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EVERY ČECH-ANALYTIC BAIRE SEMITOPOLOGICAL GROUP IS A TOPOLOGICAL GROUP

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Abstract. Among other things, we prove the assertion given in the title. This solves a problem of Pfister.

1. Introduction

A semitopological (respectively, paratopological group) is a group endowed with a topology for which the product is separately (respectively, jointly) continuous. In 1957 R. Ellis [6] showed that every locally compact paratopological group is a topological group. This answered a question posed by A. D. Wallace in [19]. Moreover, in [20] W. Zelazko has shown that every completely metrizable paratopological group is a topological group. Later, in 1982, N. Brand [3] generalized both Ellis' and Zelazko's results by proving that every Čech-complete paratopological group is a topological group. A new and short proof of this result was given by H. Pfister [17] three years later. It had been well known for many years that every locally compact or completely metrizable semitopological group is a paratopological group ([7], [16] respectively). These results motivated Pfister in [17, Remarks] to ask whether every Čech-complete semitopological group is a paratopological group and, hence by Brand's result, a topological group.

In [2, Theorem 4.3] we show the following: Let G be a Čech-complete semitopological group. Then G is a topological group if and only if G is paracompact. (The 'only if' part is a result by L. G. Brown [4].) Then, to answer Pfister's question it suffices to prove that every Čech-complete semitopological group is paracompact. The purpose of the present paper is to give a complete answer to Pfister's problem by a different method and in a general form. We prove that every Čech-analytic Baire semitopological group is a topological group (Theorem 3.3). The class of Čech-analytic spaces was introduced by D. H. Fremlin in an unpublished note of 1980 (cf. [12]). This class is sufficiently large to include all completely metrizable or locally compact spaces, and more generally, all Čech-complete spaces.

Theorem 3.3 is stated in Section 3 as a corollary of a somewhat more general statement (Theorem 3.2). Theorem 3.2 and Theorem 3.1 are settled in terms of p-σ-fragmentability. In Section 2, the concept of p-σ-fragmentability is introduced and some auxiliary results are established.

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2. Definitions and preliminaries

All topological spaces considered in this paper are supposed to be completely regular. Let X be a topological space. We shall say that a subset Y of X is fragmented by a collection $\mathcal{U}$ of subsets of X if each nonvoid subset of Y has a nonvoid relatively open subset contained in some member of $\mathcal{U}$. The space X is σ-fragmented by a cover $\mathcal{U}$ of X (not necessarily related to the topology of X) if we can write $X = \bigcup_{n\in\mathbb{N}} X_n$ where, for each $n \in \mathbb{N}$, X_n is fragmented by $\mathcal{U}$.

Recall that a sequence of covers $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of X is complete (cf. [12, p. 278]) if, whenever $\mathcal{F}$ is a filter base on X such that each $\mathcal{U}_n$ has a member containing some member of $\mathcal{F}$, then $\bigcap\{\overline{F} : F \in \mathcal{F}\} \neq \varnothing$. We shall say that a sequence of covers $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of X is point-complete if, whenever $\mathcal{F}$ is a filter base on X such that each $\mathcal{U}_n$ has a member U_n containing some member of $\mathcal{F}$ such that $\bigcap_{n\in\mathbb{N}} U_n \neq \varnothing$, then $\bigcap\{\overline{F} : F \in \mathcal{F}\} \neq \varnothing$. It is clear that a complete sequence of covers of X is a point-complete sequence of covers of X. We now introduce a concept which plays a fundamental role in this paper.

Definition 2.1. The space X is called p-σ-fragmentable if X has a point-complete sequence of covers $(\mathcal{U}_n)_{n\in\mathbb{N}}$ such that:

(1) X is σ-fragmented by each $\mathcal{U}_n$,
(2) the collection $\{\overline{U} : U \in \mathcal{U}_{n+1}\}$ is a refinement of $\mathcal{U}_n$ for each $n \in \mathbb{N}$.

We shall say that a sequence of covers of X satisfying Definition 2.1 is associated to the p-σ-fragmentable space X.

Examples 2.2. (1) All metrizable and all Čech-complete spaces are p-σ-fragmentable. More generally all p-spaces are p-σ-fragmentable. Recall that a space X is a p-space ([10]) if X has a sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ of open covers such that if $x \in X$ and for each $n \in \mathbb{N}$ there is $U_n \in \mathcal{U}_n$ such that $x \in U_n$, then the set $K = \bigcap_{n\in\mathbb{N}} \overline{U}_n$ is compact and the sequence $(\bigcap_{i\leq n} \overline{U}_i)_{n\in\mathbb{N}}$ is an outer network for K. A family $\mathcal{N}$ of subsets of X is an outer network for K if for any open subset U such that $K \subset U$ there exists $N \in \mathcal{N}$ such that $K \subset N \subset U$. The concept of a p-space was introduced by A. V. Arhangel'skii [1] in a different but equivalent form.

(2) Let X be a space σ-fragmented by a lower semicontinuous metric (cf. [13]). If the metric topology is finer than the original one, then X is p-σ-fragmented. One can use Lemma 2.4 below to show that not all spaces fragmented by a metric are p-σ-fragmented; this answers a question asked by the referee. This is the case of the Sorgenfrey line (see the paragraph after Lemma 2.4).

(3) Čech-analytic spaces are p-σ-fragmentable. (Recall that a space X is Čech-analytic [12, Theorem 5.3] if X is the projection on some compactification X^* of X of the intersection of a closed set and a G_δ subset of $X^* \times \mathbb{N}^{\mathbb{N}}$.) In fact, Theorem 5.7 of [12] says that a Čech-analytic space X has a complete sequence of covers $(\mathcal{U}_n)_{n\in\mathbb{N}}$ satisfying (2) of 2.1; moreover, for each $n \in \mathbb{N}$ one can write $\mathcal{U}_n = \bigcup_{m\in\mathbb{N}} \mathcal{U}_{n,m}$, where for each $m \in \mathbb{N}$, $\mathcal{U}_{n,m}$ is an open cover of the subspace $X_m = \bigcup \mathcal{U}_{n,m}$ of X. Hence the space X is σ-fragmented by each $\mathcal{U}_n$.

Following [15] a point $x \in X$ is called a q-point if it has a sequence of neighborhoods $(U_n)_{n\in\mathbb{N}}$ such that if $x_n \in U_n$, then the sequence $(x_n)_{n\in\mathbb{N}}$ has a cluster point in X. The space X is called a q-space if every $x \in X$ is a q-point. We shall also need the following generalizations of continuity. Let X and Y be topological spaces, and let $f : X \to Y$. The mapping f is called quasicontinuous at $x \in X$ [14]

if for each neighborhood V of $f(x)$ in Y and for each neighborhood U of x in X there is a nonvoid open set $O \subset U$ such that $f(O) \subset V$. The mapping f is quasicontinuous if f is quasicontinuous at every point $x \in X$. The mapping $f : X \to Y$ is subcontinuous at the point $x \in X$ [9] if for each net (x_α) in X which converges to x, the net $(f(x_\alpha))$ has a cluster point in Y.

The main result of this section is Theorem 2.7. To establish this result we need the following lemmas.

Lemma 2.3. *Let X be a Baire space, Y a topological space and $f : X \to Y$ a quasicontinuous mapping. Suppose that Y is σ-fragmented by the cover $\mathcal{U}$. Then the set A of points $x \in X$ for which there exists $U \in \mathcal{U}$ and a neighborhood V such that $f(V) \subset \overline{U}$ is a dense open subset of X.*

Proof. It is clear that A is open. Put $Y = \bigcup_{n\in\mathbb{N}} Y_n$ where each Y_n is fragmented by $\mathcal{U}$. Let Ω be a nonvoid open subset of X and let us show that there is a nonvoid open subset $O \subset \Omega$ of X and $U \in \mathcal{U}$ such $f(O) \subset \overline{U}$. This will imply that A is dense in X. Since Ω is a Baire space and since $\Omega \subset \bigcup_{n\in\mathbb{N}} f^{-1}(Y_n)$, there are a nonvoid open set $O_1 \subset \Omega$ and $n \in \mathbb{N}$ such that $O_1 \subset \overline{f^{-1}(Y_n)}$. In particular $f(O_1) \cap Y_n \neq \varnothing$. As Y_n is fragmented by $\mathcal{U}$, there are an open set W of Y and $U \in \mathcal{U}$ such that $\varnothing \neq f(O_1) \cap Y_n \cap W \subset U$. Pick a nonvoid open subset O of X such that $O \subset O_1$ and $f(O) \subset W$; this is possible since f is quasicontinuous. Let us show that $f(O) \subset \overline{f(O_1) \cap Y_n \cap W}$; this implies that $f(O) \subset \overline{U}$. Let $y \in f(O)$ and V be a neighborhood of y in Y; pick $x \in O$ such that $y = f(x)$. The mapping f is quasicontinuous at x, hence there is a nonvoid open set $O_2 \subset O$ of X such that $f(O_2) \subset V \cap W$. It follows from $O_2 \subset O \subset \overline{f^{-1}(Y_n)}$ that $O_2 \cap f^{-1}(Y_n) \neq \varnothing$; hence

$$\varnothing \neq f(O_2) \cap Y_n \cap f(O) \subset V \cap f(O_1) \cap Y_n \cap W. \quad \square$$

Lemma 2.4. *Let $f : X \to Y$ be a quasicontinuous mapping where X is a Baire space and Y a p-σ-fragmentable space. Then the set of points of subcontinuity of f is a dense subset of X.*

Proof. Take a sequence of covers $(\mathcal{U}_n)_{n\in\mathbb{N}}$ associated to the p-σ-fragmentable space Y. For each $n \in \mathbb{N}$ let A_n be the set of points $x \in X$ for which there is a neighborhood V in X and a member $U \in \mathcal{U}_n$ such that $f(V) \subset \overline{U}$. Let $A = \bigcap_{n\in\mathbb{N}} A_n$; by Lemma 2.3 the set A is a dense G_δ subset of X. We show that f is subcontinuous at every $x \in A$. Let $x \in A$ and $(x_\alpha)_{\alpha\in\Lambda}$ be a net in X which converges to x. Let $\mathcal{F} = \{\{f(x_\alpha) : \beta \leq \alpha\} : \beta \in \Lambda\}$; we must verify that $\bigcap\{\overline{F} : F \in \mathcal{F}\} \neq \varnothing$. For each $n \in \mathbb{N}$ pick $U_n \in \mathcal{U}_{n+1}$ and $\beta_n \in \Lambda$ such that $f(x) \in \overline{U}_n$ and $\{f(x_\alpha) : \beta_n \leq \alpha\} \subset \overline{U_n}$; since, for every $n \in \mathbb{N}$, the collection $\{\overline{U} : U \in \mathcal{U}_{n+1}\}$ is a refinement of $\mathcal{U}_n$, and since $(\mathcal{U}_n)_{n\in\mathbb{N}}$ is point-complete, we have $\bigcap\{\overline{F} : F \in \mathcal{F}\} \neq \varnothing$. $\square$

Denote by L the Sorgenfrey line ([8]). The space L is fragmented by the usual metric. The mapping $x \in \mathbb{R} \to x \in L$ is quasicontinuous but has no point of subcontinuity. Hence, by Lemma 2.4, L is not p-σ-fragmentable as is mentioned in Examples 2.2.2.

Remarks 2.5. Let X be a Baire p-σ-fragmentable space and let $(\mathcal{U}_n)$ be a sequence of covers of X associated to this space. For each $n \in \mathbb{N}$, let A_n be the union of interiors of all elements in $\mathcal{U}_n$. By Lemma 2.3, the set $A = \bigcap_{n\in\mathbb{N}} A_n$ is a dense G_δ subset of X.

(1) Every point $x \in A$ is a q-point of X. This follows from the point-completeness of the sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$.

(2) Suppose moreover that the sequence $(\mathcal{U}_n)_{n\in\mathbb{N}}$ is complete. Then the subspace A of X is Čech-complete. To show this fact, for each $n \in \mathbb{N}$ and for each $x \in A$ pick by Lemma 2.3 a neighborhood V_n^x of x in X and $U_n^x \in \mathcal{U}_n$ such that $\overline{V_n^x} \subset U_n^x \subset A_n$; then the sequence of open covers $(\mathcal{V}_n)_{n\in\mathbb{N}}$ of the space A, defined by $\mathcal{V}_n = \{V_n^x \cap A : x \in A\}$, is complete.

In this paper we investigate continuity of separately continuous group operations. The following lemma implies in particular that every separately continuous mapping $f : X \times Y \to Z$, where X is a Baire p-space and Y a q-space, is quasicontinuous. Other results of the same type have been obtained in the past (see [11] and the bibliography in this paper). To establish this lemma we use the following topological game.

Christensen's game (cf. [5])**.** Let X be a topological space. The game $\mathcal{G}_\sigma$ is a two-player game. An instance of $\mathcal{G}_\sigma$ is a sequence of triplets $((U_n, V_n, x_n))_{n\in\mathbb{N}}$ defined inductively as follows: Player β begins and chooses a nonempty open set U_0 of X; player α then chooses a nonempty open set $V_0 \subset U_0$ and a point $x_0 \in X$. When (U_i, V_i, x_i), $0 \le i \le n-1$, have been defined, player β picks a nonempty open set $U_n \subset V_{n-1}$ and player α chooses a nonempty open set $V_n \subset U_n$ and a point $x_n \in X$. Player α wins if

$$\left(\bigcap U_n\right) \cap \overline{\{x_n : n \in \mathbb{N}\}} \neq \varnothing.$$

In [2, Proposition 3.6] we demonstrate that every Baire p-space X is σ-β-defavorable, which means player β has no winning strategy in the game $\mathcal{G}_\sigma$ on X. By Lemma 2.3 (see also 2.5) the same proof of [2, Proposition 3.6] allows more generally that every p-σ-fragmentable Baire space is σ-β-defavorable. We shall use this fact in this paper.

Lemma 2.6. *Let X be a σ-β-defavorable space, Y a space with a dense subset of q-points, Z a topological space and $f : X \times Y \to Z$ a separately continuous mapping. Then f is quasicontinuous.*

Proof. Let us suppose the opposite. Choose a point $(a, b) \in X \times Y$ such that f is not quasicontinuous at (a, b). Let W be an open set of Z which contains $f(a, b)$ and let $U \times V$ be an open paving of $X \times Y$ which contains (a, b), such that for each nonvoid open set $O \subset U \times V$ one has $f(O) \not\subset W$. Without loss of generality we may assume that b is a q-point of Y. Let $\varphi : Z \to \mathbb{R}$ be a continuous mapping such that $\varphi(f(a, b)) = 1$ and $\varphi(W^c) \subset \{0\}$ (recall that Z is a completely regular space). Let ψ denote the separately continuous map $f \circ \varphi : X \times Y \to \mathbb{R}$. Pick a sequence $(O_n)_{n\in\mathbb{N}}$ of neighborhoods associated to the q-point $b \in Y$. We shall define a strategy τ for player β in the game $\mathcal{G}_\sigma$ on the space X as follows: To begin β plays the nonempty open subset $\tau(\varnothing) = U \cap \{x \in X : \psi(x, b) > 0\}$ of X and chooses a point $(x_0, y_0) \in \tau(\varnothing) \times (O_0 \cap V)$ such that $\psi(x_0, y_0) = 0$. At the $(n+1)$th stroke, if player α has played $((V_0, a_0), \ldots, (V_n, a_n))$, then β chooses $x_{n+1} \in V_n$ and $y_{n+1} \in O_{n+1} \cap V$, satisfying the conditions

$$|\psi(a_i, b) - \psi(a_i, y_{n+1})| \le 1/(n+1) \quad \text{for each } i = 0, \ldots, n,$$
$$\psi(x_{n+1}, y_{n+1}) = 0,$$

and β plays the nonempty open set

$$\tau((V_0, a_0), \dots, (V_n, a_n)) = \{x \in V_n \, |\psi(x, y_{n+1})| < 1/(n+1)\}.$$

Since the space X is σ-β-defavorable, there is for α a winning game $(V_n, a_n)_{n\in\mathbb{N}}$ against the strategy τ. Let $x \in (\bigcap_{n\in\mathbb{N}} V_n) \cap \overline{\{a_n : n \in \mathbb{N}\}}$ and let $y \in Y$ be some cluster point of the sequence $(y_n)_{n\in\mathbb{N}}$ in Y. Then we easily obtain the contradiction $0 = \psi(x, y) = \psi(x, b) = 1$. □

The key for the proof of our main result (Theorem 3.1) is

Theorem 2.7. *Let X be a σ-β-defavorable space, Y a space with a dense set of q-points and Z a p-σ-fragmentable space. Suppose that the product $X \times Y$ is a Baire space. Then every separately continuous mapping $f : X \times Y \to Z$ is subcontinuous at each point of a dense G_δ subset of $X \times Y$.*

Proof. This follows from Lemma 2.6 and Lemma 2.4. □

3. Applications to semitopological groups

Let G be a group endowed with a topology. Let us recall that the group G is said to be semitopological if for every $g \in G$ the mappings $h \in G \to hg \in G$ and $h \in G \to gh \in G$ are continuous. It is called paratopological if the mapping $(g, y) \in G \times G \to gh \in G$ is continuous.

Theorem 3.1. *Let G be a semitopological group and suppose that the product $G \times G$ is a Baire space. If G is p-σ-fragmentable, then G is a paratopological group.*

Proof. We must show that the product mapping $(g, h) \in G \times G \to gh \in G$ is continuous. It suffices to prove that for every net $((g_\alpha, h_\alpha))_{\alpha\in\Lambda}$ in $G \times G$ which converges to the point (g, h), the point gh is a cluster point of the net $(g_\alpha h_\alpha)_{\alpha\in\Lambda}$ in G. Let $(g_\alpha, h_\alpha)_{\alpha\in\Lambda}$ be such a net. Consider by Remarks 2.5 and Theorem 2.7 a point $(a, b) \in G \times G$ of subcontinuity of the product mapping in G. We have $\lim ag^{-1}g_\alpha = a$ and $\lim h_\alpha h^{-1}b = b$, hence the net $(ag^{-1}g_\alpha h_\alpha h^{-1}b)_{\alpha\in\Lambda}$ has a cluster point in G; as the multiplication in G is separately continuous the net $(g_\alpha h_\alpha)_{\alpha\in\Lambda}$ must have a cluster point $y \in G$. To end the proof we show that $y = gh$. Since G is completely regular, it is sufficient to show that $f(gh) = f(y)$ for any continuous real function on G. Let $f : G \to \mathbb{R}$ be such a function. Let B denote the set of q-points of G; B is a dense subset of G by Remarks 2.5. (Since G is homogeneous, we have $B = G$; but we do not use this fact.) Pick $c \in B$. The mapping $\varphi : (u, v) \in G \times G \to f(uv) \in \mathbb{R}$ is separately continuous, hence by [2, Theorem 2.3] there is a dense subset A of G such that φ is continuous at every point of $A \times \{c\}$. Let $u \in A$. We have $\lim ug^{-1}g_\alpha = u$ and $\lim h_\alpha h^{-1}c = c$, hence $\lim f(ug^{-1}g_\alpha h_\alpha c) = f(uc)$. Then $f(ug^{-1}yh^{-1}c) = f(uc)$ for each $u \in A$. Since A is a dense subset of G, it follows that $f(yh^{-1}c) = f(gc)$; and since B is also dense in G, we obtain $f(y) = f(gh)$. This completes the proof. □

Let G be a p-σ-fragmentable semitopological group. Suppose that G is p-σ-fragmented by a complete sequence of covers. If G is a Baire space, then by Remarks 2.5 G has a dense Čech-complete subspace. It follows that $G \times G$ is a Baire space, and then by Theorem 3.1 G is a paratopological group. Now, by [2, Theorem 4.2] G is a topological group. This proves the following result.

Theorem 3.2. *Let G be a semitopological Baire group. If G is p-σ-fragmentable by a complete sequence of covers, then G is a topological group.*

Since every Čech-analytic space is p-σ-fragmentable by a complete sequence of covers (cf. 2.2(3)), the following is a corollary of 3.2.

Theorem 3.3. *Every Čech-analytic Baire semitopological group is a topological group.*

The next particular case of 3.3 answers affirmatively Pfister's problem mentioned in the introduction.

Corollary 3.4. *Every Čech-complete semitopological group is a topological group.*

Remark 3.5. Brand proves in [3] that every locally Čech-complete paratopological group is a topological group. Hence, as asked by the referee, is it natural to try to prove that every locally Čech-complete semitopological group is paratopological and hence a topological group? In a private conversation J. P. Troallic solves this question as follows: Let G be a locally Čech-complete semitopological group and note that G is a q-space and a σ-β-defavorable space. Let W be a nonvoid open Čech-complete subspace of G. By Lemma 2.6 the group multiplication $\pi : (g, h) \in G \times G \to gh \in G$ is quasicontinuous, hence there exists a nonvoid open paving $U \times V$ of $G \times G$ such that $\pi(U \times V) \subset W$. Then, by Theorem 2.7, the mapping $\pi : U \times V \to W$ has at least a point of subcontinuity. Now, by the proof of Theorem 3.1, it follows that the group multiplication is continuous.

Note. In his (her) comments on a second version of this paper, the referee pointed out to us that E. A. Reznichenko has announced without proof in [18] our Corollary 3.4.

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