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ON THE EXISTENCE OF POSITIVE SOLUTIONS OF NONLINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

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Dedicated to Professor Taro Yoshizawa on his 75th birthday

ABSTRACT. Under suitable conditions on $f(t,u)$, the boundary value problem

$$\text{(BVP)} \qquad \begin{cases} (E)\ u''(t) + f(t,u(t)) = 0 \text{ in } (0,1), \\ (BC) \begin{cases} \alpha u(0) - \beta u'(0) = 0, \\ \gamma u(1) + \delta u'(1) = 0 \end{cases} \end{cases}$$

has at least one positive solution. Moreover, we also apply this main result to establish several existence theorems of multiple positive solutions for some nonlinear (elliptic) differential equations.

1. Introduction

There has recently been an increased interest in studying the existence of positive solutions of the following boundary value problem

$$\text{(BVP)} \qquad \begin{cases} (E)\ u''(t) + f(t,u(t)) = 0 \text{ in } (0,1), \\ (BC) \begin{cases} \alpha u(0) - \beta u'(0) = 0, \\ \gamma u(1) + \delta u'(1) = 0 \end{cases} \end{cases}$$

in the last fifteen to twenty-five years; see, for example, Bandle, Coffman and Marcus [1], Bandle and Kwong [2], Coffman and Marcus [3], H. Dang and K. Schmit [4], Erbe [6], Erbe, Hu and Wang [7], Erbe and Wang [8], Garaizar [9], Iffland [11], Santanilla [13], Wang [14] and Wong [15].

In 1994, Erbe, Hu and Wang [7, 8] showed the following excellent results:

Theorem A (Erbe and Wang [8]). *Suppose that*
(A.1) $f \in C([0,1]\times[0,\infty);[0,\infty))$,
(A.2) $\alpha,\beta,\gamma,\delta \geq 0$ *and* $\rho := \gamma\beta + \alpha\gamma + \alpha\delta > 0$.

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Then, (BVP) has at least one positive solution in the case

(i) $\lim_{u\to 0^+} max_{t\in[0,1]} \frac{f(t,u)}{u} = 0$ *and* $\lim_{u\to\infty} min_{t\in[0,1]} \frac{f(t,u)}{u} = \infty$ *(superlinear), or*

(ii) $\lim_{u\to 0^+} min_{t\in[0,1]} \frac{f(t,u)}{u} = \infty$ *and* $\lim_{u\to\infty} max_{t\in[0,1]} \frac{f(t,u)}{u} = 0$ *(sublinear).*

Theorem B (Erbe, Hu and Wang [7]). *Assume that* (A_1), (A_2) *and the following assumptions hold:*

(A.3) $\lim_{u\to 0^+} min_{t\in[0,1]} \frac{f(t,u)}{u} = \lim_{u\to\infty} min_{t\in[0,1]} \frac{f(t,u)}{u} = \infty$,

(A.4) *there is a* $p > 0$ *such that*

$$f(t,u) \leq \eta p \text{ on } [0,1]\times[0,p],$$

where $\eta = (\int_0^1 G(s,s)ds)^{-1} = \frac{6\rho}{6\delta\beta+3\gamma\beta+\alpha\gamma+3\alpha\delta}$. *Here* $G(t,s)$ *is the Green's function of*

$$u''(t) = 0 \text{ in } (0,1)$$

with respect to the boundary value condition (BC).

Then, (BVP) *has at least two positive solutions* u_1 *and* u_2 *such that*

$$0 < ||u_1|| < p < ||u_2||.$$

Theorem C (Erbe, Hu and Wang [7]). *Assume that* (A_1), (A_2) *and the following assumptions hold:*

(A.5) $\lim_{u\to 0^+} max_{t\in[0,1]} \frac{f(t,u)}{u} = \lim_{u\to\infty} max_{t\in[0,1]} \frac{f(t,u)}{u} = 0$,

(A.6) *there is a* $p > 0$ *such that*

$$f(t,u) \geq \lambda p \text{ on } [\frac{1}{4},\frac{3}{4}]\times[\sigma p, p],$$

where $\lambda = (\int_{\frac{1}{4}}^{\frac{3}{4}} G(\frac{1}{2},s)ds)^{-1}$ *and* $\sigma = min\{\frac{\gamma+4\delta}{4(\gamma+\delta)}, \frac{\alpha+4\beta}{4(\alpha+\beta)}\}$.

Then, (BVP) *has at least two positive solutions* u_1 *and* u_2 *such that*

$$0 < ||u_1|| < p < ||u_2||.$$

Let

$$maxf_0 := \lim_{u\to 0^+} max_{t\in[0,1]} \frac{f(t,u)}{u},$$

$$minf_0 := \lim_{u\to 0^+} min_{t\in[0,1]} \frac{f(t,u)}{u},$$

$$maxf_\infty := \lim_{u\to\infty} max_{t\in[0,1]} \frac{f(t,u)}{u},$$

and

$$minf_\infty := \lim_{u\to\infty} min_{t\in[0,1]} \frac{f(t,u)}{u}.$$

Then, it follows from Theorems A, B and C that

(I) $maxf_0 = 0$ and $minf_\infty = \infty$ implies (BVP) has at least one positive solution,

(II) $minf_0 = \infty$ and $maxf_\infty = 0$ implies (BVP) has at least one positive solution,

(III)

$$\begin{cases} minf_0 = minf_\infty = \infty, \\ f(t,u) \le \eta p \text{ on } [0,1]\times[0,p] \end{cases}$$

implies (BVP) has at least two positive solutions,

(IV)

$$\begin{cases} maxf_0 = maxf_\infty = 0, \\ f(t,u) \ge \lambda p \text{ on } [0,1]\times[\sigma p,p] \end{cases}$$

implies (BVP) has at least two positive solutions.

Seeing such a fact, we can not but ask **"whether or not we can obtain a similar conclusion, if** $maxf_0, minf_0, maxf_\infty, minf_\infty \notin \{0,\infty\}$**."** Inspired by the above-mentioned results, we attempt to establish a simple criterion for the existence of positive solutions of (BVP), which is a generalization of Theorems A, B, and C and gives a positive answer to the question stated above.

2. Main results

Let $\alpha,\beta,\gamma,\delta \ge 0$ and $\rho := \gamma\beta+\alpha\gamma+\alpha\delta > 0$. In order to abbreviate our discussion, throughout this paper, we suppose that the following assumptions hold:

(C_1) $k(t,s)$ is the Green's function of the differential equation

$$u''(t) = 0 \text{ in } (0,1) \tag{1}$$

with respect to the boundary value condition (BC);

(C_2) $0 < M := min\{\frac{\gamma+4\delta}{4(\gamma+\delta)}, \frac{\alpha+4\beta}{4(\alpha+\beta)}\} < 1$;

(C_3) $f \in C([0,1]\times[0,\infty);[0,\infty))$.

In order to discuss our main result (Theorem 1 below), we need the following two useful lemmas:

Lemma D. *Suppose that $k(t,s)$ is defined as in (C_1). Then we have the following results:*

$$\begin{cases} (R_1)\ \dfrac{k(t,s)}{k(s,s)} \le 1, \quad for\ t\in[0,1]\ and\ s\in[0,1], \\ (R_2)\ \dfrac{k(t,s)}{k(s,s)} \ge M, \quad for\ t\in[\frac{1}{4},\frac{3}{4}]\ and\ s\in[0,1]. \end{cases}$$

Proof. Let

$$\varphi(t) := (\gamma+\delta-\gamma t) \text{ and } \psi(t) := \beta+\alpha t \text{ for } t\in[0,1].$$

Then,

$$k(t,s) = \begin{cases} \dfrac{1}{\rho}\varphi(t)\psi(s),\ 0\le s\le t\le 1, \\ \dfrac{1}{\rho}\varphi(s)\psi(t),\ 0\le t\le s\le 1, \end{cases}$$

which implies

$$\frac{k(t,s)}{k(s,s)} = \begin{cases} \dfrac{\varphi(t)}{\varphi(s)},\ 0\le s\le t\le 1, \\ \dfrac{\psi(t)}{\psi(s)},\ 0\le t\le s\le 1. \end{cases}$$

Hence, we obtain the following desired results:

$$\frac{k(t,s)}{k(s,s)} \leq 1 \text{ for } t \in [0,1],$$

and

$$\frac{k(t,s)}{k(s,s)} \geq \begin{cases} \dfrac{\gamma+4\delta}{4(\gamma+\delta)} \ \geq \ M, \ 0 \leq s \leq t \leq \dfrac{3}{4}. \\ \dfrac{\alpha+4\beta}{4(\alpha+\beta)} \ \geq \ M, \ \dfrac{1}{4} \leq t \leq s \leq 1. \end{cases}$$

Lemma E (Deimling [5] and Krasnoselskii [12]). *Let E be a Banach space, and let $K \subset E$ be a cone in E. Assume Ω_1, Ω_2 are open subsets of E with $0 \in \Omega_1, \overline{\Omega_1} \subset \Omega_2$, and let*

$$A : K \cap (\overline{\Omega_2} \backslash \Omega_1) \longrightarrow K$$

be a completely continuous operator such that either
(i) $||Au|| \leq ||u||, \ u \in K \cap \partial\Omega_1$ *and* $||Au|| \geq ||u||, \ u \in K \cap \partial\Omega_2$; *or*
(ii) $||Au|| \geq ||u||, \ u \in K \cap \partial\Omega_1$ *and* $||Au|| \leq ||u||, \ u \in K \cap \partial\Omega_2$.
Then A has a fixed point in $K \cap (\overline{\Omega_2} \backslash \Omega_1)$.

Now, we can state and prove our main result.

Theorem 1. *Assume that there exist two distinct positive constants λ, η such that*

$$f(t,u) \leq \lambda\left(\int_0^1 k(s,s)ds\right)^{-1} \ on \ [0,1] \times [0,\lambda], \tag{2}$$

and

$$f(t,u) \geq \eta\left(\int_{\frac{1}{4}}^{\frac{3}{4}} k(\frac{1}{2},s)ds\right)^{-1} \ on \ [\frac{1}{4},\frac{3}{4}] \times [M\eta,\eta]. \tag{3}$$

Then (BVP) has at least one positive solution u such that $||u||$ between λ and η, where $||u|| := sup_{t\in[0,1]}|u(t)|$.

Proof. Without loss of generality, we may assume that $\lambda < \eta$. It is clear that (BVP) has a solution $u = u(t)$ if, and only if, u is the solution of the operator equation

$$u(t) = \int_0^1 k(t,s)f(s,u(s))ds := Au(t), \ u \in C[0,1].$$

Let K be a cone in $C[0,1]$ given by

$$K = \{u \in C[0,1] | u(t) \geq 0, min_{t\in[\frac{1}{4},\frac{3}{4}]} u(t) \geq M||u||\}.$$

It follows from the definition of K and Lemma D that

$$\begin{aligned} min_{t\in[\frac{1}{4},\frac{3}{4}]}(Au)(t) &= min_{t\in[\frac{1}{4},\frac{3}{4}]} \int_0^1 k(t,s)f(s,u(s))ds \\ &\geq M \int_0^1 k(s,s)f(s,u(s))ds \ (using(R_2)) \\ &\geq M \int_0^1 k(t,s)f(s,u(s))ds \ (using(R_1)). \end{aligned}$$

Hence, $min_{t\in[\frac{1}{4},\frac{3}{4}]}(Au)(t) \geq M||Au||$, which implies $AK \subset K$. Furthermore, it is easy to check that $A : K \to K$ is completely continuous. In order to complete the proof, we separate the rest of the proof into the following two steps:

Step (I) Let $\Omega_1 := \{u \in K| ||u|| < \lambda\}$. It follows from (2) and Lemma D that for $u \in \partial\Omega_1$,

$$\begin{aligned}(Au)(t) &= \int_0^1 k(t,s)f(s,u(s))ds \\ &\leq \int_0^1 k(s,s)f(s,u(s))ds \\ &\leq \lambda(\int_0^1 k(s,s)ds)^{-1}(\int_0^1 k(s,s)ds)\frac{||u||}{\lambda} \\ &= ||u||.\end{aligned}$$

Hence,

$$||Au|| \leq ||u|| \text{ for } u \in \partial\Omega_1.$$

Step (II) Let $\Omega_2 := \{u \in K| ||u|| < \eta\}$. It follows from the definitions of $||u||$ and K that

$$\begin{cases} u(t) \leq ||u|| = \eta \text{ for } t \in [0,1], \\ u(t) \geq min_{t\in[\frac{1}{4},\frac{3}{4}]}u(t) \geq M||u|| = M\eta \text{ for } t \in [\frac{1}{4},\frac{3}{4}], \end{cases}$$

for $u \in \partial\Omega_2$, which implies

$$M\eta \leq u(t) \leq \eta \text{ for } t \in [\frac{1}{4},\frac{3}{4}].$$

Hence, by (3),

$$\begin{aligned}(Au)(\frac{1}{2}) &= \int_0^1 k(\frac{1}{2},s)f(s,u(s))ds \\ &\geq \int_{\frac{1}{4}}^{\frac{3}{4}} k(\frac{1}{2},s)f(s,u(s))ds \\ &\geq \eta(\int_{\frac{1}{4}}^{\frac{3}{4}} k(\frac{1}{2},s)ds)^{-1}(\int_{\frac{1}{4}}^{\frac{3}{4}} k(\frac{1}{2},s)ds)\frac{||u||}{\eta} \\ &= ||u||.\end{aligned}$$

Thus,

$$||Au|| \geq ||u|| \text{ for } u \in \partial\Omega_2.$$

Therefore, by the first part of Lemma E, we complete the proof.

Remark 2. Since

$$(\int_0^1 k(s,s)ds)^{-1} := A = \frac{6\rho}{6\delta\beta + 3\gamma\beta + \alpha\gamma + 3\alpha\delta},$$

and

$$(\int_{\frac{1}{4}}^{\frac{3}{4}} k(\frac{1}{2},s)ds)^{-1} := B = \frac{8\rho}{4\beta\delta + 2\beta\gamma + \alpha\gamma + 2\alpha\delta},$$

then, we have the following results:

(a) Suppose that $maxf_0 := C_1 \in [0, A)$. Taking $\epsilon = A - C_1 > 0$, there exists $\lambda_1 > 0$ (λ_1 can be chosen small arbitrarily) such that

$$\max_{t\in[0,1]} \frac{f(t,u)}{u} \leq \epsilon + C_1 = A \ on \ (0, \lambda_1].$$

Hence,

$$f(t,u) \leq Au \leq A\lambda_1 \ on \ [0,1] \times [0, \lambda_1],$$

which satisfies the hypothesis (2) of Theorem 1.

(b) Suppose that $minf_\infty := C_2 \in (\frac{B}{M}, \infty]$. Taking $\epsilon = C_2 - \frac{B}{M} > 0$, there exists $\eta_1 > 0$ (η_1 can be chosen large arbitrarily) such that

$$\min_{t\in[0,1]} \frac{f(t,u)}{u} \geq -\epsilon + C_2 = \frac{B}{M} \ on \ [M\eta_1, \infty).$$

Hence,

$$f(t,u) \geq \frac{B}{M}u \geq \frac{B}{M}M\eta_1 = B\eta_1 \ on \ [\frac{1}{4}, \frac{3}{4}] \times [M\eta_1, \eta_1] \subseteq [0,1] \times [M\eta_1, \infty),$$

which satisfies the hypothesis (3) of Theorem 1.

(c) Suppose that $minf_0 := C_3 \in (\frac{B}{M}, \infty]$. Taking $\epsilon = C_3 - \frac{B}{M} > 0$, there exists $\eta_2 > 0$ (η_2 can be chosen small arbitrarily) such that

$$\min_{t\in[0,1]} \frac{f(t,u)}{u} \geq -\epsilon + C_3 = \frac{B}{M} \ on \ (0, \eta_2].$$

Hence,

$$f(t,u) \geq \frac{B}{M}u \geq \frac{B}{M}M\eta_2 = B\eta_2 \ on \ [\frac{1}{4}, \frac{3}{4}] \times [M\eta_2, \eta_2] \subseteq [0,1] \times [0, \eta_2],$$

which satisfies the hypothesis (3) of Theorem 1.

(d) Suppose that $maxf_\infty := C_4 \in [0, A)$. Taking $\epsilon = A - C_4 > 0$, there exists $\delta > 0$ (δ can be chosen large arbitrarily) such that

$$\max_{t\in[0,1]} \frac{f(t,u)}{u} \leq \epsilon + C_4 = A \ on \ [\delta, \infty). \tag{4}$$

Hence, we have the following two cases:

Case (I). Assume that $max_{t\in[0,1]}f(t,u)$ is bounded, say,

$$f(t,u) \leq L \ on \ [0,1] \times [0, \infty).$$

Taking $\lambda_2 = \frac{L}{A}$ (since L can be chosen large arbitrarily, λ_2 can be chosen large arbitrarily, too),

$$f(t,u) \leq L = A\lambda_2 \ on \ [0,1] \times [0, \lambda_2] \subseteq [0,1] \times [0, \infty).$$

Case (II). Assume that $max_{t\in[0,1]}f(t,u)$ is unbounded, hence, there exists $\lambda_2 \geq \delta$ (λ_2 can be chosen large arbitrarily) and $t_0 \in [0,1]$ such that

$$f(t,u) \leq f(t_0, \lambda_2) \ on \ [0,1] \times [0, \lambda_2].$$

It follows from $\lambda_2 \geq \delta$ and (4) that

$$f(t,u) \leq f(t_0, \lambda_2) \leq A\lambda_2 \ on \ [0,1] \times [0, \lambda_2].$$

By Cases (I) and (II), the hypothesis (2) of Theorem 1 is satisfied.

It follows from Remark 2 that the following corollaries hold. They are generalizations of Theorems A, B and C, respectively.

Corollary 3. *Let A and B be defined as in Remark 2. Then, (BVP) has at least one positive solution in the case*
(1) $max f_0 = C_1 \in [0, A)$ *and* $min f_\infty = C_2 \in (\frac{B}{M}, \infty]$, *or*
(2) $min f_0 = C_3 \in (\frac{B}{M}, \infty]$ *and* $max f_\infty = C_4 \in [0, A)$.

Proof. It follows from Remark 2 and Theorem 1 that the desired result holds, immediately.

Corollary 4. *Let A and B be defined as in Remark 2. Then, (BVP) has at least two positive solutions u_1 and u_2 such that*

$$0 < ||u_1|| < \lambda^* < ||u_2||,$$

if the following hypotheses hold:
(H_1) $min f_\infty = C_2,\ min f_0 = C_3 \in (\frac{B}{M}, \infty]$,
(H_2) *there exists $\lambda^* > 0$ such that*

$$f(t, u) \le A\lambda^* \ on\ [0, 1] \times [0, \lambda^*].$$

Proof. It follows from Remark 2 that there exist two real numbers η_1 and η_2 satisfying

$$0 < \eta_2 < \lambda^* < \eta_1,$$

$$f(t, u) \ge B\eta_1 \text{ on } [\frac{1}{4}, \frac{3}{4}] \times [M\eta_1, \eta_1],$$

and

$$f(t, u) \ge B\eta_2 \text{ on } [\frac{1}{4}, \frac{3}{4}] \times [M\eta_2, \eta_2].$$

Hence, by Theorem 1, we see that (BVP) has two positive solutions u_1 and u_2 such that

$$\eta_2 < ||u_1|| < \lambda^* < ||u_2|| < \eta_1.$$

Thus, we complete the proof.

Corollary 5. *Let A and B be defined as in Remark 2. Then, (BVP) has at least two positive solutions u_1 and u_2 such that*

$$0 < ||u_1|| < \eta^* < ||u_2||,$$

if the following hypotheses hold:
(H_3) $max f_0 = C_1,\ max f_\infty = C_4 \in [0, A)$,
(H_4) *there exists $\eta^* > 0$ such that*

$$f(t, u) \ge B\eta^* \ on\ [\frac{1}{4}, \frac{3}{4}] \times [M\eta^*, \eta^*].$$

Proof. It follows from Remark 2 that there exist two real numbers λ_1 and λ_2 satisfying

$$0 < \lambda_1 < \eta^* < \lambda_2,$$

$$f(t, u) \le A\lambda_1 \text{ on } [0, 1] \times [0, \lambda_1],$$

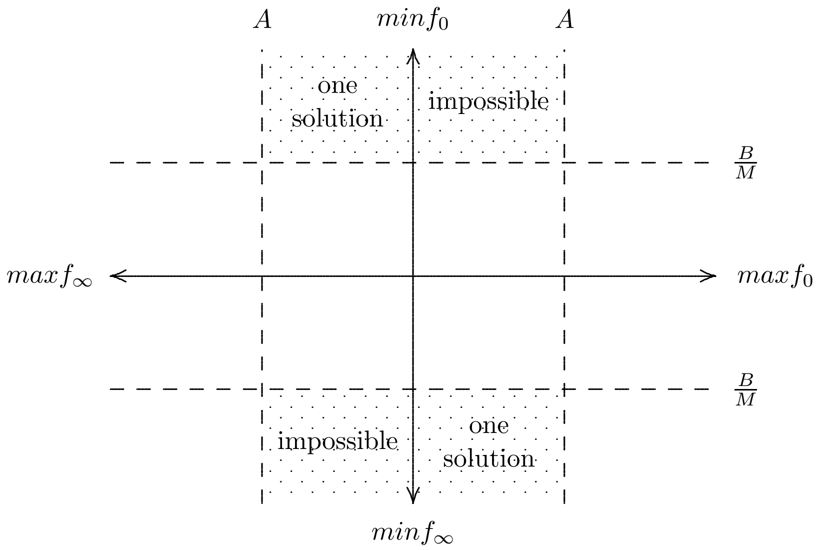

FIGURE 2.1

and

$$f(t,u) \le A\lambda_2 \text{ on } [0,1] \times [0,\lambda_2].$$

Hence, by Theorem 1, we see that (BVP) has two positive solutions u_1 and u_2 such that

$$\lambda_1 < ||u_1|| < \eta^* < ||u_2|| < \lambda_2.$$

Thus, we complete the proof.

Remark 6. There are many functions $f(t,u)$ which do not satisfy "$maxf_0,\ minf_0,\ maxf_\infty, minf_\infty \in \{0,\infty\}$", for example, $f(t,u) := \frac{e^u-1}{1+t^2}$ ($maxf_0 = 1$ and $minf_0 = \frac{1}{2}$), $f(t,u) := (t+1)sinhu$ ($maxf_0 = 2$ and $minf_0 = 1$), $f(t,u) := u+t^2e^{-u}$ ($maxf_0 = \infty,\ minf_0 = maxf_\infty = minf_\infty = 1$).

Remark 7. It follows from Corollaries 3, 4, 5 that we obtain Figure 2.1.

Remark 8. If we consider the radial solutions to the boundary value problem of the following semilinear elliptic equation of the form

$$\text{(BVP.1)} \qquad \begin{cases} \Delta u + g(|x|)f(u) = 0\ , 0 < R_1 < |x| < R_2,\ N \ge 2, \\ (BC.1) \begin{cases} \alpha u(x) - \beta \dfrac{\partial u}{\partial n}(x) = 0,\ |x| = R_1, \\ \gamma u(x) - \delta \dfrac{\partial u}{\partial n}(x) = 0,\ |x| = R_2, \end{cases} \end{cases}$$

then (BVP .1) can be reduced to the following boundary problem:

$$\text{(BVP.2)} \qquad \begin{cases} u''(r) + \dfrac{N-1}{r}u'(r) + g(r)f(u(r)) = 0 \ , 0 < R_1 < r < R_2, \\ (BC.2) \begin{cases} \alpha u(R_1) - \beta \dfrac{\partial u}{\partial n}(R_1) = 0, \\ \gamma u(R_2) - \delta \dfrac{\partial u}{\partial n}(R_2) = 0. \end{cases} \end{cases}$$

Setting $s = -\int_r^{R_2}(1/t^{N-1})dt, v(s) = u(r(s)), m = -\int_{R_1}^{R_2}(1/t^{N-1})dt$, (BVP .2) is equivalent to

$$\text{(BVP.3)} \qquad \begin{cases} v''(s) + r^{2(N-1)}g(r(s))f(v(s)) = 0, \ m < s < 0, \\ (BC.3) \begin{cases} \alpha v(m) + \beta {R_1}^{1-N}v'(m) = 0, \\ \gamma v(0) - \delta {R_2}^{1-N}v'(0) = 0. \end{cases} \end{cases}$$

Furthermore, if $t = \frac{(m-s)}{m}, z(t) = v(s)$, then (BVP .3) can be transformed into

$$\text{(BVP.4)} \qquad \begin{cases} z''(t) + m^2 r^{2(N-1)}[m(1-t)]g[r(m(1-t))]f(z(t)) = 0, \ 0 < t < 1, \\ (BC.4) \begin{cases} \alpha z(0) - \beta \dfrac{{R_1}^{1-N}}{m}z'(0) = 0, \\ \gamma z(1) + \delta \dfrac{{R_1}^{1-N}}{m}z'(1) = 0. \end{cases} \end{cases}$$

Hence, we can apply Theorem 1 and Corollaries 3, 4, 5 to (BVP .4), and thereby study the existence of positive radial solutions for (BVP .1).

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