

CHARACTERIZATIONS OF THE GELFAND-SHILOV SPACES VIA FOURIER TRANSFORMS

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ABSTRACT. We give symmetric characterizations, with respect to the Fourier transformation, of the Gelfand-Shilov spaces of (generalized) type S and type W . These results explain more clearly the invariance of these spaces under the Fourier transformations.

1. INTRODUCTION

The purpose of this paper is to give new characterizations of the Gelfand-Shilov spaces of (generalized) type S and type W by means of the Fourier transformation. Gelfand and Shilov introduced the above spaces in [6] to study the uniqueness of the Cauchy problems of partial differential equations. In [1] the Schwartz space \mathcal{S} is characterized by the estimates

$$(1.1) \quad \sup_x |x^\alpha \varphi(x)| < \infty, \quad \sup_x |\partial^\beta \varphi(x)| < \infty,$$

or by the estimates

$$(1.2) \quad \sup_x |x^\alpha \varphi(x)| < \infty, \quad \sup_\xi |\xi^\beta \hat{\varphi}(\xi)| < \infty,$$

where the Fourier transform $\hat{\varphi}$ is defined by $\hat{\varphi}(\xi) = \int e^{-ix \cdot \xi} \varphi(x) dx$.

In addition, the Sato space \mathcal{F} of test functions for the Fourier hyperfunctions is characterized by the estimates

$$(1.3) \quad \sup_x |\varphi(x)| \exp k|x| < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp h|\xi| < \infty$$

for some $h, k > 0$ in [2].

Generalizing the above results in a similar manner we give more symmetric characterizations of the Gelfand-Shilov spaces in terms of the Fourier transformations as follows.

I. For the space S_r^s the following statements are equivalent.

- (1) $\varphi \in S_r^s$;
- (2) $\sup_x |\varphi(x)| \exp k|x|^{1/r} < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp h|\xi|^{1/s} < \infty$ for some $h, k > 0$.

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II. For the space $S_{M_p}^{N_p}$ the following statements are equivalent.

- (1) $\varphi \in S_{M_p}^{N_p}$;
- (2) $\sup_x |\varphi(x)| \exp M(ax) < \infty$, $\sup_\xi |\hat{\varphi}(\xi)| \exp N(b\xi) < \infty$ for some $a, b > 0$, where $M(x)$ and $N(\xi)$ are associated functions of M_p and N_p , respectively (see (2.1) for the definition).

III. For the space W_M^Ω the following statements are equivalent.

- (1) $\varphi \in W_M^\Omega$;
- (2) $\sup_x |\varphi(x)| \exp M(a|x|) < \infty$, $\sup_\xi |\hat{\varphi}(\xi)| \exp \Omega^*(b|\xi|) < \infty$ for some $a, b > 0$, where Ω^* is the Young conjugate of Ω (see Definition 3.3).

2. CHARACTERIZATION OF GELFAND–SHILOV SPACES OF (GENERALIZED) TYPE S

In this section we characterize the Gelfand–Shilov spaces of type S and generalized type S in a more symmetric way by means of the Fourier transformation, which are generalizations of (1.2) and (1.3).

We first introduce the *Gelfand–Shilov spaces of generalized type S* and *type S*. Let $M_p, p = 0, 1, 2, \dots$, be a sequence of positive numbers. We impose the following conditions on M_p :

- (M.1) (logarithmic convexity) $M_p^2 \leq M_{p-1}M_{p+1}$, $p = 1, 2, \dots$;
- (M.2) (stability under differential operators) there are constants A and H such that $M_{p+q} \leq AH^{p+q}M_pM_q$, $p, q = 0, 1, 2, \dots$.

Definition 2.1. Let M_p and $N_p, p = 0, 1, 2, \dots$, be sequences of positive numbers. Then the Gelfand–Shilov spaces S_{M_p}, S^{N_p} and $S_{M_p}^{N_p}$ consist of all infinitely differentiable functions $\varphi(x)$ on \mathbb{R}^n satisfying the following estimates, respectively,

$$\begin{aligned} \sup_x |x^\alpha \partial^\beta \varphi(x)| &\leq C_\beta A^{|\alpha|} M_{|\alpha|}, \\ \sup_x |x^\alpha \partial^\beta \varphi(x)| &\leq C_\alpha B^{|\beta|} N_{|\beta|}, \\ \sup_x |x^\alpha \partial^\beta \varphi(x)| &\leq CA^{|\alpha|} B^{|\beta|} M_{|\alpha|} N_{|\beta|} \end{aligned}$$

for some positive constants A and B for all multi-indices α and β .

Remark. In particular, if $M_p = p!^r$ and $N_p = p!^s$, then we denote the spaces S_{M_p}, S^{N_p} and $S_{M_p}^{N_p}$ by S_r, S^s and S_r^s , respectively, and call these spaces the *Gelfand–Shilov spaces of type S*.

Definition 2.2. Let M_p and N_p be sequences of positive numbers satisfying (M.1). Then we write $M_p \subset N_p$ ($M_p \prec N_p$, respectively) if there are constants $L, C > 0$ (for any $L > 0$ there is a constant $C > 0$, respectively) such that $M_p \leq CL^p N_p$, $p = 0, 1, 2, \dots$.

Also, M_p and N_p are said to be equivalent if $M_p \subset N_p$ and $M_p \supset N_p$ hold.

Theorem 2.3. *If M_p and N_p satisfy (M.1) and (M.2) and $M_p N_p \supset p!$, then the following conditions are equivalent.*

- (i) $\varphi \in S_{M_p}^{N_p}$.
- (ii) *There exist positive constants A, B and C such that*

$$\sup_x |x^\alpha \varphi(x)| \leq CA^{|\alpha|} M_{|\alpha|}, \quad \sup_x |\partial^\beta \varphi(x)| \leq CB^{|\beta|} N_{|\beta|}$$

for all multi-indices α and β .

(iii) *There exist positive constants A, B and C such that*

$$\sup_x |x^\alpha \varphi(x)| \leq CA^{|\alpha|} M_{|\alpha|}, \quad \sup_\xi |\xi^\beta \hat{\varphi}(\xi)| \leq CB^{|\beta|} N_{|\beta|}$$

for all multi-indices α and β .

Proof. The implications (i) \Rightarrow (ii) and (i) \Rightarrow (iii) follow immediately from the equality $\widehat{S_{M_p}^{N_p}} = S_{N_p}^{M_p}$. We now prove the implication (ii) \Rightarrow (i). We can use the L^2 -norm instead of the supremum norm. Using integration by parts, the Leibniz formula and the Schwarz inequality we obtain

$$\begin{aligned} \|x^\alpha \partial^\beta \varphi(x)\|_{L^2}^2 &= \int_{\mathbb{R}^n} [x^{2\alpha} \partial^\beta \varphi(x)] \partial^\beta \varphi(x) dx \\ &\leq \sum_{\substack{\gamma \leq 2\alpha \\ \gamma \leq \beta}} \binom{\beta}{\gamma} \binom{2\alpha}{\gamma} \gamma! \|\partial^{2\beta-\gamma} \varphi(x)\|_{L^2} \|x^{2\alpha-\gamma} \varphi(x)\|_{L^2} \\ &\leq C^2 \sum_\gamma \binom{\beta}{\gamma} \binom{2\alpha}{\gamma} \gamma! A^{2(|\alpha|+|\beta|-|\gamma|)} M_{|2\alpha-\gamma|} N_{|2\beta-\gamma|}. \end{aligned}$$

It follows from the conditions (M.1), (M.2) and $M_p N_p \supset p!$ that

$$\begin{aligned} \|x^\alpha \partial^\beta \varphi(x)\|_{L^2}^2 &\leq C^2 A^{2(|\alpha|+|\beta|)} M_{|2\alpha|} N_{|2\beta|} \sum_\gamma \binom{\beta}{\gamma} \binom{2\alpha}{\gamma} \gamma! / (M_{|\gamma|} N_{|\gamma|}) \\ &\leq C^2 (2AH)^{2(|\alpha|+|\beta|)} M_{|\alpha|}^2 N_{|\beta|}^2, \end{aligned}$$

which implies that $\varphi(x)$ belongs to $S_{M_p}^{N_p}$. Therefore, it remains to prove (iii) \Rightarrow (ii). The inequality $|\xi^\beta \hat{\varphi}(\xi)| \leq CB^{|\beta|} N_{|\beta|}$ means that

$$|\hat{\varphi}(\xi)| \leq C \exp[-N(|\xi|/B)],$$

where $N(\rho)$ is the associated function of N_p defined by

$$(2.1) \quad N(\rho) = \sup_p \log \frac{\rho^p}{N_p}.$$

Therefore, by the conditions (M.1) and (M.2) of N_p we have

$$\begin{aligned} |\partial^\beta \varphi(x)| &\leq (2\pi)^{-n} \int |e^{ix \cdot \xi} \xi^\beta \hat{\varphi}(\xi)| d\xi \\ &\leq C_1 \int |\xi|^{|\beta|} \exp[-N(|\xi|/B)] d\xi \\ &\leq C_1 \sup_\xi [|\xi|^{2|\beta|} \exp(-N(|\xi|/B))]^{1/2} \int \exp[-N(|\xi|/B)/2] d\xi \\ &\leq C_2 B^{|\beta|} N_{2|\beta|}^{1/2} \leq C_2 B_1^{|\beta|} N_{|\beta|}, \end{aligned}$$

which completes the proof.

Using the associated function as in (2.1) we can characterize $S_{M_p}^{N_p}$ as follows.

Corollary 2.4. *If M_p and N_p satisfy (M.1) and (M.2) and $M_p N_p \supset p!$, then the following conditions are equivalent.*

- (i) $\varphi \in S_{M_p}^{N_p}$.
- (ii) *There exist positive constants a and b such that*

$$\sup_x |\varphi(x)| \exp M(ax) < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp N(b\xi) < \infty.$$

In particular, putting $M_p = p!^r$ and $N_p = p!^s$ we can give simple characterizations for the Gelfand–Shilov spaces of type S as corollaries.

Corollary 2.5. *If $r + s \geq 1$, then the following are equivalent:*

- (i) $\varphi \in S_r^s$.
- (ii) *There exist positive constants h and k such that*

$$\sup_x |\varphi(x)| \exp k|x|^{1/r} < \infty, \quad \sup_\xi |\hat{\varphi}(\xi)| \exp h|\xi|^{1/s} < \infty.$$

Remark. We can easily prove the similar results on the characterization of S_{M_p} and S^{N_p} .

3. CHARACTERIZATION OF GELFAND–SHILOV SPACES OF TYPE W

In this section we characterize the Gelfand–Shilov spaces of type W in a more symmetric way by means of the Fourier transformation.

Let $M(x)$ and $\Omega(y)$ be differentiable functions on $[0, \infty)$ satisfying the condition (K): $M(0) = \Omega(0) = M'(0) = \Omega'(0) = 0$ and their derivatives are continuous, increasing and tending to infinity.

We now define the *Gelfand–Shilov spaces of type W* as in [6].

Definition 3.1. (i) The space W_M consists of all infinitely differentiable functions $\varphi(x)$ on \mathbb{R}^n satisfying the estimate $|\partial^\beta \varphi(x)| \leq C_\beta \exp(-M(a|x|))$ for some $a > 0$.

(ii) The space W^Ω consists of all entire analytic functions $\varphi(\zeta)$ on \mathbb{C}^n satisfying the estimate $|\zeta^\alpha \varphi(\zeta)| \leq C_\alpha \exp \Omega(b|\eta|)$ for some $b > 0$, where $\zeta = \xi + i\eta \in \mathbb{R}^n + i\mathbb{R}^n$.

(iii) The space W_M^Ω consists of all entire analytic functions $\varphi(\zeta)$ on \mathbb{C}^n satisfying the estimate $|\varphi(\xi + i\eta)| \leq C \exp(-M(a|\xi|) + \Omega(b|\eta|))$ for some $a, b > 0$.

In order to relate the sequences M_p and the functions $M(x)$ we need the following definitions.

Definition 3.2. If $M(\rho)$ is an increasing convex function in $\log \rho$ and increases more rapidly than $\log \rho^p$ for any p as ρ tends to infinity, we define its *defining sequence* by

$$M_p = \sup_{\rho > 0} \rho^p / \exp M(\rho), \quad p = 0, 1, 2, \dots$$

Definition 3.3. Let $M : [0, \infty) \rightarrow [0, \infty)$ be a convex and increasing function with $M(0) = 0$ and $\lim_{x \rightarrow \infty} x/M(x) = 0$. Then we define its *Young conjugate* M^* by $M^*(\rho) = \sup_x (x\rho - M(x))$.

To prove the main theorem on the characterizations of the Gelfand–Shilov spaces of type W we need the following relations between the defining sequences and the associated functions as in [3, 5, 9].

Proposition 3.4 ([9]). *If M_p satisfies (M.1), then M_p is the defining sequence of the associated function of itself.*

Proposition 3.5 ([5]). *Let $M : [0, \infty) \rightarrow [0, \infty)$ be a function satisfying the condition (K). Then $M(x)$ is equivalent to the associated function of the defining sequence of itself.*

Here, $M(x)$ and $N(x)$ are said to be *equivalent* if there exist constants A and B such that $M(Ax) \leq N(x) \leq M(Bx)$.

Proposition 3.6 ([9]). *Let $m_p = M_p/M_{p-1}$, $p = 1, 2, \dots$, and let $m(\lambda)$ be the number of m_p such that $m_p \leq \lambda$. Then we have $M(\rho) = \int_0^\rho m(\lambda)/\lambda d\lambda$.*

Lemma 3.7. *Let $M(\rho)$ be a function satisfying the condition (K). Then the defining sequence M_p^* of the Young conjugate $M^*(\rho)$ of $M(\rho)$ is equivalent to $p!/M_p$, where M_p is the defining sequence of $M(\rho)$. In fact, $M_p^* = (p/e)^p/M_p$. Consequently, M_p satisfies the following conditions:*

- (M.1)' (strong logarithmic convexity) $m_p = M_p/M_{p-1}$ is increasing and tends to infinity as $p \rightarrow \infty$;
- (M.1)* (duality) $p!/M_p$ satisfies (M.1)'.

Conversely, if M_p satisfies (M.1)' and (M.1)*, then the associated function $M^\#(\rho)$ of $p!/M_p$ and the Young conjugate $M^*(\rho)$ of the associated function $M(\rho)$ of M_p are equivalent.

Proof. We may assume that $M'(\rho)$ is strictly increasing. Then it is easy to see that $M^*(\rho) = \int_0^\rho M'^{-1}(t) dt$ where M'^{-1} is the inverse function of M' . Let $g(t) = t^p / \exp M^*(t)$. To find the maximum of $g(t)$ for $t > 0$, taking logarithm, differentiating and equating the result to zero, we obtain

$$(3.1) \quad p/t = M'^{-1}(t).$$

Let t_0 be the root of the equation (3.1). Then there exists $\rho_0 > 0$ such that $M'(\rho_0) = t_0$. Putting $t = M'(\rho_0)$ in (3.1) we have $\rho_0 M'(\rho_0) = p$. Thus we have

$$M^*(M'(\rho_0)) = \sup_x (xM'(\rho_0) - M(x)) = \rho_0 M'(\rho_0) - M(\rho_0)$$

since the function $h(x) = xM'(\rho_0) - M(x)$ takes its maximum at $x = \rho_0$. Therefore we have

$$\begin{aligned} M_p^* &= \sup_{t>0} \frac{t^p}{\exp M^*(t)} = \frac{t_0^p}{\exp M^*(t_0)} = \frac{[M'(\rho_0)]^p}{M^*(M'(\rho_0))} \\ &= \frac{[M'(\rho_0)]^p}{\exp[\rho_0 M'(\rho_0) - M(\rho_0)]} = \frac{[M'(\rho_0)]^p}{\exp(p - M(\rho_0))} \\ &= \left(\frac{p}{e}\right)^p \frac{\exp M(\rho_0)}{\rho_0^p} = \left(\frac{p}{e}\right)^p \frac{1}{M_p}. \end{aligned}$$

For the converse, by Stirling's formula it is easy to see that $p!$ and $(p/e)^p$ are equivalent. So we have for any $t, \rho > 0$

$$M(t) + M^\#(\rho) = \sup_p \log \frac{t^p}{M_p} + \sup_p \log \frac{\rho^p M_p}{p!} \geq \sup_p \log \frac{(t\rho)^p}{p!} \geq At\rho$$

for some $A > 0$, where $M^\#(\rho)$ is the associated functions of $p!/M_p$.

Thus we have

$$(3.2) \quad M^\#(\rho) \geq At\rho - M(t).$$

Taking the supremum for t in the right-hand side of (3.2), we have $M^\#(\rho) \geq M^*(A\rho)$. Since M_p satisfies (M.1)' and (M.1)*, we may assume that the sequences $m_p = M_p/M_{p-1}$ and p/m_p are strictly increasing.

We denote by $m(\lambda)$ the number of m_p such that $m_p \leq \lambda$. Then we have by Proposition 3.6

$$M^*(\rho) = \sup_x \int_0^x (\rho - m(\lambda)/\lambda) d\lambda.$$

Putting $\rho = p/m_p$ and $x = m_p$, we obtain

$$\begin{aligned} M^*(p/m_p) &\geq \int_0^{m_p} (p/m_p - m(\lambda)/\lambda) d\lambda \\ &= p - \int_0^{m_p} m(\lambda)/\lambda d\lambda \\ &= p - \sum_{j=1}^{p-1} \int_{m_j}^{m_{j+1}} j/\lambda d\lambda \\ &= p + \log \frac{m_1 \cdots m_{p-1}}{m_p^{p-1}}. \end{aligned}$$

On the other hand, let $m^\#(\lambda)$ be the number of p/m_p such that $p/m_p \leq \lambda$. Then we have

$$\begin{aligned} M^\#(p/m_p) &= \int_0^{p/m_p} m^\#(\lambda)/\lambda d\lambda \\ &= \sum_{j=1}^{p-1} \int_{j/m_j}^{(j+1)/m_{j+1}} j/\lambda d\lambda \\ &= \log \frac{p^p m_1 \cdots m_{p-1}}{p! \cdot m_p^{p-1}} \\ &\leq p + \log \frac{m_1 \cdots m_{p-1}}{m_p^{p-1}} \\ &\leq M^*(p/m_p). \end{aligned}$$

Now, for any $\rho > 0$ such that $p/m_p < \rho < (p+1)/m_{p+1}$ we have

$$\begin{aligned} M^*(\rho) &\geq M^*(p/m_p) \geq M^\#(p/m_p) \\ &\geq M^\# \left(\frac{1}{2}(p+1)/m_{p+1} \right) \geq M^\# \left(\frac{1}{2}\rho \right), \end{aligned}$$

which completes the proof.

We are now in a position to state and prove the main theorems on the characterizations of the Gelfand–Shilov spaces of type W .

Theorem 3.8. *If there is a constant L such that $M(x) \leq \Omega(Lx)$, then the space W_M^Ω is characterized by the following estimates*

$$(3.3) \quad |\varphi(x)| \leq C \exp(-M(a|x|)), \quad |\hat{\varphi}(\xi)| \leq C \exp(-\Omega^*(b|\xi|)).$$

Proof. If $\varphi \in C^\infty(\mathbb{R}^n)$ satisfies (3.3), then $\varphi(x)$ satisfies

$$\sup_x |x^\alpha \varphi(x)| \leq CA^{|\alpha|} M_{|\alpha|}, \quad \sup_\xi |\xi^\beta \hat{\varphi}(\xi)| \leq CB^{|\beta|} N_{|\beta|}$$

for some $A, B > 0$, where M_p and N_p are the defining sequences of $M(x)$ and $\Omega^*(y)$, respectively. Then the sequences M_p and N_p satisfy (M.1)' and (M.1)* by Lemma 3.7 and the condition $M(x) \leq \Omega(Lx)$ implies $M_p N_p \supset p!$. Therefore $\varphi(x)$ belongs to $S_{M_p}^{N_p}$ by Theorem 2.3, since (M.1)' and (M.1)* are stronger than (M.1) and (M.2), respectively. We now prove that $S_{M_p}^{N_p} \subset W_M^\Omega$. Let $\varphi \in S_{M_p}^{N_p}$. Then for every $\alpha, \beta \in \mathbb{N}_0^n$ we obtain

$$(3.4) \quad |\xi^\alpha \partial^\beta \varphi(\xi)| \leq CA^{|\alpha|} B^{|\beta|} M_{|\alpha|} N_{|\beta|}$$

for some $A, B > 0$. Since N_p satisfies (M.1)* or $p!/N_p$ satisfies (M.1)', it is easy to see that $N_p \prec p!$. Hence the function $\varphi(\xi)$ can be continued analytically into the complex domain as an entire analytic function. Applying the Taylor expansion and the inequality (3.4) we have

$$(3.5) \quad \begin{aligned} |\xi^\alpha \varphi(\xi + i\eta)| &\leq \sum_{\gamma \in \mathbb{N}_0^n} \frac{|\xi^\alpha \partial^\gamma \varphi(\xi)|}{\gamma!} |\eta|^{|\gamma|} \\ &\leq C \sum_{\gamma \in \mathbb{N}_0^n} A^{|\alpha|} M_{|\alpha|} N_{|\gamma|} |B\eta|^{|\gamma|} / \gamma! \\ &\leq 2^n CA^{|\alpha|} M_{|\alpha|} \exp N^\#(2B|\eta|). \end{aligned}$$

Dividing $|\xi|^{|\alpha|}$ in both sides of the inequality (3.5) and taking infimum for $|\alpha|$ in the right-hand side of (3.5), we have

$$|\varphi(\xi + i\eta)| \leq 2^n C \exp [- M(|\xi|/A) + N^\#(2B|\eta|)].$$

Note that we may use $|\xi|^{|\alpha|} \partial^\beta \varphi(\xi)$ instead of $|\xi^\alpha \partial^\beta \varphi(\xi)|$ in (3.4). Also, Lemma 3.7 implies

$$N^\#(2B|\eta|) \leq N^*(B'|\eta|) \leq (\Omega^*)^*(B''|\eta|) = \Omega(B''|\eta|)$$

for some $B', B'' > 0$, where $N^\#$ is the associated function of $p!/M_p$ and N^* is the Young conjugate of the associated function of N_p . Thus, we have

$$|\varphi(\xi + i\eta)| \leq C_1 \exp [- M(|\xi|/A) + \Omega(B''|\eta|)],$$

which completes the proof.

Remark. We can prove the characterization theorem for W_M and W^Ω similarly.

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