 1.4 implies that, if d_1 divides the order of an eigenvalue of monodromy of f and d_2 divides the order of an eigenvalue of monodromy of x_n^k, then $\operatorname{lcm}(d_1, d_2)$ divides the order of an eigenvalue of monodromy of $f + x_n^k$. So for any pair (d_1, d_2) such that $d \mid \operatorname{lcm}(d_1, d_2)$ we have that

(∗) d_1 does not divide the order of any eigenvalue of monodromy of f, or d_2 does not divide the order of any eigenvalue of monodromy of x_n^k.

Now remember that a decomposition $\chi = \chi_1 \cdot \chi_2$ in characters χ_1 and χ_2 satisfies $d \mid \operatorname{lcm}(\text{order } \chi_1, \text{order } \chi_2)$. Therefore (∗) and the hypothesis on f imply the following:

(∗∗) For any pair of characters (χ_1, χ_2) such that $\chi = \chi_1 \cdot \chi_2$ we have that $Z(s, \chi_1, f)$ is holomorphic (for almost all completions K of F) or that $Z(s, \chi_2, x_n^k)$ is holomorphic.

Then Corollary 2.8 and (∗∗) yield that $Z(s, \chi, f + x_n^k)$ must be holomorphic, since we have trivially that also $A(s, \chi_2) Z(s, \chi_2, x_n^k + c)$ is holomorphic for any $c \neq 0$. □

3.2. *Remark.* In fact, we proved above the following stronger statement: if the holomorphy conjecture is true for $f(x_1, \dots, x_{n-1})$ *over K*, then it is true for $f(x_1, \dots, x_{n-1}) + x_n^k$ *over K*, where $k \geq 3$.

So if the conjecture would be true for f over any completion K, it would also be true for $f + x_n^k$ over any completion K (where $k \geq 3$). In this context it is important to notice that up to now no f with a 'failing' K is known; the conjecture might eventually be true for all K.

3.3. *Remark.* Motivated by Remark 1.6 we call an eigenvalue of monodromy of f 'bad' if it has order $r \equiv 2 \bmod 4$ and if there exists no other eigenvalue of f with order $r' \not\equiv 2 \bmod 4$ such that $r \mid r'$. If f has no 'bad' eigenvalues of monodromy, then Theorem 3.1 is also true for $k = 2$.

3.4. **Corollary.** *Let $f(x, y) \in F[x, y]$ for some number field F. The holomorphy conjecture is true for $f(x, y) + z^k$, where $k \geq 3$.*

Proof. Immediate by Theorem 3.1 and [V1, Theorem 3.1]. □

3.5. **Corollary.** *The holomorphy conjecture is true for diagonal forms, i.e. for $f = \sum_{i=1}^n a_i x_i^{m_i}$ where all a_i belong to some number field and all $m_i \geq 1$.*

Proof. This follows from Theorem 3.1 and the fact that the holomorphy conjecture is true for $f = \sum_{i=1}^{r} a_i x_i^2$, which is for example implied by the formula of [D1, Theorem 2.2] for $Z(s, \chi, f)$ and by [D1, Theorem 1.1]. □

3.6. In view of generalizing Theorem 3.1 to more general functions instead of x_n^k we introduce the following concept, which is inspired by the essential properties of x_n^k, needed in the proof of the theorem. Let $g(y) \in \mathbb{C}[y]$, $y = (y_1, \dots, y_m)$. We say that g has *'good' monodromy* when the following conditions are satisfied:

(i) the function $g(y)$ admits only 0 as critical value;
(ii) if $e^{2\pi i \cdot \frac{1}{k}}$ is an eigenvalue of monodromy of g, then $e^{2\pi i \cdot \frac{\ell}{k}}$ is an eigenvalue of monodromy of g for all $\ell = 1, \dots, k-1$; and
(iii) if -1 is an eigenvalue of monodromy of g, then there exists an eigenvalue of monodromy of g with *even* order at least 4.

3.7. **Proposition.** *Let $f(x) \in F[x]$, $x = (x_1, \dots, x_n)$, and $g(y) \in F[y]$, $y = (y_1, \dots, y_m)$, for some number field F. Suppose that $g(y)$ has 'good' monodromy as in* 3.6. *If the holomorphy conjecture is true for $f(x)$, then it is true for $f(x) + g(y)$.*

3.8. This condition on $g(y)$ is of course quite restrictive; we have however the following nontrivial examples:

(i) $g(y)$ homogeneous of degree $k \geq 3$ such that 0 is its only critical point (or equivalently such that $\{g = 0\}$ is nonsingular in $\mathbb{P}^{m-1}$).

(ii) $g(y)$ such that 0 is its only critical value and 1 is its only eigenvalue of monodromy; for instance, $g = \sum_{i=1}^{m} y_i^2$ with m even, and $g = \prod_{i=1}^{m} y_i$.

References

[D1] J. Denef, *Local zeta functions and Euler characteristics*, Duke Math. J. **63** (1991), 713–721.

[D2] ——, *Report on Igusa's local zeta function*, Sém. Bourbaki 741, Astérisque **201/202/203** (1991), 359–386.

[D3] ——, *Degree of local zeta functions and monodromy*, Compositio Math. **89**, (1994), 207–216.

[DM] J. Denef and D. Meuser, *A functional equation of Igusa's local zeta function*, Amer. J. Math. **113** (1991), 1135–1152.

[I1] J. Igusa, *Complex powers and asymptotic expansions* I, J. Reine Angew. Math. **268/269** (1974), 110–130; II, **278/279** (1975), 307–321.

[I2] ——, *Lectures on forms of higher degree*, Tata Inst. Fund. Res., Bombay, 1978.

[I3] ——, *Some aspects of the arithmetic theory of polynomials*, Discrete Groups in Geometry and Analysis, Prog. Math., vol. 67, Birkhaüser, Boston, MA, 1987, pp. 20–47.

[I4] ——, *B-functions and p-adic integrals*, Algebraic Analysis, Academic Press, New York, 1988, pp. 231–241.

[KSZ] T. Kimura, F. Sato, and X. Zhu, *On the poles of p-adic complex powers and the b-functions of prehomogeneous vector spaces*, Amer. J. Math. **122** (1990), 423–437.

[L1] F. Loeser, *Fonctions d'Igusa p-adiques et polynômes de Bernstein*, Amer. J. Math. **110** (1988), 1–22.

[L2] ——, *Fonctions d'Igusa p-adiques, polynômes de Bernstein, et polyèdres de Newton*, J. Reine Angew. Math. **412** (1990), 75–96.

[M] J. Milnor, *Singular points of complex hypersurfaces*, Princeton Univ. Press, Princeton, NJ, 1968.

[S] K. Sakamoto, *Milnor fiberings and their characteristic maps*, Proc. Internat. Conf. on Manifolds and Related Topics in Topology (A. Hattori, ed.), Univ. of Tokyo Press, Tokyo, 1973, pp. 145–150.

[ST] M. Sebastiani and R. Thom, *Un résultat sur la monodromie*, Invent. Math. **13** (1971), 90–96.

[SGA7] A. Grothendieck, P. Deligne, and N. Katz, *Groupes de monodromie en géométrie algébrique* I, Lecture Notes in Math., vol. 288, Springer, New York, 1972.

[V1] W. Veys, *Holomorphy of local zeta functions for curves*, Math. Ann. **295** (1993), 635–641.

[V2] ——, *Poles of Igusa's local zeta function and monodromy*, Bull. Soc. Math. France **121** (1993), 545–598.

[V3] ——, *On Euler characteristics associated to exceptional divisors*, Trans. Amer. Math. Soc. (to appear).

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