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BANACH SPACES IN WHICH EVERY p-WEAKLY SUMMABLE SEQUENCE LIES IN THE RANGE OF A VECTOR MEASURE

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Abstract. Let X be a Banach space. For $1 < p < +\infty$ we prove that the identity map I_X is $(1,1,p)$-summing if and only if the operator $x^* \in X^* \to \sum \langle x_n, x^* \rangle e_n \in l_q$ is nuclear for every unconditionally summable sequence (x_n) in X, where q is the conjugate number for p. Using this result we find a characterization of Banach spaces X in which every p-weakly summable sequence lies inside the range of an X^{**}-valued measure (equivalently, every p-weakly summable sequence (x_n) in X, satisfying that the operator $(\alpha_n) \in l_q \to \sum \alpha_n x_n \in X$ is compact, lies in the range of an X-valued measure) with bounded variation. They are those Banach spaces such that the identity operator I_{X^*} is $(1,1,p)$-summing.

Let X be a Banach space. In [AD] it is proved that every sequence (x_n) in X satisfying $\sum_n |\langle x_n, x^* \rangle|^2 < +\infty$ for all $x^* \in X^*$ lies inside the range of an X-valued measure. Nevertheless, they show a sequence which does not lie in the range of an X-valued measure with bounded variation. In [PR] the authors proved that X is finite dimensional if and only if every nul sequence (equivalently, everyt compact set) in X lies inside the range of an X-valued measure having bounded variation. The purpose of this paper is to characterize, given a real number $p \in (1, +\infty)$, the Banach spaces in which every p-weakly summable sequence lies inside the range of an X^{**}-valued measure with bounded variation. We start by explaining some basic notation used in this paper. In general, our operator and vector measure terminology and notation follow [Ps] and [DU]. We only consider real Banach spaces. If X is a such space, B_X will denote its closed unit ball. The phrase "range of an X-valued measure" always means a set of the form $rg(F) = \{F(A) : A \in \Sigma\}$, where Σ is a σ-algebra of subsets of a set Ω and $F\colon \Sigma \to X$ is countably additive. Given $p \geq 1$, $l_w^p(X)$ will denote the vector space of all sequences (x_n) in X such that $\sum_{n=1}^{\infty} |\langle x_n, x^* \rangle|^p < +\infty$ for all $x^* \in X^*$. It is easy to see that if $(x_n) \in l_w^p(X)$, then

$$\varepsilon_p((x_n)) = \sup \left\{ \left(\sum_{n=1}^{\infty} |\langle x_n, x^* \rangle|^p \right)^{1/p} : x^* \in B_{X^*} \right\} < +\infty$$

and $(l_w^p(X), \varepsilon_p)$ is itself a Banach space.

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If $\hat{x} = (x_n) \in l_w^p(X)$ and P is a finite subset of $\mathbb{N}$, $\hat{x}(P) = (x_n(P))$ is the sequence defined by

$$x_n(P) = \begin{cases} x_n & \text{if } n \in P, \\ 0 & \text{if } n \notin P \end{cases}$$

for all $n \in \mathbb{N}$. $l_u^p(X)$ will denote the subspace of $l_w^p(X)$ consisting of the sequences $\hat{x} = (x_n)$ such that the net $(\hat{x}(P))_{P \in \mathcal{F}(\mathbb{N})}$ converges to (x_n) in $l_w^p(X)$, where $\mathcal{F}(\mathbb{N})$ is the set of all finite subsets of $\mathbb{N}$. Recall that $l_u^1(X)$ is formed by the unconditionally summable sequences in X. We need the following propositions that list some privileges that membership in $l_w^p(X)$ or in $l_u^p(X)$ entail.

Proposition A. *Let $p > 1$ and X be a Banach space. The following statements are equivalent:*

(i) $(x_n) \in l_w^p(X)$.

(ii) *The series $\sum_{n=1}^\infty \alpha_n x_n$ converges unconditionally for every sequence $(\alpha_n) \in l_q$.*

(iii) *The map $(\alpha_n) \in l_q \to \sum_{n=1}^\infty \alpha_n x_n \in X$ defines a bounded operator.*

Proposition B. *Let $p \geq 1$. If $(x_n) \in l_u^p(X)$, then the operator $(\alpha_n) \in l_q \to \sum_{n=1}^\infty \alpha_n x_n \in X$ is compact.*

1. Main result

Throughout this section X will be a Banach space and $p \in (1, +\infty)$.

Theorem 1. *The following statements are equivalent*:

(i) *For every unconditionally sequence (x_n) in X the operator $x^* \in X^* \to \sum_{n=1}^\infty \langle x_n, x^* \rangle e_n \in l_q$ is nuclear.*

(ii) *There exists a constant $c > 0$ such that*

$$\begin{aligned} \left| \sum_{k=1}^n \langle x_k, x_k^* \rangle \right| \leq & c \sup \left\{ \sum_{k=1}^n |\langle x_k, x^* \rangle| : \|x^*\| \leq 1 \right\} \\ & \cdot \sup \left\{ \left(\sum_{k=1}^n |\langle x, x_k^* \rangle|^p \right)^{1/p} : \|x\| \leq 1 \right\} \end{aligned} \tag{1}$$

for all $\{x_1, \ldots, x_n\} \subset X$ and $\{x_1^*, \ldots, x_n^*\} \subset X^*$.

Proof. (i)⇒(ii) We consider the linear map

$$\hat{x} = (x_n) \in l_u^1(X) \to T_{\hat{x}} \in \mathcal{N}(X^*, l_q)$$

defined by $T_{\hat{x}}(x^*) = \sum \langle x_n, x^* \rangle e_n$ for all $x^* \in X^*$($\{e_n : n \in \mathbb{N}\}$ is the unit basis of l_q). It has closed graph, so there exists a positive constant c so that

$$\nu \left(\sum_{n=1}^\infty x_n \otimes e_n : X^* \to l_q \right) \leq c \sup \left\{ \sum_{n=1}^\infty |\langle x_n, x^* \rangle| : \|x^*\| \leq 1 \right\}$$

for all $(x_n) \in l_u^1(X)$. By a standard argument we obtain

$$\nu \left(\sum_{n=1}^m x_n \otimes e_n : X^* \to l_q^m \right) \leq c \sup \left\{ \sum_{n=1}^m |\langle x_n, x^* \rangle| : \|x^*\| \leq 1 \right\} \tag{2}$$

for all $m \in \mathbb{N}$ and $\{x_1, \ldots, x_m\} \subset X$.

Now, given $\{x_1, \ldots, x_m\} \subset X$ and $\{x_1^*, \ldots, x_m^*\} \subset X^*$, define two operators $v\colon l_q^m \to X^*$ and $u\colon X^* \to l_q^m$ by

$$v(\alpha_i) = \sum_{i=1}^{m} \alpha_i x_i^* \quad \text{and} \quad u(x^*) = \sum_{i=1}^{m} \langle x_i, x^* \rangle e_i.$$

Note that $tr(u \circ v) = \sum_{i=1}^{m} \langle x_i, x_i^* \rangle$, so we have

$$\begin{aligned} \left| \sum_{i=1}^{m} \langle x_i, x_i^* \rangle \right| &\le \nu(u \circ v) \le \nu(u) \|v\| \\ &= \nu(u) \sup \left\{ \left(\sum_{i=1}^{m} |\langle x, x_i^* \rangle|^p \right)^{1/p} : \|x\| \le 1 \right\} \end{aligned}$$

and using (2) we obtain

$$\begin{aligned} \left| \sum_{k=1}^{n} \langle x_k, x_k^* \rangle \right| \le & c \sup \left\{ \sum_{k=1}^{n} |\langle x_k, x^* \rangle| \colon \|x^*\| \le 1 \right\} \\ & \cdot \sup \left\{ \left(\sum_{k=1}^{n} |\langle x, x_k^* \rangle|^p \right)^{1/p} : \|x\| \le 1 \right\}. \end{aligned}$$

(ii)$\Rightarrow$(i) Given $(x_n^*) \in l_w^p(X^*)$, we define a linear form ϕ by

$$(x_n) \in l_u^1(X) \to \sum_{n=1}^{\infty} \langle x_n, x_n^* \rangle \in \mathbb{R}.$$

By (ii) $\phi \in l_u^1(X)^*$ and $\|\phi\| \le c \|(x_n^*)\|_p$. So, the linear map $x \in X \to (\langle x, x_n^* \rangle) \in l_\infty$ is integral (see [DU, p. 232]). Equivalently, $x \in X \to (\langle x, x_n^* \rangle) \in c_0$ is integral. Then, so is its adjoint $(\alpha_n) \in l_1 \to \sum \alpha_n x_n^* \in X^*$. Therefore, the linear map ψ defined by

$$(x_n^*) \in l_w^p(X^*) \to \sum_{n=1}^{\infty} e_n \otimes x_n^* \in I(l_1, X^*)$$

is well defined and $\|\psi\| \le c$. Now denote the restriction map of ψ to $l_u^p(X^*)$ by ψ_u. Since $\hat{x}^* = \lim_P \hat{x}^*(P)$ for all $\hat{x}^* \in l_u^p(X^*)$, it follows that ψ_u takes all its values in $\mathcal{N}(l_1, X^*)$ (note that $\mathcal{N}(l_1, X^*)$ is a subspace of $I(l_1, X^*)$ because $(l_1)^*$ has the metric approximation property). If we also denote the operator

$$(x_n^*) \in l_u^p(X^*) \to \sum_{n=1}^{\infty} e_n \otimes x_n^* \in \mathcal{N}(l_1, X^*)$$

by ψ_u, then $(\psi_u)^*$ maps $B(X^*, l_1)$ into $l_u^p(X^*)^*$. In particular, for all $(x_n) \in l_u^1(X)$, the operator $x^* \in X^* \to \sum \langle x_n, x^* \rangle e_n \in l_q$ is integral. This completes the proof because nuclear and integral operators into a reflexive space are the same.

Recall that an operator $T\colon X \to Y$ is called (r, q, p)-summing if there is a constant $c \geq 0$ such that

$$\left(\sum_{k=1}^{n} |\langle Tx_k, y_k^*\rangle|^r\right)^{1/r} \leq c \sup_{x^* \in B_{X^*}} \left(\sum_{k=1}^{n} |\langle x_k, x^*\rangle|^q\right)^{1/q} \cdot \sup_{y \in B_Y} \left(\sum_{k=1}^{n} |\langle y, y_k^*\rangle|^p\right)^{1/p}$$

for all finite families of elements $x_1, \dots, x_n \in X$ and functionals $y_1^*, \dots, y_n^* \in Y^*$. So, Theorem 1 gives us a characterization of Banach spaces X for which I_X is $(1,1,p)$-summing.

An operator $T\colon X \to Y$ is (p, q)-summing if there is a constant $c \geq 0$ such that

$$\left(\sum_{k=1}^{n} \|Tx_k\|^p\right)^{1/p} \leq c \sup\left\{\left(\sum_{k=1}^{n} |\langle x_k, x^*\rangle|^q\right)^{1/q} : \|x^*\| \leq 1\right\}$$

for all finite subset $\{x_1, \dots, x_n\}$ of X.

Following [Ps] we will say that a Banach space X satisfies Grothendieck's Theorem (in short, X is a G.T. space) if $B(X, l_2) = \Pi_1(X, l_2)$. The next proposition shows the relationship between the Banach spaces X for which I_X is absolutely $(1,1,p)$-summing and the above classes. $\square$

Proposition 2. (i) *If X is a G.T. space, then I_X is $(1,1,p)$-summing for $1 < p \leq 2$.*

(ii) *If $1 < p < +\infty$ and $T \in B(X, Y)$, then*

$$T \text{ is } (1,1,p)\text{-summing } \Rightarrow T \text{ is } (q,1)\text{-summing.}$$

Proof. If $(x_n) \in l_u^1(X)$, then the operator $T\colon x^* \in X^* \to \sum \langle x_n, x^*\rangle e_n \in l_q$ admits the following factorization

$$\begin{tikzcd} X^* \arrow[rr, "T"] \arrow[dr, "J"'] & & l_q \\ & l_1 \arrow[ur, "I"'] & \end{tikzcd}$$

where $I\colon l_1 \to l_q$ is the natural inclusion and $J\colon X^* \to l_1$ is defined by $Jx^* = (\langle x_n, x^*\rangle)$ for all $x^* \in X^*$. I is obviously 1-summing and J is 2-summing by [Ps, 6.6.2], so T is nuclear.

(ii) If T is $(1,1,p)$-summing there is a constant $c \geq 0$ such that

$$\left|\sum_{i=1}^{n} \langle Tx_i, y_i^*\rangle\right| \leq c \sup\left\{\sum_{i=1}^{n} |\langle x_i, x^*\rangle| \colon \|x^*\| \leq 1\right\} \cdot \varepsilon_p((y_i^*)_{i=1}^n) \tag{3}$$

for all $\{x_1, \dots, x_n\} \subset X$ and $\{y_1^*, \dots, y_n^*\} \subset Y^*$. Given $\{x_1, \dots, x_n\} \subset X$, choose $y_i^* \in B_{Y^*}$ so that $|\langle Tx_i, y_i^*\rangle| = \|Tx_i\|$ for each $i \leq n$. By (3) we have

$$\begin{aligned} \sum_{i=1}^{n} |\alpha_i| \, \|Tx_i\| = \sum_{i=1}^{n} |\alpha_i| \, |\langle Tx_i, y_i^*\rangle| &\leq c\varepsilon_1((x_i)_{i=1}^n)\varepsilon_p((\alpha_i y_i^*)_{i=1}^n) \\ &\leq c\varepsilon_1((x_i)_{i=1}^n)\|(\alpha_i)\|_p \end{aligned}$$

for all $(\alpha_i) \in l_p^n$. Then

$$\left(\sum_{i=1}^{n} \|Tx_i\|^q\right)^{1/q} \leq c \sup\left\{\sum_{i=1}^{n} |\langle x_i, x^*\rangle| \colon \|x^*\| \leq 1\right\}$$

for all $\{x_1, \ldots, x_n\} \subset X$.

In [P, 17.1.6], Pietsch formulated the following conjecture: for $1/r > 1/q + 1/p - 1/2$, I_X is (r, q, p)-summing if and only if X is finite dimensional. The conjecture is true for $q = r = 1$. Certainly, let $p > 2$. If I_X is $(1, 1, p)$-summing, it follows from Proposition 2(ii) that I_x is $(q, 1)$-summing. By [P, Theorem 17.2.7.] X has to be finite dimensional since $q < 2$. □

2. Sequences in the range of a vector measure with bounded variation

In this section we use Theorem 1 to obtain a characterization of Banach spaces X for which every p-weakly summable sequence (x_n) in X lies inside the range of an X^{**}-valued measure having bounded variation. The following lemma collects some elementary facts we need (see [Pi 2]).

Lemma 3. *Let X be a Banach space. If $\hat{x} = (x_n)$ is a bounded sequence in X, we consider the linear operator $T_{\hat{x}} \colon l_1 \to X$ defined by $T_{\hat{x}}(\alpha_n) = \sum \alpha_n x_n$ for all $(\alpha_n) \in l_1$. Then the following assertions hold:*

(i) *(x_n) lies inside the range of an X^{**}-valued measure with bounded variation iff $T_{\hat{x}}$ is integral.*

(ii) *(x_n) lies inside the range of an X-valued measure with bounded variation iff $T_{\hat{x}}$ is Pietsch-integral.*

Now we are ready to face our problem.

Theorem 4. *Let X a Banach space and $1 < p < +\infty$. The following statements are equivalent:*

(i) *Every sequence $(x_n) \in l_w^p(X)$ lies inside the range of an X^{**}-valued measure with bounded variation.*

(ii) *Every sequence $(x_n) \in l_w^p(X)$, satisfying that the operator $(\alpha_n) \in l_q \to \sum \alpha_n x_n \in X$ is compact, lies inside the range of an X-valued measure with bounded variation.*

(iii) *Every sequence $(x_n) \in l_u^p(X)$ lies inside the range of an X-valued measure with bounded variation.*

(iv) *I_{X^*} is $(1, 1, p)$-summing.*

Proof. (i)⇒(ii) By Lemma 3, we can consider the linear map

$$\phi \colon \hat{x} \in l_w^p(X) \to T_{\hat{x}} \in I(l_1, X).$$

It is continuous because its graph is closed. Since $(l_q)^*$ has the approximation property, for each sequence $\hat{x} = (x_n) \in l_w^p(X)$ satisfying that the operator $(\alpha_n) \in l_q \to \sum \alpha_n x_n \in X$ is compact, there exists a sequence $(\hat{y}_k)$ in $l_w^p(X)$ such that $\hat{x} = \lim_{k \to +\infty} \hat{y}_k$ in $l_w^p(X)$ and each sequence $\hat{y}_k$ is finite dimensional. Then $\phi(\hat{y}_k)$ belongs to $\mathcal{N}(l_1, X)$ for all $k \in \mathbb{N}$. By continuity, so does $\phi(\hat{x})$ (recall that $\mathcal{N}(l_1, X)$ is a closed subspace of $I(l_1, X)$). Hence, we have proved that such a sequence

$(x_n) \in l_w^p(X)$ actually lies inside a sum of segments

$$\sum[-z_n, z_n] = \left\{\sum \alpha_n z_n \colon (\alpha_n) \in l_\infty, \|(\alpha_n)\|_\infty \leq 1\right\}$$

where $\sum \|z_n\| < +\infty$ (see [Pi 1]).

(ii)⇒(iii) It is obvious because the operator

$$(\alpha_n) \in l_q \to \sum \alpha_n x_n \in X$$

is compact for each sequence $(x_n) \in l_n^p(X)$.

(iii)⇒(iv) Now we consider the linear map

$$\psi \colon \hat{x} \in l_u^p(X) \to T_{\hat{x}} \in I(l_1, X).$$

Having a closed graph, ψ is continuous. Since $\hat{x} = \lim_P \hat{x}(P)$ for every sequence $\hat{x} \in l_u^p(X)$, it follows that ψ takes its values into $\mathcal{N}(l_1, X)$. As mentioned earlier, using the trace duality it is easy to prove that ψ^* takes every $(x_n^*) \in l_u^1(X^*)$ in $\sum x_n^* \otimes e_n \in I(X, l_q)$. Again the reflexivity of l_q yields (iv).

(iv)⇒(i) In the same way as in the proof of Theorem 1 we can prove that the linear map

$$(x_n^{**}) \in l_w^p(X^{**}) \to \sum_{n=1}^{\infty} e_n \otimes x_n^{**} \in I(l_1, X^{**})$$

is well defined and continuous. In particular, it follows from the above lemma that every $(x_n) \in l_w^p(X)$ lies inside the range of an X^{**}-valued measure of bounded variation.

In view of Theorem 4 and the notes at the end of section 1, for $p > 2$, only finite-dimensional Banach spaces X have the property that every sequence $(x_n) \in l_w^p(X)$ lies inside the range of an X-valued measure having bounded variation. That is why from now on we only consider $p \in [1, 2]$. □

3. Final notes and examples

It is well known that every sequence $(x_n) \in l_w^1(X)$ lies inside the range of an X-valued measure with bounded variation. In fact, the vector measure F defined by

$$F(A) = 2\sum_{n=1}^{\infty} \left(\int_A r_n(t)dt\right) x_n,$$

for any Lebesgue measurable subset A of $[0, 1]$, has bounded variation whenever $(x_n) \in l_w^1(X)$. In [AD] it is proved that $\{x_n \colon n \in \mathbb{N}\} \subset rg(F)$. Then, given an infinite-dimensional Banach space X, we can consider the set $r(X)$ formed by all real numbers $r \in [1, 2]$ such that every sequence $(x_n) \in l_w^r(X)$ lies inside the range of an X-valued measure having bounded variation. Then $r(X)$ is an interval whose bounds are 1 and $\sup(r(X))$. In the following we will determine the set $r(X)$ for some classical Banach spaces.

(i) $r(X) = [1, 2]$ for every Banach space X satisfying:
  (a) X^* is a G.T. space,
  (b) X is a dual space.

By Proposition 2(i), I_{X^*} is $(1,1,r)$-summing for all $r \in (1,2]$. Then Theorem 4 implies that $r \in r(X)$ for all $r \in [1,2]$.

In particular, if μ is a σ-finite positive measure, $r(L^\infty(\mu)) = [1,2]$.

(ii) $r(l_p) = \{1\}$ for $1 \le p < +\infty$.

I_p will denote the identity map $l_p \to l_p$. First, we consider the case $p = 1$. If (e_n^*) denotes the unit basis of $l_\infty = (l_1)^*$, then $(e_n^*) \in l_w^1(l_\infty)$. Since $\sum \|e_n^*\|^s = \infty$ for $s \ge 1$ it follows that I_∞ cannot be $(s,1)$-summing for $s \ge 1$. So, Proposition 2(ii) tells us that I_∞ is not $(1,1,r)$-summing for $r > 1$. By Theorem 4, $r(l_1) = \{1\}$.

Now suppose $1 < p < +\infty$.

Claim. $r(l_p) \cap (1,q) = \emptyset$. Let $r \in r(l_p) \cap (1,q)$. Theorems 1 and 4 assure us that there is a constant $c \ge 0$ such that

$$\sum_{i=1}^{n} |\langle x_i, x_i^* \rangle| \le c\varepsilon_1((x_i^*)_{i=1}^n)\varepsilon_r((x_i)_{i=1}^n) \tag{4}$$

for all $\{x_1, \ldots, x_n\} \subset l_p$ and $\{x_1^*, \ldots, x_n^*\} \subset l_q$. Given $(\alpha_n) \in l_q$ and $(\beta_n) \in l_u$ with $u = rq(q-r)^{-1}$, define $x_n^* = \alpha_n e_n^*$ and $x_n = \beta_n e_n$ for all $n \in \mathbb{N}$. From (4) we get

$$\sum_{i=1}^{m} |\alpha_i|\,|\beta_i| \le c\varepsilon_1((\alpha_i e_i^*)_{i=1}^m)\varepsilon_r((\beta_i e_i)_{i=1}^m)$$

for all $m \in \mathbb{N}$. Applying Holder's inequality we obtain

$$\varepsilon_1((\alpha_i e_i^*)_{i=1}^m) \le \|(\alpha_n)\|_q \varepsilon_p((e_n^*)) = \|(\alpha_n)\|_q$$

and

$$\varepsilon_r((\beta_i e_i)_{i=1}^m) \le \varepsilon_q((e_n))\|(\beta_n)\|_u = \|(\beta_n)\|_u.$$

Then, for all $m \in \mathbb{N}$ and $(\alpha_n) \in l_q$, we have

$$\sum_{i=1}^{m} |\alpha_i|\,|\beta_i| \le c\|(\alpha_n)\|_q \|(\beta_n)\|_u.$$

This implies that $(\beta_n) \in l_p = (l_q)^*$. Choosing $(\beta_n) \in l_u \backslash l_p$ we fall in a contradiction since $rq(q-r)^{-1} > p$.

With our claim established we already have proved that $r(l_p) = \{1\}$ for $p < 2$. Finally, we are going to show that $r(l_p) \cap [q,2] = \emptyset$ for $p \ge 2$. This is the easy part. Certainly, the identity map $l_1 \to l_p$ is not nuclear, hence Lemma 3(ii) allows us to conclude that the sequence (e_n) does not lie inside the range of an l_p-valued measure of bounded variation. Nevertheless, $(e_n) \in l_w^r(l_p)$ for all $r \ge q$. Thus $[q,2] \cap r(l_p) = \emptyset$.

(iii) $r(X) = \{1\}$ for all infinite-dimensional $\mathcal{L}_p$-space X with $1 \le p < +\infty$.

By [LP, Proposition 7.3], X has a complemented subspace H isomorphic to l_p. Then $r(X) \subset r(H) = r(l_p) = \{1\}$.

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