

ON PERTURBATIONS OF M-ACCRETIVE OPERATORS IN BANACH SPACES

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ABSTRACT. In this paper, we consider the solvability of nonlinear equations of the form

$$Au + Cu \ni p$$

where A is an m -accretive operator on a Banach space X , C is a mapping on X and $p \in X$.

1. INTRODUCTION

Let X be a real Banach space, $A: X \supset D(A) \rightarrow X$ be an m -accretive operator, $C: X \supset \overline{D(A)} \rightarrow X$ is a continuous mapping and $p \in X$. In the last decade, the solvability of the nonlinear equation

$$(P) \quad Au + Cu \ni p$$

has been studied by Hirano [3], Kartsatos [4, 6], Morales [8], and other authors (cf. Kartsatos [5] for further references). The abstract results for (P) are applied to partial differential equations (cf. [5]).

Our purpose in this paper is to consider the solvability of problem (P) in the case that $C: X \supset D(A) \rightarrow X$ is not necessarily continuous. The results for noncontinuous cases can be applied to nonlinear boundary value problems (cf. the Example below).

In what follows, X stands for a real Banach space with norm $\|\cdot\|$, and J stands for the normalized duality mapping. For each $x \in X^*$ and $x \in X$, we denote by $\langle x, x^* \rangle$ the value of x^* at x . An operator $A: X \supset D(A) \rightarrow 2^X$ is said to be accretive if for every $x, y \in D(A)$, there exists $j \in J(x - y)$ such that

$$\langle u - v, j \rangle \geq 0, \quad \text{for all } u \in Ax, v \in Ay.$$

An accretive operator is said to be m -accretive if $R(I + \lambda A) = X$ for all $\lambda \in (0, \infty)$. For an m -accretive operator A , the resolvent $J_\lambda: X \rightarrow D(A)$ is defined by $J_\lambda = (I + \lambda A)^{-1}$ for all $\lambda \in (0, \infty)$. We denote by A_λ the operator defined as $A_\lambda = (1/\lambda)(I - J_\lambda)$ for $\lambda \in (0, \infty)$. It is easy to see that $A_\lambda x \in AJ_\lambda x$ for $x \in X$.

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For each $x \in D(A)$, we set $|Ax| = \inf\{\|v\| : v \in Ax\}$. For each bounded set $S \subset X$, the measure of noncompactness $\alpha(S)$ of S is defined by

$$\alpha(S) = \inf\{\delta > 0 : S \text{ can be covered with a finite number of sets of diameter less than } \delta\}.$$

A continuous mapping $F: X \supset D(F) \rightarrow X$ is said to be condensing if, for every bounded noncompact subset S of $D(F)$, $F(S)$ is bounded and $\alpha(F(S)) < \alpha(S)$ (cf. [7]). It is obvious from the definition that F is condensing if F is compact or Lipschitz continuous with Lipschitz constant $k < 1$. It is also easy to see that the sum of a condensing mapping and a compact mapping is a condensing mapping. A mapping $F: X \supset D(F) \rightarrow X$ is said to be bounded if F maps each bounded set of $D(F)$ to bounded sets of X . We denote by $B_r(x)$ the open ball with center at x and radius $r > 0$. For each subset D of X , we denote by ∂D the boundary of D . We denote by Γ the set of all functions $\beta: R^+ \rightarrow R^+$ such that $\beta(r) \rightarrow 0$ as $r \rightarrow \infty$. We first consider the case that the m -accretive operator A has a compact resolvent.

Theorem 1. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive with $(A + I)^{-1}$ compact. Let $C: X \supset D(A) \rightarrow X$ be bounded and satisfy that $C(I + \lambda A)^{-1}: X \rightarrow X$ is condensing for some $\lambda \in (0, 1]$. Let $p \in X$ and assume that there exist positive constants b, r and $z \in D(A)$ satisfying that $\|z\| < b$ and for every $x \in D(A)$ with $\|x\| \geq b$,*

$$(*) \quad \langle u + Cx - p, j \rangle \geq 0$$

for all $u \in Ax$ and all $j \in J(x - z)$. Then $p \in R(A + C)$.

Proof. Let $\lambda \in (0, 1]$ be such that $C(I + \lambda A)^{-1}$ is condensing. Let $y \in X$ satisfy the equality

$$(1.1) \quad A_\lambda y + C J_\lambda y = p.$$

Then recalling that $A_\lambda y \in A J_\lambda y$, we have that $x = J_\lambda y$ is a solution of the problem (P). Then we seek a solution of (1.1). Since $A_\lambda y = (1/\lambda)(I - J_\lambda)y$, equation (1.1) can be rewritten as

$$(1.2) \quad y = \lambda p + J_\lambda y - \lambda C J_\lambda y.$$

We now define a mapping $T: X \rightarrow X$ by

$$(1.3) \quad T y = \lambda p + J_\lambda y - \lambda C J_\lambda y.$$

Then by (1.2), the solution of (1.1) is a fixed point of the mapping T . Since J_λ is compact and $C J_\lambda$ is condensing, we have that T is condensing on X . Here we fix $w \in X$ such that $w \in z + \lambda A z$. Since C is bounded, we can choose $M > 0$ such that

$$(1.4) \quad \sup_{x \in D(A), \|x\| \leq b} \|\lambda p + x - \lambda C x - w\| < M.$$

We choose $r > 0$ so large that $r > \max\{(1 + \lambda)b, \|w\|\}$ and

$$(1.5) \quad r - 2b - \|w\| > M.$$

By Theorem 6.3.2 of [7], T possesses a fixed point in $\overline{B_r(0)}$ if

$$(1.6) \quad T(y) - w \neq \alpha(y - w), \quad \text{for all } y \in \partial B_r(0) \text{ and } \alpha > 1.$$

Suppose that $y \in \partial B_r(0)$ and $T(y) - w = \alpha(y - w)$ for some $\alpha > 1$. Then we have by (1.3) that

$$(1.7) \quad \alpha(y - w) + w = \lambda p + J_\lambda y - \lambda C J_\lambda y.$$

We put $x = J_\lambda y$. Then $x + \lambda Ax \ni y$. Let $v \in Ax$ such that $x + \lambda v = y$. Then

$$(1.8) \quad \alpha x + \alpha(\lambda v - w) = \lambda p + x - \lambda Cx - w.$$

Now suppose that $\|x\| < b$; then by (1.4),

$$\|\lambda p + x - \lambda Cx - w\| < M.$$

On the other hand, noting that $\lambda\|v\| = \|y - x\| \geq r - b$,

$$\begin{aligned} \|\alpha x + \alpha(\lambda v - w)\| &\geq \alpha\lambda\|v\| - \alpha(\|x\| + \|w\|) \\ &\geq \alpha((r - b) - b - \|w\|) \\ &\geq r - 2b - \|w\| > M. \end{aligned}$$

This contradicts (1.8). Therefore we have that $\|x\| \geq b$.

On the other hand, recalling that $v \in Ax$, we have by the accretivity of A that there exists $j \in J(x - z)$ and

$$(1.9) \quad \alpha\langle x + \lambda v - w, j \rangle > \langle x + \lambda v - w, j \rangle \geq \|x - z\|^2 > 0.$$

Then by (1.8), we have

$$\langle \lambda p + x - \lambda Cx - w, j \rangle > \langle x + \lambda v - w, j \rangle.$$

Thus we find

$$(1.10) \quad \langle v + Cx - p, j \rangle < 0.$$

Since $\|x\| \geq b$, (1.10) contradicts the condition (*). Therefore (1.6) holds and then (1.1) has a solution. This completes the proof. \square

Theorem 2. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive with $(A + I)^{-1}$ compact. Let $C: X \supset \overline{D(A)} \rightarrow X$ be continuous and bounded, and let $p \in X$. Assume that there exist positive constants b, r and $z \in D(A)$ satisfying that $\|z\| < b$ and, for every $x \in D(A)$ with $\|x\| \geq b$, (*) holds for all $u \in Ax$ and all $j \in J(x - z)$. Then $p \in R(A + C)$.*

Proof. Since $C: \overline{D(A)} \rightarrow X$ is bounded and continuous, we have that $C(I + A)^{-1}$ is compact. Then the assertion follows from Theorem 1. \square

We can treat the case that C is not defined on $\overline{D(A)}$ if the dual space X^* of X is uniformly convex:

Theorem 2'. *Let X^* be uniformly convex. Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive with $(A + I)^{-1}$ compact. Let $C: X \supset D(A) \rightarrow X$ be continuous and bounded, and let $p \in X$. Assume that there exist positive constants b, r and $z \in D(A)$ satisfying that $\|z\| < b$ and, for every $x \in D(A)$ with $\|x\| \geq b$, (*) holds for all $u \in Ax$ and, for $j = J(x - z)$. Then $p \in R(A + C)$.*

Proof. It is sufficient to show that $C(I + A)^{-1}$ is compact. Let $\{x_n\} \subset X$ be a bounded sequence and put $y_n = (I + A)^{-1}x_n$ for $n \geq 1$. We may assume that x_n converges weakly to $x \in X$ and y_n converges strongly to $y \in X$. Then since the m -accretive operator A is demiclosed (cf. Proposition 3.5 of Barbu [1]), we find that $y \in D(A)$ and $x \in Ay$. Then by the continuity of C on $D(A)$, we find that $Cy_n \rightarrow Cy$. That is, $C(I + A)^{-1}$ is compact. \square

Remark. Theorem 2 (Theorem 2') should be compared with Theorem 5 of [6]. It is known that the duality mapping J is single valued if X^* is strictly convex. If J is single valued, then condition (*) of Kartsatos [6] is the condition (*) with $z = 0$. We also note that as we have seen in the proof of Theorem 2', our argument does not require the mapping C to be defined on $\overline{D(A)}$ if the m -accretive operator A is demiclosed. The known results so far demand C to be defined and continuous on $\overline{D(A)}$.

We next treat the case that C is not bounded.

Theorem 3. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive with $(A + I)^{-1}$ compact. Let $C: X \supset D(A) \rightarrow X$ satisfy that $C(I + \lambda A)^{-1}: X \rightarrow X$ is condensing for some $\lambda \in (0, 1]$, and let $p \in X$. Assume that there exist positive constants b, r and $z \in D(A)$ satisfying that $\|z\| < b$, Az is bounded and, for every $x \in D(A)$ with $\max\{\|x\|, \lambda|Ax|\} \geq b$, (*) holds for all $u \in Ax$ and all $j \in J(x - z)$. Then $p \in R(A + C)$.*

Proof. We put $M = \sup\{\|v\|: v \in Az\}$. Let $r > 0$ be so large that

$$(1.11) \quad r > b + \lambda \max\{b, M\}.$$

Let x, y, v and z be as in the proof of Theorem 1. Then since $\|y\| = r$ and $y = x + \lambda v$, we find from (1.11) that

$$(1.12) \quad \|x\| > b \quad \text{or} \quad \|v\| > \max\{b, M\}.$$

This implies that $x \neq z$. Then by (1.9), we find that (1.10) holds. On the other hand, it follows from (1.12) that (*) holds. This is a contradiction and the proof is complete. \square

We next consider the solvability of (P) under a condition introduced in Guan and Karsatos [2]. In the next result, we impose the following condition on C :

(c) If $\{x_n\} \subset X$ is a convergent sequence such that $x = \lim_{n \rightarrow \infty} x_n$ and there exists a bounded sequence $\{v_n\} \subset X$ with $v_n \in Ax_n$ for $n \geq 1$, then $Cx = \lim_{n \rightarrow \infty} Cx_n$.

Theorem 4. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive with $(A + I)^{-1}$ compact. Let $C: X \supset D(A) \rightarrow X$ be bounded, satisfy (c) and satisfy that $C(I + \lambda A)^{-1}: X \rightarrow X$ is condensing for some $\lambda \in (0, 1]$. Let $S \subset X$ be such that for every $p \in S$ there exists $K(p) > 0, \beta = \beta_s \in \Gamma$ and $z \in D(A)$ such that*

$$(**) \quad \langle v + Cx - p, j \rangle \geq -K(p) - \beta(\|x\|)\|x\|,$$

for all $x \in \overline{D(A)}$ with $\|x\|$ sufficiently large, all $v \in Ax$ and for all $j \in J(x - z)$. Then $S \subset \overline{R(A + B)}$ and $\text{int } S \subset R(A + B)$.

Proof. Let $p \in S$. We set

$$C_n x = Cx + (1/n)(x - z) \quad \text{for } x \in D(A) \text{ and } n \geq 1.$$

Then C_n is bounded. Since $(I + A)^{-1}$ is compact, we can see that $C_n(I + \lambda A)^{-1} = C(I + \lambda A)^{-1} + (1/n)(I + \lambda A)^{-1} - (1/n)z$ is condensing. From (**), we have that for each $n \geq 1$,

$$\langle v + C_n x - p, j \rangle \geq (1/n)\|x - z\|^2 - K(p) - \beta(\|x\|)\|x\|$$

for all $x \in D(A)$ with $\|x\|$ sufficiently large, all $v \in Ax$ and for all $j = j_{x,s} \in J(x-z)$. It then follows that for each $n \geq 1$, there exists $b_n > 0$ such that

$$\langle v + C_n x - p, j \rangle \geq 0$$

for all $x \in D(A)$ with $\|x\| \geq b_n$, all $v \in Ax$ and for all $j = j_{x,s} \in J(x-z)$. Then by Theorem 1, we find that $p \in R(A+C_n)$ for all $n \geq 1$. That is, there exist sequences $\{x_n\} \subset X$ and $\{v_n\} \subset X$ such that $v_n \in Ax_n$ and

$$(1.13) \quad v_n + Cx_n + \frac{1}{n}(x_n - z) = p \quad \text{for all } n \geq 1.$$

If $\{x_n\}$ contains a bounded subsequence, then from (1.13), we have that $p \in \overline{R(A+C)}$. We now assume that $\lim_{n \rightarrow \infty} \|x_n\| = \infty$. Then we have by (**) and (1.13) that

$$(1/n)\|x_n - z\|^2 \leq K(p) + \beta(\|x_n\|)\|x_n\|$$

for n sufficiently large. Then since

$$\lim_{n \rightarrow \infty} \|x_n\|/\|x_n - z\| = 1$$

and

$$\lim_{n \rightarrow \infty} \beta(\|x_n\|) = 0,$$

we find that $\lim_{n \rightarrow \infty} (1/n)\|x_n - z\| = 0$. It then follows from (1.13) that $p \in \overline{R(A+C)}$. Thus we have shown that $S \subset \overline{R(A+C)}$.

We next show $\text{int } S \subset R(A+C)$. Suppose that $p \in \text{int } S$ and let $\{x_n\}$ be the sequence of solutions of (1.13). Suppose that there exists a bounded subsequence $\{x_m\}$ of $\{x_n\}$. By (1.13), we find that

$$(1.14) \quad x_m = (I + A)^{-1} \left(x_m + p - \frac{1}{n}(x_m - z) - Cx_m \right).$$

Since $\{x_m\}$ is bounded, we have by the hypothesis that $\{Cx_m\}$ is bounded. Then by (1.14), we have that $\{x_m\}$ is precompact. Then we may assume that x_n converges to $x \in X$. It also follows from (1.13) that $\{v_n\}$ is bounded. Then by (c), $\lim_{n \rightarrow \infty} Cx_n = Cx$. This implies that $v_n \rightarrow v \in X$ and then $v \in Ax$ (cf. Proposition 3.4 of [1]). Thus we find that $Ax + Cx \ni p$. We next suppose that $\lim_{n \rightarrow \infty} \|x_n\| = \infty$. Let $j_n \in J(x_n - z)$. We put

$$\tilde{j}_n = j_n / \max\{\beta(\|x_n\|)\|x_n\|, 1\} \quad \text{for each } n \geq 1.$$

Since $\beta \in \Gamma$ we have that $\lim_{n \rightarrow \infty} \|\tilde{j}_n\| = \infty$. Since $p \in \text{int } S$, there exists $r > 0$ such that $B_r(p) \subset S$. Let $h \in B_r(0)$. Then since

$$v_n + Cx_n + \frac{1}{n}(x_n - z) - (p + h) = -h,$$

we have by (**),

$$\langle h, j_n \rangle \leq \beta(\|x_n\|)\|x_n\| + K(h + p)$$

for each $n \geq 1$. Then

$$\langle h, \tilde{j}_n \rangle \leq 1 + K(h + p) \quad \text{for all } n \geq 1.$$

Since $h \in B_r(0)$ is arbitrary, we have by the Banach-Steinhaus theorem that $\{\tilde{j}_n\}$ is bounded in X^* . This is a contradiction. Thus we have that $\{x_n\}$ is bounded in X and then $Ax + Cx \ni p$. This completes the proof. \square

Corollary 5. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive with $(A+I)^{-1}$ compact. Let $C: X \supset \overline{D(A)} \rightarrow X$ be bounded and continuous. Let $S \subset X$ be such that for every $p \in S$ there exists $K(p) > 0$, $\beta = \beta_s \in \Gamma$ and $z \in D(A)$ such that $(**)$ holds for all $x \in D(A)$ with $\|x\|$ sufficiently large, all $v \in Ax$ and for all $j \in J(x-z)$. Then $S \subset \overline{R(A+B)}$ and $\text{int } S \subset R(A+B)$.*

Proof. Since C is bounded and continuous, C satisfies (c) and that $C(I+A)^{-1}$ is compact. Then the assertions follow from Theorem 4. \square

We next consider the case that the mapping $C(I+\lambda A)^{-1}$ is compact for some $\lambda > 0$.

Theorem 6. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive with $J_1 = (A+I)^{-1}$ condensing. Let $C: X \supset D(A) \rightarrow X$ be bounded and satisfy that $C(I+\lambda A)^{-1}$ is compact for some $\lambda \in (0, 1]$. Let $p \in X$ and assume that there exist positive constants b, r and $z \in D(A)$ such that $\|z\| < b$ and, for every $x \in D(A)$ with $\|x\| \geq b$, $(*)$ holds for all $u \in Ax$ and all $j \in J(x-z)$. Then $p \in R(A+C)$.*

Proof. The proof of Theorem 6 is the same as that of Theorem 1. In fact, by the hypothesis, the mapping T defined in the proof of Theorem 1 is condensing. \square

Theorem 7. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive. Let $C: X \supset D(A) \rightarrow X$ be bounded and satisfy that $C(I+\lambda A)^{-1}$ is compact for some $\lambda \in (0, 1]$. Let $p \in X$ and assume that there exist positive constants b, r and $z \in D(A)$ such that $\|z\| < b$ and, for every $x \in D(A)$ with $\|x\| \geq b$, $(*)$ holds for all $u \in Ax$ and all $j \in J(x-z)$. Then $p \in \overline{R(A+C)}$.*

Proof. For each $n \geq 1$, we put $A(n) = A + (1/n)I$. Then the resolvent $J_1(n) = (I+A(n))^{-1}$ is Lipschitz mapping with Lipschitz constant $n/(1+n)$. That is, $J_1(n)$ is condensing. Then by applying Theorem 6 with A replaced by $A(n)$, we find that there exist sequences $\{x_n\} \subset X$ and $\{v_n\} \subset X$ such that $v_n \in Ax_n$ and

$$(1.15) \quad v_n + \frac{1}{n}x_n + Cx_n = p \quad \text{for all } n \geq 1.$$

To show the assertion, it is sufficient to show that $\{x_n\}$ contains a subsequence such that $\lim_{n \rightarrow \infty} x_n/n = 0$. If $\{x_n\}$ contains a bounded subsequence $\{x_n\}$, then $\lim_{m \rightarrow \infty} x_m/m = 0$ and the assertion follows. Suppose that $\lim_{n \rightarrow \infty} \|x_n\| = \infty$. Then from (1.15) and $(*)$, we find that

$$(1.16) \quad \left\langle -\frac{1}{n}x_n, j_n \right\rangle \geq 0$$

for sufficiently large n , where $j_n \in J(x_n - z)$. Then it follows from the definition of j_n that

$$\frac{1}{n}\|x_n - z\| \leq \frac{1}{n}\|z\|$$

for n sufficiently large. Then we obtain that $\lim_{n \rightarrow \infty} x_n/n = 0$ and the proof is complete. \square

Corollary 8. *Let $A: X \supset D(A) \rightarrow 2^X$ be m -accretive and $C: X \supset D(A) \rightarrow X$ be compact. Let $p \in X$ and assume that there exist positive constants b, r and $z \in D(A)$ such that $\|z\| < b$ and, for every $x \in D(A)$ with $\|x\| \geq b$, $(*)$ holds for all $u \in Tx$ and all $j \in J(x-z)$. Then $p \in \overline{R(A+C)}$.*

Proof. It is obvious that if C is compact, then $C(I + A)^{-1}$ is compact. Then the assertion follows from Theorem 7. \square

Remark. Corollary 5 and Corollary 8 should be compared with Theorem 4.1 of [2] and Theorem 3 of [6], respectively. We note that we do not know if the assertion of Theorem 4 holds in the case that A is m -accretive, $C(I + \lambda A)^{-1}$ is compact for some $\lambda > 0$ and $p \in X$ satisfies the condition (**).

Example. Let $\Omega \subset R^N$ be a bounded domain with a smooth boundary $\partial\Omega$. We consider the solvability of a nonlinear elliptic boundary value problem of the form

$$(BV) \quad \begin{cases} -\sum_{i=1}^N \frac{\partial}{\partial x_i} a_i \left(\frac{\partial u}{\partial x_i} \right) + g \left(x, \frac{\partial u}{\partial x_1}, \dots, \frac{\partial u}{\partial x_N}, u \right) = p(x) & \text{on } \Omega, \\ u(x) = 0 & \text{on } \partial\Omega. \end{cases}$$

Here we assume that $a_i \in C^1(R)$ is a function such that $0 < \inf_{t \in R} a_i'(t) \leq \sup_{t \in R} a_i'(t) < \infty$ for each $i = 1, \dots, N$. We define a nonlinear operator A by

$$Au = -\sum_{i=1}^N \frac{\partial}{\partial x_i} a_i \left(\frac{\partial u}{\partial x_i} \right) \quad \text{for } u \in D(A) = H^2(\Omega) \cap H_0^1(\Omega).$$

Then A is an m -accretive operator on $X = L^2(\Omega)$ (cf. Barbu [1]). Let $p \in L^2(\Omega)$ and $g \in C(\Omega \times R^N \times R, R)$ satisfy that there exist positive constants c_1, c_2 and

$$(g1) \quad |g(x, s, t)| \leq c_1|t| + c_2 \quad \text{for all } x \in \Omega, s \in R^N \text{ and } t \in R,$$

$$(g2) \quad \int_{\Omega} (g(x, Du(x), u(x)) - p(x))u(x) dx \geq 0$$

for all $u \in D(A)$ with $\|u\|_{L^2} \geq b$, where $(Du)(x) = (\partial u/\partial x_1, \dots, \partial u/\partial x_N)$ for $x \in \Omega$.

Here we put $(Cu)(x) = g(x, Du(x), u(x))$ for $u \in D(A)$ and $x \in \Omega$. Then from (g1), we have that $C: X \supset D(A) \rightarrow X$ is bounded. From the assumption, the mapping $(I + A)^{-1}$ maps bounded sets of $L^2(\Omega)$ to bounded sets of $H^2(\Omega)$. Then recalling that $H^2(\Omega)$ is compactly embedded in $H^1(\Omega)$, we find that $C(I + A)^{-1}$ is compact in $L^2(\Omega)$. It is easy to see from the definition of A and (g2) that (*) holds. Then by Theorem 1, problem (BV) has a solution.

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