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THE ADDITIVITY OF POROSITY IDEALS

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Abstract. We show that several σ-ideals related to porous sets have additivity ω_1 and cofinality 2^ω. This answers a question addressed by Miroslav Repický.

1. Introduction

This work is about the set-theoretic side of certain "thin" sets of reals, the *porous sets*, which play a role in real analysis (see [Za] for a survey).

Let $\mathbb{I} = [0,1]$ be the unit interval. Given a set of reals $A \subseteq \mathbb{I}$, the *porosity* and the *symmetric porosity* of A at a real $r \in \mathbb{I}$ are defined by

$$p(A,r) = \lim_{\varepsilon \to 0^+} \sup \frac{\lambda(A,(r-\varepsilon,r+\varepsilon))}{\varepsilon}$$

and

$$s(A,r) = \lim_{\varepsilon \to 0^+} \sup \frac{\lambda^*(A,(r-\varepsilon,r+\varepsilon))}{\varepsilon},$$

respectively, where $\lambda(A,I)$ denotes the maximal length of an open subinterval of the interval I which is disjoint from A; similarly $\lambda^*(A,(c,d))$ stands for the maximal $\delta \geq 0$ such that $(c,c+\delta) \cup (d-\delta,d)$ is disjoint from A. For $a \in A$ we have $s(A,a) \leq p(A,a) \leq 1$. A is *porous* (*strongly porous*, respectively) iff $p(A,a) > 0$ ($p(A,a) = 1$, resp.) for every $a \in A$; similarly, A is *symmetrically porous* (*strongly symmetrically porous*, respectively) iff $s(A,a) > 0$ ($s(A,a) = 1$, resp.) for every $a \in A$. Next, we let

$\mathcal{P}$:= the σ-ideal generated by the strongly porous sets;
$\mathcal{P}^*$:= the σ-ideal generated by the porous sets;
$\mathcal{S}$:= the σ-ideal generated by the strongly symmetrically porous sets;
$\mathcal{S}^*$:= the σ-ideal generated by the symmetrically porous sets.

The elements of $\mathcal{P}$ ($\mathcal{P}^*, \mathcal{S}, \mathcal{S}^*$, respectively) are called *σ-strongly porous sets* (*σ-porous sets*, etc., respectively).

Given a σ-ideal $\mathcal{I}$ on the reals $\mathbb{I}$ we define the following four cardinal invariants:
$add(\mathcal{I})$:= the least κ such that $\exists \mathcal{F} \in [\mathcal{I}]^\kappa$ ($\bigcup \mathcal{F} \notin \mathcal{I}$);
$cov(\mathcal{I})$:= the least κ such that $\exists \mathcal{F} \in [\mathcal{I}]^\kappa$ ($\bigcup \mathcal{F} = \mathbb{I}$);
$unif(\mathcal{I})$:= the least κ such that $[\mathbb{I}]^\kappa \setminus \mathcal{I} \neq \emptyset$;

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$cof(\mathcal{I}) :=$ the least κ such that $\exists \mathcal{F} \in [\mathcal{I}]^\kappa\ \forall A \in \mathcal{I}\ \exists B \in \mathcal{F}\ (A \subseteq B)$.
Here, $[S]^\kappa$ denotes the set of subsets of size κ of a set S (see [Je] for any set-theoretic notion left undefined in our work).

The goal of this note is to show (in section 2) $add(\mathcal{P}) = add(\mathcal{P}^*) = add(\mathcal{S}) = add(\mathcal{S}^*) = \omega_1$ and $cof(\mathcal{P}) = cof(\mathcal{P}^*) = cof(\mathcal{S}) = cof(\mathcal{S}^*) = 2^\omega$. In section 3 we make some remarks and state a question concerning the invariants $unif(\mathcal{I})$ and $cov(\mathcal{I})$, where $\mathcal{I}$ is again one of the four ideals defined above. For a survey concerning known results about these cardinals, as well as further references, we refer the reader to the survey article [Re].

2. Proof of the main result

We shall work in a space of the form $X := \prod_n g(n)$, where $g \in (\omega \setminus 2)^\omega$; i.e. $f \in X$ iff $f \in \omega^\omega$ and for all $n \in \omega$, $f(n) < g(n)$. Using the Cantor expansion, we can almost identify X and the unit interval $\mathbb{I}$ in a canonical way: reals $r \in \mathbb{I}$ correspond to reals $f \in X$ via the map $\phi\colon X \to \mathbb{I}$ defined by

$$r = \phi(f) = \sum_{n\in\omega} \frac{f(n)}{g(0)\cdot \cdots \cdot g(n)}.$$

However, we shall be more interested in a slightly different identification. Let $g \in \omega^\omega$ be strictly increasing and taking odd values such that there are strictly increasing sequences of natural numbers $\langle m_n; n \in \omega\rangle$ and $\langle \ell_n; n \in \omega\rangle$ such that
(i) $\ell_n^{m_n}/g(n) \to 0$ for $n \to \infty$;
(ii) $\ell_{n-1}/\ell_n \to 0$ for $n \to \infty$.
Next, given $n \in \omega$, we let μ_n be the unique measure on $P(g(n))$ satisfying
(a) $\mu_n(g(n)) = 1$;
(b) $\mu_n(\{i\}) = \mu_n(\{j\})$ for $m_n \le i \le j \le g(n) - m_n - 1$;
(c) $\mu_n(\{i\}) = \mu_n(\{g(n) - i - 1\}) = \mu_n(\{i + 1\})/\ell_n$ for $i < m_n$.
We equip X with the product measure of the μ_n. This gives another almost-identification of X and $\mathbb{I}$: the idea is that the open set $[\sigma] := \{f \in X; \sigma \subseteq f\}$ corresponds to an interval in $\mathbb{I}$ of length $\prod_{n<\ell} \mu_n(\{\sigma(n)\})$ for $\sigma \in \prod_{n<\ell} g(n)$. We call this almost-correspondence $\tilde{\phi}\colon X \to \mathbb{I}$, and say $A \subseteq X$ is porous (or strongly porous, etc.) iff $\tilde{\phi}(A)$ is. Since $\tilde{\phi} \circ \tilde{\phi}^{-1}(B) = B$ and $\tilde{\phi}^{-1} \circ \tilde{\phi}(A) = A \cup C$ for some at most countable $C \subseteq X$, the σ-ideals of σ-porous (σ-strongly porous, etc.) sets can be identified.

Let $\langle A_\alpha; \alpha < 2^\omega\rangle$ be a sequence of almost-disjoint subsets of $\{10n; n \in \omega\}$. For $\alpha < 2^\omega$ we define $B_\alpha := \{f \in X; \forall n \in A_\alpha\ (f(n) = \frac{g(n)-1}{2})\}$. Each B_α is easily seen to be strongly symmetrically porous. We claim:

Theorem. *Whenever $C \subseteq X$ is σ-porous, then for all but countably many $\alpha \in 2^\omega, B_\alpha \not\subseteq C$.*

Corollary. *For $\mathcal{I} \in \{\mathcal{P}, \mathcal{P}^*, \mathcal{S}, \mathcal{S}^*\}$, $add(\mathcal{I}) = \omega_1$ and $cof(\mathcal{I}) = 2^\omega$.*

Proof of the Corollary from the Theorem. As $\mathcal{S} \subseteq \mathcal{S}^*, \mathcal{P} \subseteq \mathcal{P}^*$, $\{B_\alpha; \alpha \in \omega_1\}$ witnesses $add(\mathcal{I}) = \omega_1$ for any of the σ-ideals. Furthermore, any family $\mathcal{F}$ of sets from $\mathcal{I}$ of size $< 2^\omega$ cannot be cofinal, because some B_α will not be contained in any member of $\mathcal{F}$. $\square$

Proof of the Theorem. Let $C=\bigcup_{i\in\omega} C_i$, where each C_i is porous. Fix $\sigma\in\prod_{n<\ell} g(n)$ (for some $\ell\in\omega$), $m<[\frac{\ell}{2}]$ and $\Gamma\subseteq 2^\omega$ finite. We set

$$B(\sigma,m,\Gamma):=\Bigg\{f\in X;\sigma\subseteq f\wedge\forall n\geq\ell\ (m\leq f(n)\leq g(n)-m-1)$$
$$\wedge\forall\alpha\in\Gamma\ \forall n\in A_\alpha\ \left(n\geq\ell\to f(n)=\frac{g(n)-1}{2}\right)\Bigg\}$$

and

$$\widehat{B}(\sigma,m,\Gamma):=\Bigg\{\tau\supseteq\sigma;\forall n\ (\ell\leq n<lh(\tau)\to m\leq\tau(n)\leq g(n)-m-1)$$
$$\wedge\forall\alpha\in\Gamma\ \forall n\in A_\alpha\ \left(\ell\leq n<lh(\tau)\to\tau(n)=\frac{g(n)-1}{2}\right)\Bigg\}.$$

Given $\sigma\in\prod_{n<\ell} g(n)$ and $i,m\in\omega$ with $m+1<[\frac{\ell}{2}]$, we say Γ is (σ,m,i)-*funny* iff **either**

(i) there are uncountably many Δ_α ($\alpha<\omega_1$) which are pairwise disjoint so that for all $\alpha<\omega_1$

$$B(\sigma,m+1,\Gamma\cup\Delta_\alpha)\cap C_i=\emptyset;$$

or

(ii) for some $\tau\supseteq\sigma$ ($[\tau]\cap C_i=\emptyset\wedge\tau\in\widehat{B}(\sigma,m,\Gamma)$).

Main Observation. *Given σ,m,i, as above, there is $\Omega\subseteq 2^\omega$ countable so that whenever $\Gamma\subseteq 2^\omega\setminus\Omega$ is finite, then Γ is* (σ,m,i)-funny.

Proof of Main Observation. Suppose not. Then we can easily construct a sequence $\langle\Delta_\alpha;\alpha<\omega_1\rangle$ of pairwise disjoint finite sets, none of which is (σ,m,i)-funny. By clause (I) of the definition of funniness applied to Δ_0 there is $\alpha<\omega_1$ so that $B(\sigma,m+1,\Delta_0\cup\Delta_\alpha)\cap C_i\neq\emptyset$. Choose f from the latter set. Let $p:=p(\tilde{\phi}(C_i),\tilde{\phi}(f))>0$. Find ε so small that ε and $n=n_\varepsilon$, which is unique with $\mu([f\restriction n])\geq\varepsilon$ and $\mu([f\restriction n+1])<\varepsilon$, satisfy:

(A) $\lambda:=\lambda(\tilde{\phi}(C_i),(\tilde{\phi}(f)-\varepsilon,\tilde{\phi}(f)+\varepsilon))>\frac{p}{2}\cdot\varepsilon$;
(B) $\ell_n^{m_n}/g(n)$ is small compared to p;
(C) ℓ_{n-1}/ℓ_n is small compared to p;
(D) $(\bigcup_{\beta\in\Delta_0} A_\beta)\cap(\bigcup_{\beta\in\Delta_\alpha} A_\beta)\subseteq n-5$.

Without loss $I:=(\tilde{\phi}(f)-\varepsilon,\tilde{\phi}(f)-\varepsilon+\lambda)$ is disjoint from $\tilde{\phi}(C_i)$. Clearly $\tilde{\phi}(f)-\varepsilon\notin\tilde{\phi}[f\restriction n+1]$. Also we either have

$$\tilde{\phi}(f)-\varepsilon\in\tilde{\phi}[f\restriction n\hat{\ }\langle j\rangle]$$

for some $j<f(n)$ or

$$\tilde{\phi}(f)-\varepsilon\in\tilde{\phi}[f\restriction(n-1)\hat{\ }\langle j\rangle]$$

for some $j<f(n-1)$ or

$$\tilde{\phi}(f)-\varepsilon\in\tilde{\phi}[f\restriction(n-2)\hat{\ }\langle f(n-2)-1\rangle].$$

[This is because $\mu([f\restriction n-1])/(g(n-1)-2\cdot m_{n-1})>\mu([f\restriction n])\geq\varepsilon$, which implies $\mu([f\restriction(n-2)\hat{\ }\langle f(n-2)-1\rangle])>\varepsilon\cdot(g(n-1)-2\cdot m_{n-1})/\ell_{n-2}>\varepsilon$.] The core of the proof is to construct $\tau\supseteq f\restriction n-2$ so that $lh(\tau)\leq n+2,\forall j\ (n-2\leq j<lh(\tau)\to m\leq\tau(j)\leq g(j)-m-1)$ and $[\tau]\cap C_i=\emptyset$ (♣).

It is easy to see that one of the following three cases must hold.

Case 1. *For $\sigma_j = f \restriction j-1$, where $j \in \{n, n+1\}$, we have $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_j \hat{}\langle k\rangle]$, where $m \le k < f(j-1)$ ($k < f(j-1)-1$ in case $j = n$).*

Note that for any such k we have $\mu([\sigma_j \hat{}\langle k\rangle]) \le \mu([\sigma_j \hat{}\langle k+1\rangle]) < \varepsilon \cdot \ell_{j-1}^{m_{j-1}}$. [In case $j = n$, this follows because the assumption $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_n \hat{}\langle k\rangle]$ implies $\mu([\sigma_n \hat{}\langle f(n-1)-1\rangle]) < \varepsilon$; in case $j = n+1$, this is immediate from $\mu([f \restriction n+1]) < \varepsilon$.] Let ℓ be such that $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_j \hat{}\langle k\rangle \hat{}\langle \ell\rangle]$. In case $\ell < g(j) - m - 1$ find $0 < \ell' \le m$ such that $m \le \ell + \ell' \le g(j) - m - 1$, and we have

$$\sum_{\tilde\ell \le \ell'} \mu([\sigma_j \hat{}\langle k\rangle \hat{}\langle \ell + \tilde\ell\rangle]) < \varepsilon \cdot \frac{(m+1) \cdot \ell_{j-1}^{m_{j-1}}}{g(j) - 2 \cdot m_j} < \varepsilon \cdot \frac{p}{2} < \lambda.$$

This entails $\tilde\phi[\sigma_j \hat{}\langle k\rangle \hat{}\langle \ell + \ell'\rangle] \subseteq I$, and so $[\sigma_j \hat{}\langle k\rangle \hat{}\langle \ell + \ell'\rangle] \cap C_i = \emptyset$. Hence $\tau = \sigma_j \hat{}\langle k\rangle \hat{}\langle \ell + \ell'\rangle$ will work. In case $\ell \ge g(j) - m - 1$ we compute

$$\begin{aligned} &\sum_{g(j) > \tilde\ell \ge \ell} \mu([\sigma_j \hat{}\langle k\rangle \hat{}\langle \tilde\ell\rangle]) + \sum_{\tilde\ell \le m} \mu([\sigma_j \hat{}\langle k+1\rangle \hat{}\langle \tilde\ell\rangle]) \\ &\qquad < \varepsilon \cdot \frac{2 \cdot (m+1) \cdot \ell_{j-1}^{m_{j-1}}}{g(j) - 2 \cdot m_j} < \varepsilon \cdot \frac{p}{2} < \lambda. \end{aligned}$$

Thus $\tau = \sigma_j \hat{}\langle k+1\rangle \hat{}\langle m\rangle$ is as required.

Case 2. *For $\sigma_j = f \restriction j-2$, where $j \in \{n, n+1\}$, we have:*

either $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_j \hat{}\langle f(j-2)-1\rangle \hat{}\langle g(j-1)-m-1\rangle \hat{}\langle g(j)-k-1\rangle]$, where $k \le m$,

or $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_j \hat{}\langle f(j-2)-1\rangle \hat{}\langle g(j-1)-k-1\rangle]$, where $k < m$,

or $\tilde\phi(f) - \varepsilon \in \tilde\phi[f \restriction (j-1) \hat{}\langle k\rangle]$, where $k < m$.

In this case we necessarily have $\mu([f \restriction (j-1) \hat{}\langle m\rangle]) < \varepsilon$, and hence

$$\mu([\sigma_j \hat{}\langle f(j-2)-1\rangle \hat{}\langle g(j-1)-m-1\rangle]) < \varepsilon \cdot \ell_{j-2}.$$

Thus (putting $\tilde\sigma_j = \sigma_j \hat{}\langle f(j-2)-1\rangle$ and $\hat\sigma_j = f \restriction j-1$) we have

$$\begin{aligned} &\sum_{k \le m} \mu([\tilde\sigma_j \hat{}\langle g(j-1)-m-1\rangle \hat{}\langle g(j)-k-1\rangle]) \\ &\qquad + \sum_{k<m} \mu([\tilde\sigma_j \hat{}\langle g(j-1)-k-1\rangle]) \\ &\qquad + \sum_{k<m} \mu([\hat\sigma_j \hat{}\langle k\rangle]) + \sum_{k \le m} \mu([\hat\sigma_j \hat{}\langle m\rangle \hat{}\langle k\rangle]) \\ &\quad < \varepsilon \cdot 2 \cdot \left(\frac{(m+1) \cdot l_{j-2}}{g(j) - 2 \cdot m_j} + \sum_{k<m} \frac{\ell_{j-2}}{\ell_{j-1}^{k+1}} \right) < \varepsilon \cdot \frac{p}{2} < \lambda. \end{aligned}$$

Hence $\tau = f \restriction (j-1) \hat{}\langle m\rangle \hat{}\langle m\rangle$ is as required.

Case 3. *For $\sigma_j = f \restriction (j-2) \hat{}\langle f(j-2)-1\rangle$, where $j \in \{n, n+1\}$, we have:*

either $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_j \hat{}\langle k\rangle]$, where $k < g(j-1) - m - 1$,

or $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_j \hat{}\langle g(j-1)-m-1\rangle \hat{}\langle k\rangle]$, where $k < g(j) - m - 1$.

As before we necessarily have $\mu([f \restriction (j-1) \hat{}\langle m\rangle]) < \varepsilon$, and hence $\mu([\sigma_j \hat{}\langle k\rangle]) < \varepsilon \cdot \ell_{j-2} \cdot \ell_{j-1}^{m_{j-1}}$ (for $k \le g(j-1) - m - 1$). If $k \ge m$ we can finish similarly to Case 1 [i.e., we let ℓ be such that $\tilde\phi(f) - \varepsilon \in \tilde\phi[\sigma_j \hat{}\langle k\rangle \hat{}\langle \ell\rangle]$ and split into the two subcases

$\ell < g(j) - m - 1$ and $\ell \geq g(j) - m - 1$]. Note that

$$\sum_{\ell < m} \mu([\sigma_j \hat{\ } \langle \ell \rangle]) + \sum_{\tilde{\ell} \leq m} \mu([\sigma_j \hat{\ } \langle m \rangle \hat{\ } \langle \tilde{\ell} \rangle])$$

$$< \varepsilon \cdot \left(\sum_{\ell < m} \frac{\ell_{j-2}}{\ell_{j-1}^{\ell+1}} + \frac{(m+1) \cdot \ell_{j-2}}{g(j) - 2 \cdot m_j} \right) < \varepsilon \cdot \frac{p}{2} < \lambda.$$

Thus if $k < m, \tau = \sigma_j \hat{\ } \langle m \rangle \hat{\ } \langle m \rangle$ will work (in fact this is similar to Case 2).

Thus we have found τ as required in (♣). By almost-disjointness and the choice of n we necessarily have either $\tau \in \widehat{B}(\sigma, m, \Delta_0)$ or $\tau \in \widehat{B}(\sigma, m, \Delta_\alpha)$. Thus either Δ_0 or Δ_α is funny, and we reach a contradiction. This proves the Main Observation. □

Conclusion. By the Main Observation, there is $\Omega \subseteq 2^\omega$ countable such that whenever $\Gamma \subseteq 2^\omega \backslash \Omega$ is finite, then Γ is (σ, m, i)-funny for all $\sigma \in \prod_{n<\ell} g(n)$ (all $\ell \in \omega$), $m < [\frac{\ell}{2}]$, and $i \in \omega$.

Fix $\alpha \in 2^\omega \backslash \Omega$. We construct recursively $\sigma_i \in \prod_{n<\ell(i)} g(n)$, where $\ell(i) \in \omega$, $m(i) < [\frac{\ell(i)}{2}]$, and $\Gamma_i \subseteq 2^\omega \backslash \Omega$ finite such that

(1) $\{\alpha\} = \Gamma_0, \Gamma_i \subseteq \Gamma_{i+1}$;
(2) $\ell(0) = 2$ and $\ell(i) < \ell(i+1)$;
(3) $m(0) = 0$ and $m(i) \leq m(i+1)$;
(4) $\sigma_{i+1} \in \widehat{B}(\sigma_i, m(i), \Gamma_i)$;
(5) $B(\sigma_{i+1}, m(i+1), \Gamma_{i+1}) \cap C_i = \emptyset$.

$i = 0$. Trivial.

$i \to i+1$. Γ_i is $(\sigma_i, m(i), i)$-funny. If it satisfies the first clause of the definition of *funny*, we find $\Gamma_i \subseteq \Gamma_{i+1} \subseteq 2^\omega \backslash \Omega$ such that $B(\sigma_i, m(i)+1, \Gamma_{i+1}) \cap C_i = \emptyset$. In this case we put $\ell(i+1) = \ell(i)+2$, we choose $\sigma_{i+1} \in \widehat{B}(\sigma_i, m(i)+1, \Gamma_{i+1})$ of length $\ell(i+1)$, and we let $m(i+1) = m(i)+1$. Then (1)–(5) are satisfied. If it satisfies the second clause, we find $\tau \supseteq \sigma_i$ such that $[\tau] \cap C_i = \emptyset$ and $\tau \in \widehat{B}(\sigma_i, m(i), \Gamma_i)$. In case $\tau \supset \sigma_i$ let $\sigma_{i+1} = \tau$ and $\ell(i+1) = lh(\tau)$; otherwise let $\ell(i+1) = \ell(i)+1$ and choose $\sigma_{i+1} \in \widehat{B}(\sigma_i, m(i), \Gamma_i)$ of length $\ell(i+1)$. We let $\Gamma_{i+1} = \Gamma_i$ and $m(i+1) = m(i)$. Again (1)–(5) are satisfied.

This concludes the construction. Let $f = \bigcup_{i \in \omega} \sigma_i$. It is easily seen that $f \in B_\alpha \backslash C$, thus proving the Theorem. □

3. Porosity and evasion

We were motivated to prove the above result by our discussion of evasion ideals [Br, section 4] which seem to be closely related to porosity ideals.

As in section 2, fix $g \in (\omega \backslash 2)^\omega$, and let $X := \prod_n g(n)$. Following Blass [Bl, section 4] (see also [Br, 4.1]), an *X-predictor* is a pair $\pi = (D_\pi, (\pi_n; n \in D_\pi))$ such that $D_\pi \subseteq \omega$ is infinite and for every $n \in D_\pi, \pi_n \colon \prod_{k<n} g(k) \to g(n)$; π *predicts* $f \in X$ iff $\forall^\infty n \in D_\pi$ $(\pi_n(f \restriction n) = f(n))$; otherwise f *evades* π. $\mathbf{e}_X := \min\{|\mathcal{F}|; \mathcal{F} \subseteq X \wedge \forall X\text{-predictors } \pi \, \exists f \in X \ (f \text{ evades } \pi)\}$ is the *evasion number.* Furthermore, let $\mathcal{I}_X := \{A \subseteq X;$ there is a countable set of X-predictors Π such that for all $f \in A$ there is $\pi \in \Pi$ predicting $f\}$ [Br, 4.5]. Making again a standard identification between X and $\mathbb{I}$ as at the beginning of section 2, we see that $\mathcal{I}_{2^\omega} \subseteq \mathcal{P}^*$ and $\mathcal{I}_X \subseteq \mathcal{P}$ for $X = \prod_n g(n)$, where g converges to infinity. [Moreover, the sets B_α which are crucial for the proof in section 2 are elements of $\mathcal{I}_X$.] Thus $\mathbf{e}_X \leq unif(\mathcal{I}_X) \leq unif(\mathcal{P})$ and $\mathbf{e}_{2^\omega} \leq unif(\mathcal{I}_{2^\omega}) \leq unif(\mathcal{P}^*)$ as well as $cov(\mathcal{P}) \leq cov(\mathcal{I}_X)$ and

$cov(\mathcal{P}^*) \leq cov(\mathcal{I}_{2^\omega})$. We believe it is an interesting line of research to investigate further the relationship between evasion and porosity. In particular, we would like to know whether some of these cardinals can be shown to be equal in ZFC (note that $\omega_1 = \mathbf{e}_X = unif(\mathcal{I}_X) < \mathbf{e}_{2^\omega} = unif(\mathcal{I}_{2^\omega}) = \omega_2$ holds in the Mathias real model [Br, 4.2]).

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