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LOCAL JORDAN *-DERIVATIONS OF STANDARD OPERATOR ALGEBRAS

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ABSTRACT. We prove that on standard operator algebras every local Jordan *-derivation is a Jordan *-derivation.

There is an extensive study of algebraic reflexivity of certain subsets of linear transformations on vector spaces and algebras (e.g. [1], [2], [6], [8] and see also the references therein). Its main problem is concerned with the question of how the transformations under consideration are determined by their local actions. As an illustration we mention the following fundamental theorem of Larson and Sourour. Let $\mathcal{B}(X)$ denote the algebra of all bounded linear operators on the complex Banach space X. If φ is a so-called local derivation of $\mathcal{B}(X)$, that is, a linear mapping from $\mathcal{B}(X)$ into itself with the property that for every $A \in \mathcal{B}(X)$ there exists a derivation θ_A of $\mathcal{B}(X)$ such that $\varphi(A) = \theta_A(A)$, then φ is a derivation.

The aim of this paper is to contribute to this study by presenting a result of the same spirit for a new class of transformations. Our main objects are the Jordan *-derivations. If $\mathcal{B}$ is a *-ring and $\mathcal{A} \subset \mathcal{B}$ is a subring, then the additive function $J : \mathcal{A} \to \mathcal{B}$ is called a Jordan *-derivation if

$$J(T^2) = TJ(T) + J(T)T^* \qquad (T \in \mathcal{A}).$$

It is easy to verify that for every $A \in \mathcal{B}$, the mapping J defined by $J(T) = TA - AT^* \quad (T \in \mathcal{A})$ is a Jordan *-derivation. If $A \in \mathcal{A}$, this is called an inner Jordan *-derivation. The importance of these mappings relies upon the fact that their structure plays an essential role in the problem of representability of quadratic functionals by sesquilinear forms on modules [10], [11].

However, to point out a significant difference between our result below and the other works on this field, we emphasize that although the underlying structure is an algebra and real-linearity would be reasonable to assume, the involved mappings are supposed to be merely additive. Investigations of this kind concerning isomorphisms and derivations of semi-simple Banach algebras were initiated by Kaplansky [7] and then followed, for example, by Johnson and Sinclair [5].

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We begin with the notation. If H is a Hilbert space, then let $\mathcal{B}(H)$ denote the algebra of all bounded linear operators on H and let $\mathcal{S}(H)$ be the set of self-adjoint elements of $\mathcal{B}(H)$. $\mathcal{F}(H)$ stands for the algebra of bounded finite-rank operators on H. A subalgebra of $\mathcal{B}(H)$ which contains $\mathcal{F}(H)$ is called a standard operator algebra.

If H is complex and $\mathcal{A}$ is a standard operator algebra, then [12, Theorem] together with a direct computation in the case of $\dim H = 1$ implies that the additive mapping $J : \mathcal{A} \to \mathcal{B}(H)$ is a Jordan *-derivation if and only if there exists an $A \in \mathcal{B}(H)$ such that

$$J(T) = TA - AT^* \qquad (T \in \mathcal{A}).$$

The additive mapping $J : \mathcal{A} \to \mathcal{B}$ is said to be a local Jordan *-derivation [locally inner Jordan *-derivation] if for every $T \in \mathcal{A}$ there exists a Jordan *-derivation $J_T : \mathcal{A} \to \mathcal{B}$ [an element $S_T \in \mathcal{A}$] such that $J(T) = J_T(T)$ [$J(T) = TS_T - S_T T^*$].

Our main result is formulated in the following theorem.

Theorem. *Let H be a complex Hilbert space, $\dim H > 1$ and suppose that $\mathcal{A} \subset \mathcal{B}(H)$ is a standard operator algebra. Then every local Jordan *-derivation from $\mathcal{A}$ into $\mathcal{B}(H)$ is a Jordan *-derivation.*

In the proof we shall need the next proposition.

Lemma. *Let $J : \mathcal{A} \to \mathcal{B}(H)$ be an additive mapping with the property that for every $T \in \mathcal{A}$ there is a self-adjoint operator $A_T \in \mathcal{B}(H)$ such that $J(T) = TA_T - A_T T^*$. Assume that P is an (orthogonal) projection of finite rank, $t, s \in \mathbb{R} \setminus \{0\}$ and $A, B \in \mathcal{S}(H)$ satisfy*

$$J(tP) = t(PA - AP) \quad \text{and} \quad J(isP) = is(PB + BP).$$

Then $PA - AP = PB - BP$.

Proof of Lemma. If $P = I$, there is nothing to prove. So exclude this possibility and let

$$P = \sum_{i=1}^{n} e_i \otimes e_i,$$

where $\{e_i\}_{i=1}^n$ is an orthonormal system of vectors. Let $e \in H$ be a unit vector which is orthogonal to this system and for an $1 \leq k \leq n$ define

$$Q = e_k \otimes e, \quad R = e \otimes e_k.$$

Suppose that $C, D \in \mathcal{S}(H)$ are such that

$$J(R) = RC - CQ, \quad J(iR) = i(RD + DQ).$$

Choose nonzero rational numbers $\lambda_1, \lambda_2, \mu_1, \mu_2$. The additivity of J implies that there exist $X, Y \in \mathcal{S}(H)$ for which

$$\begin{aligned} &\lambda_1 t(PA - AP) + i\mu_1 s(PB + BP) + \lambda_2(RC - CQ) = \\ &\lambda_1 t(PX - XP) + i\mu_1 s(PX + XP) + \lambda_2(RX - XQ) \end{aligned} \tag{1}$$

and

$$\begin{aligned} &\lambda_1 t(PA - AP) + i\mu_1 s(PB + BP) + i\mu_2(RD + DQ) = \\ &\lambda_1 t(PY - YP) + i\mu_1 s(PY + YP) + i\mu_2(RY + YQ). \end{aligned} \tag{2}$$

First consider equation (1). If we take the operators on both sides at e_k and then form inner product with e_k, we obtain

$$\langle Be_k, e_k\rangle = \langle Xe_k, e_k\rangle. \tag{1–1}$$

Applying the same operators on e and calculating the inner product of the obtained vectors with e_k we arrive at

$$\begin{aligned}\lambda_1 t\langle Ae, e_k\rangle + i\mu_1 s\langle Be, e_k\rangle - \lambda_2\langle Ce_k, e_k\rangle \\ = \lambda_1 t\langle Xe, e_k\rangle + i\mu_1 s\langle Xe, e_k\rangle - \lambda_2\langle Xe_k, e_k\rangle.\end{aligned} \tag{1–2}$$

Taking the operators in (1) once again at e and then forming another inner product, this time with e, we get

$$\langle Ce, e_k\rangle - \langle Ce_k, e\rangle = \langle Xe, e_k\rangle - \langle Xe_k, e\rangle. \tag{1–3}$$

From (1–1) and (1–2) we have

$$\langle Xe, e_k\rangle = \frac{\lambda_1 t\langle Ae, e_k\rangle + i\mu_1 s\langle Be, e_k\rangle - \lambda_2\langle Ce_k, e_k\rangle + \lambda_2\langle Be_k, e_k\rangle}{\lambda_1 t + i\mu_1 s}. \tag{1–4}$$

The self-adjointness of C and X together with (1–3) and (1–4) implies that

$$\operatorname{Im}\langle Ce, e_k\rangle = \operatorname{Im}\frac{\lambda_1 t\langle Ae, e_k\rangle + i\mu_1 s\langle Be, e_k\rangle - \lambda_2\langle Ce_k, e_k\rangle + \lambda_2\langle Be_k, e_k\rangle}{\lambda_1 t + i\mu_1 s}.$$

Since this relation holds for every $0 \neq \lambda_2 \in \mathbb{Q}$, thus, sending λ_2 to 0 we have

$$\operatorname{Im}\langle Ce, e_k\rangle = \operatorname{Im}\frac{\lambda_1 t\langle Ae, e_k\rangle + i\mu_1 s\langle Be, e_k\rangle}{\lambda_1 t + i\mu_1 s}. \tag{1–5}$$

Applying the same method to equation (2), we obtain

$$\operatorname{Re}\langle De, e_k\rangle = \operatorname{Re}\frac{\lambda_1 t\langle Ae, e_k\rangle + i\mu_1 s\langle Be, e_k\rangle}{\lambda_1 t + i\mu_1 s}. \tag{2–1}$$

It follows from (1–5) and (2–1) that the value of

$$\frac{\lambda_1 t\langle Ae, e_k\rangle + i\mu_1 s\langle Be, e_k\rangle}{\lambda_1 t + i\mu_1 s}$$

does not depend on λ_1 and μ_1, which implies

$$\langle Ae, e_k\rangle = \langle Be, e_k\rangle.$$

The validity of the equation $PA - AP = PB - BP$ is now easy to check. □

Proof of Theorem. Let J be a local Jordan *-derivation on $\mathcal{A}$. Then J is a linear combination of two local Jordan *-derivation having the "self-adjointness property" described in the assumptions of Lemma. More precisely,

$$J = (1/2)(D_1 - iD_2),$$

where D_1 and D_2 are local Jordan *-derivations defined by

$$D_1(T) = J(T) - J(T)^* \quad \text{and} \quad D_2(T) = i(J(T) + J(T)^*) \quad (T \in \mathcal{A})$$

having the additional property that for every $T \in \mathcal{A}$ there is a self-adjoint operator A_i such that $D_i(T) = TA_i - A_iT^*$, $i = 1, 2$. Hence, it is enough to prove our result only for such local Jordan *-derivations.

Therefore, assume from now on that this "self-adjointness property" holds for J. In particular, $J(S)$ is a skew-Hermitian operator for every self-adjoint $S \in \mathcal{A}$. We shall prove that J is real-linear on $\mathcal{SF}(H)$, the set of all bounded self-adjoint finite-rank operators on H.

Let P be a finite-rank projection and $0 \neq t_0 \in \mathbb{R}$. There exist $A, B, C \in \mathcal{S}(H)$ such that

$$J(P) = PA - AP, \quad J(iP) = i(PB + BP), \quad \text{and} \quad J(t_0 P) = t_0(PC - CP).$$

Using Lemma with $t = s = 1$, we get $PA - AP = PB - BP$, and once again with $t = t_0,\ s = 1$, we have $PC - CP = PB - BP$. It follows that

$$J(t_0 P) = t_0 J(P), \tag{3}$$

which holds for $t_0 = 0$ as well. Since every operator in $\mathcal{SF}(H)$ is a real-linear combination of finite-rank projections, it follows from (3) that the restriction of J to $\mathcal{SF}(H)$ is real-linear.

Define a real-linear mapping $\phi : \mathcal{SF}(H) \to \mathcal{B}(H \oplus H)$ by

$$\phi(T) = \begin{bmatrix} T & J(T) \\ 0 & T \end{bmatrix} \qquad (T \in \mathcal{SF}(H)).$$

It is easy to see that $\phi(P)^2 = \phi(P)$ holds for every finite-rank projection P. Pick an arbitrary operator $T \in \mathcal{SF}(H)$. Then $T = \sum_{i=1}^n t_i P_i$, where $t_i \in \mathbb{R}$ and P_i is a finite-rank projection such that $P_i P_j = P_j P_i = 0$ if $i \neq j$. Since $P_i + P_j$ is a projection if $i \neq j$, we have $(\phi(P_i) + \phi(P_j))^2 = \phi(P_i) + \phi(P_j)$. This yields $\phi(P_i)\phi(P_j) + \phi(P_j)\phi(P_i) = 0$. Using this relation it is easy to see that $\phi(T)^2 = \phi(T^2)$ which further implies

$$J(T^2) = TJ(T) + J(T)T \qquad (T \in \mathcal{SF}(H)).$$

Linearizing this equation, i.e. replacing T by $T + S$, we obtain

$$J(TS + ST) = TJ(S) + SJ(T) + J(T)S + J(S)T \quad (T, S \in \mathcal{SF}(H)). \tag{4}$$

Next, we define a complex-linear mapping $J_1 : \mathcal{F}(H) \to \mathcal{F}(H)$ by

$$J_1(R) = J_1(T + iS) = J(T) + iJ(S) \qquad (R \in \mathcal{F}(H)),$$

where $T = (1/2)(R + R^*)$ is the real part and $S = (2i)^{-1}(R - R^*)$ is the imaginary part of R, respectively. Applying (4) it is easy to see that J_1 is a Jordan derivation, that is,

$$J_1(R^2) = RJ_1(R) + J_1(R)R \qquad (R \in \mathcal{F}(H)).$$

Using a result of Herstein [4, Theorem 3.1] we infer that J_1 is a derivation and then, by a well-known theorem of Chernoff [3, Corollary 3.4], we conclude that there exists an $A \in \mathcal{B}(H)$ such that

$$J_1(R) = RA - AR \qquad (R \in \mathcal{F}(H)). \tag{5}$$

Clearly, we have $J_1(S) = J(S)$ for every $S \in \mathcal{SF}(H)$. It follows that $SA - AS$ is a skew-Hermitian operator whenever $S \in \mathcal{SF}(H)$. A straightforward computation shows that $A - A^*$ commutes with every element of $\mathcal{SF}(H)$ and this implies that $A - A^* = itI$ for some real number t. The relation (5) holds true also in the case when we replace A by $A - (1/2)itI$. Hence, we can assume with no loss of generality that the operator A in (5) is self-adjoint.

Now let us define an additive maping $J_2 : \mathcal{A} \to \mathcal{B}(H)$ by

$$J_2(R) = J(R) - (RA - AR^*) \qquad (R \in \mathcal{A}).$$

This is obviously a local Jordan *-derivation having the "self-adjointness property". Moreover, $J_2(S) = 0$ for all $S \in \mathcal{SF}(H)$. We are going to show that J_2 is a Jordan *-derivation on $\mathcal{F}(H)$.

Let $e, f \in H$ be mutually orthogonal unit vectors and define projections $Q_s,\ 0 \leq s \leq 1$, and $R_k,\ k = 0, 1$, by

$$Q_s = (s^{1/2}e + (1-s)^{1/2}f) \otimes (s^{1/2}e + (1-s)^{1/2}f),$$
$$R_k = (1/2)(e + (-1)^k f) \otimes (e + (-1)^k f).$$

Suppose that $0 \neq t \in \mathbb{R}$ and let P and B be a rank-one projection and a self-adjoint operator, respectively. Assume that $J_2(itP) = it(PB + BP)$. It follows from Lemma and $J_2(P) = 0$ that B commutes with P and thus $J_2(itP)$ is a real scalar multiple of iP. Hence, there exists an additive function $f : \mathbb{R} \to \mathbb{R}$ such that $J_2(itP) = if(t)P\ (t \in \mathbb{R})$. Similarly, we can find additive functions $f_s, g_k : \mathbb{R} \to \mathbb{R}$, $0 \leq s \leq 1$, $k = 0, 1$, satisfying

$$J_2(itQ_s) = if_s(t)Q_s \quad \text{and} \quad J_2(itR_k) = ig_k(t)R_k \quad (t \in \mathbb{R})$$

for every $0 \leq s \leq 1$, $k = 0, 1$. We have $R_1 = Q_0 + Q_1 - R_0$ and consequently,

$$\begin{aligned} g_1(t)Q_0 + g_1(t)Q_1 - g_1(t)R_0 &= -iJ_2(itR_1) \\ &= f_0(t)Q_0 + f_1(t)Q_1 - g_0(t)R_0 \quad (t \in \mathbb{R}), \end{aligned}$$

which implies that

$$f_0 = f_1 = g_0 = g_1. \tag{6}$$

Moreover, we have

$$\begin{aligned} itQ_s = i(2t(s-s^2)^{1/2}R_0 &+ (t(1-s) - t(s-s^2)^{1/2})Q_0 \\ &+ (ts - t(s-s^2)^{1/2})Q_1) \quad (t \in \mathbb{R},\ 0 \leq s \leq 1) \end{aligned}$$

and it follows that

$$\begin{aligned} f_s(t)Q_s = &g_0(2t(s-s^2)^{1/2})R_0 + f_0(t(1-s) - t(s-s^2)^{1/2})Q_0 \\ &+ f_1(ts - t(s-s^2)^{1/2})Q_1 \end{aligned}$$

whenever $t \in \mathbb{R},\ 0 \leq s \leq 1$. Taking the operators on both sides at e and f and forming inner product with e respectively f, we obtain

$$f_s(t)s = g_0(t(s-s^2)^{1/2}) + f_1(ts - t(s-s^2)^{1/2}),$$

and

$$f_s(t)(1-s) = g_0(t(s-s^2)^{1/2}) + f_0(t(1-s) - t(s-s^2)^{1/2})$$

for every $t \in \mathbb{R}$ and $0 \leq s \leq 1$. As a consequence of (6) we have

$$\frac{f_1(ts)}{s} = \frac{f_1(t(1-s))}{1-s} = \frac{f_1(t) - f_1(ts)}{1-s} \qquad (t \in \mathbb{R},\ 0 < s < 1).$$

Hence

$$f_1(ts) = f_1(t)s \qquad (t \in \mathbb{R},\ 0 < s < 1).$$

Replacing t by 1 we get

$$f_1(s) = as \qquad (0 < s < 1),$$

where $a = f_1(1)$. Since f_1 is additive, this last equation holds for every real number s.

Let us now assume that $\dim H \geq 3$ and let K and L be arbitrary projections of rank one. Just as before, we obtain that there are real numbers a_1 and a_2 such that

$J_2(itK) = ia_1tK$ and $J_2(itL) = ia_2tL$ hold for every real t. We can find a projection M having rank one and being orthogonal to K and L. We have $J_2(itM) = ia_3tM$ $(t \in \mathbb{R})$, where a_3 is a fixed real number. From the discussion above it follows that $a_1 = a_3$ and $a_2 = a_3$. Hence, there exists a real constant a such that for every projection P with rank one we have $J_2(itP) = iatP$. But this implies that

$$J_2(T) = aT$$

for every skew-Hermitian finite-rank operator T. Thus

$$J_2(T) = T((a/2)I) - ((a/2)I)T^*$$

holds for every finite-rank operator T. Therefore, the restriction of J to $\mathcal{F}(H)$ is a Jordan *-derivation.

The proof in this case can be completed as it was done in [8, Proof of Theorem 1.2]. That is, by [12, Theorem], there is an operator $A \in \mathcal{B}(H)$ such that $J(T) = TA - AT^*$ for every $T \in \mathcal{F}(H)$. Let

$$J_3(T) = J(T) - (TA - AT^*) \qquad (T \in \mathcal{A}).$$

We have to show that $J_3 = 0$. Let $T \in \mathcal{A}$, $x \in H$ and let P denote the orthogonal projection onto the subspace generated by the vectors $x, J_3(T)x$. Since J_3 is a local Jordan *-derivation which vanishes on $\mathcal{F}(H)$, we have

$$0 = PJ_3((I-P)T(I-P))P = PJ_3(T)P$$

and this implies $J_3(T)x = 0$.

Finally, in the two-dimensional case we identify $\mathcal{B}(H)$ with the algebra of all 2×2 complex matrices. We have already proved that there is a real number a such that

$$J_2(itQ_0 + isQ_1) = a(itQ_0 + isQ_1) \qquad (t, s \in \mathbb{R}).$$

This gives together with $J_2(itR_0) = iatR_0$ that

$$J_2\left(\begin{bmatrix} 0 & it \\ it & 0 \end{bmatrix}\right) = a\begin{bmatrix} 0 & it \\ it & 0 \end{bmatrix}$$

holds for every $t \in \mathbb{R}$. Using considerations similar to those which led to (6) but replacing R_0 and R_1 by $\tilde{R}_0 = (1/2)(ie+f)\otimes(ie+f)$ and $\tilde{R}_1 = (1/2)(ie-f)\otimes(ie-f)$, respectively, we have $J_2(it\tilde{R}_0) = iat\tilde{R}_0$. This implies that

$$J_2\left(\begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix}\right) = a\begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix}$$

holds for every real number t. Thus, we have proved also in this case that $J_2(T) = aT$ for every skew-Hermitian operator T. Now, one can complete the proof just as in the previous case. □

Remark. As for the remaining case $\dim H = 1$, we assert that the theorem does not hold true in this case. To see this, let $f : \mathbb{R} \to \mathbb{R}$ be any non-continuous additive function. Then the function $J : \mathbb{C} \to \mathbb{C}$ defined by

$$J(x + iy) = if(y) \qquad (x, y \in \mathbb{R})$$

is a local Jordan *-derivation which is not a Jordan *-derivation. If $\dim H = 1$, the conclusion of the statement remains valid under the stronger assumption that J is a real-linear local Jordan *-derivation.

We also show that in our theorem the condition that $\mathcal{A}$ contains $\mathcal{F}(H)$ cannot be omitted. In fact, let $H = \mathbb{C} \oplus \mathbb{C}$,

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

and $\mathcal{A} = \mathbb{C}P$. Take arbitrary 2×2 matrices A, B and define $J : \mathcal{A} \to \mathcal{B}(H)$ by

$$J((\lambda + i\mu)P) = \lambda(PA - AP) + i\mu(PB + BP) \qquad (\lambda, \mu \in \mathbb{R}).$$

It is easy to see that J is additive and for every $T \in \mathcal{A}$ there is a matrix X such that

$$J(T) = TX - XT^*$$

which implies that J is a local Jordan *-derivation. But considering the equation characterizing the Jordan *-derivations, one can check that J is a Jordan *-derivation if and only if $PA - AP = PB - BP$ which obviously does not hold in general.

To conclude the paper we formulate the following analogues of the results in [9] for the case of Jordan *-derivations. Since the proofs can be based on arguments being very similar to those which can be found there, we omit them.

Proposition. *Let H be a complex separable Hilbert space. Then there are exactly three *-subalgebras of $\mathcal{B}(H)$ on which every Jordan *-derivation is locally inner, namely $\{0\}$, $\mathcal{F}(H)$ and $\mathcal{B}(H)$. Moreover, if $\mathcal{I}$ is a symmetric norm ideal in $\mathcal{B}(H)$, then every locally inner Jordan *-derivation on $\mathcal{I}$ is inner.*

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