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AN ENGEL CONDITION WITH DERIVATION FOR LEFT IDEALS

CHARLES LANSKI

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ABSTRACT. We generalize a number of results in the literature by proving the following theorem: Let R be a semiprime ring, D a nonzero derivation of R, L a nonzero left ideal of R, and let $[x,y]=xy-yx$. If for some positive integers $t_0,t_1,\ldots,t_n$, and all $x\in L$, the identity $[[\ldots[[D(x^{t_0}),x^{t_1}],x^{t_2}],\ldots],x^{t_n}]=0$ holds, then either $D(L)=0$ or else the ideal of R generated by $D(L)$ and $D(R)L$ is in the center of R. In particular, when R is a prime ring, R is commutative.

In this paper we prove a theorem generalizing several results, principally [20] and [9], which combine derivations with Engel type conditions. Before stating our theorem we discuss the relevant literature. If one defines $[x,y]_0=x$ and $[x,y]_1=[x,y]=xy-yx$, then an Engel condition is a polynomial $[x,y]_{n+1}=[[x,y]_n,y]$ in noncommuting indeterminates. A commutative ring satisfies any such polynomial, and a nilpotent ring satisfies one if n is sufficiently large. The question of whether a ring is commutative, or nilpotent, if it satisfies an Engel condition goes back to the well known work of Engel on Lie algebras [15, Chapter 2], and has been considered, with various modifications, by many since then (e.g. [2] or [7]). The connection of Engel type conditions and derivations appeared in a well known paper of E. C. Posner [23] which showed that for a nonzero derivation D of a prime ring R, if $[D(x),x]$ is central for all $x\in R$, then R is commutative. This result has led to many others (see [19] for various references), and in particular to a result of J. Vukman [25] showing that if $[D(x),x]_2$ is central for all $x\in R$, a prime ring with $\operatorname{char} R\neq 2,3$, then again R is commutative. We extended this result [20] by proving that if $[D(x),x]_n=0$ for all $x\in I$, an ideal of the prime ring R, then R is commutative, and if instead, this Engel type condition holds for all $x\in U$, a Lie ideal of R, then R embeds in $M_2(F)$ for F a field with $\operatorname{char} F=2$. Recently, [9] proved that for a left ideal L of a semiprime ring R, either $D(L)=0$ or R contains a nonzero central ideal if either: R is 6-torsion free and $[D(x),x]_2$ is central for all $x\in L$; or if $[D(x),x^n]$ is central for all $x\in L$ and R is $n!$-torsion free. The first of these conditions generalized [1, Theorem 3, p. 99], which assumed that $[D(x),x]$ is central for all $x\in L$, with no restriction on torsion. The second, involving powers, is related to both [12], which showed that a prime ring R is commutative if $D(x^k)=0$ for all $x\in R$, and to [8], a significant extension of [12], showing that R is commutative if it contains no nonzero nil ideal and $[D(x^{k(x)}),x^{k(x)}]_n=0$ on

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R. Other results and conditions involving the image of a derivation on a one-sided ideal of R have been appearing with increased frequency (e.g. [3], [4], [21], [24]).

Our result here combines a variant of the Engel condition and the action of a derivation on a left ideal in a semiprime ring. It generalizes or extends a number of the results mentioned above and eliminates all torsion assumptions.

Main Theorem. *Let R be a semiprime ring, D a nonzero derivation of R, and L a nonzero left ideal of R. If for some positive integers $t_0, t_1, \ldots, t_n$, and all $x \in L$, the identity $[[\ldots[[D(x^{t_0}), x^{t_1}], x^{t_2}], \ldots], x^{t_n}] = 0$ holds, then either $D(L) = 0$ or else $D(L)$ and $D(R)L$ are contained in a nonzero central ideal of R. In particular, when R is a prime ring, R is commutative.*

Note that the statement about prime rings does follow from the semiprime case since if I is a central ideal in a prime ring R, then the identity $[xy, z] = x[y, z] + [x, z]y$ shows that $0 = [IR, R] = I[R, R]$, so $I = 0$ or R is commutative. Also, when R is prime and $D(L) = 0$, then $D(R)L = D(RL) = 0$, and $D = 0$ results. Something like the conclusion that R contains a central ideal is the most that one can expect since R could be the direct sum of ideals A, B, and G, with G commutative, $I = B + G, D(A) \neq 0, D(B) = 0$ and $D(G) \subseteq G$. In this case $D(I)$ is central but $D(I) \neq 0$ and I itself is not central.

The heart of our proof of the Main Theorem is a special case for prime rings. The basic approach and ideas are like those in [20], so we first recall the basic notions required ([6] or [18]). If R is a prime ring, its extended centroid $C(R) = C$ is a field which is the center of the symmetric quotient ring $Q = Q(R)$ of R. For our purposes it suffices to know that RC and Q are prime overrings of R, for each $q \in Q$ there is a nonzero ideal I_q of R with $qI_q + I_q q \subseteq R$, and if $qI_q = 0$, then $q = 0$. Any derivation D of R extends uniquely to Q, and if on $Q, D(q) = qA - Aq$ for $A \in Q$, then D is called *inner*; otherwise D is *outer*. An important result of W. S. Martindale [22] is that R satisfies a generalized polynomial identity exactly when $H = \operatorname{soc} RC \neq 0$ and for each minimal left ideal RCe of RC with $e^2 = e$, $eRCe$ is a finite dimensional divisional algebra over C.

Theorem 1. *Let R be a prime ring, D a nonzero derivation of R, and L a nonzero left ideal of R. If for integers $k, n+1 \geq 1$, $[D(x^k), x^k]_n = 0$ for all $x \in L$, then R is commutative.*

Proof. It is easy to see that if $L \subseteq R \cap C$, then R must be commutative [14, Corollary, p. 7], so we may choose $a \in L - C$. For any $r \in R$, $[D((ra)^k), (ra)^k]_n = 0$, and it follows that

$$[D((Xa)^k), (Xa)^k]_n = \left[\sum_{i=0}^{k-1} (Xa)^i (X^D a + XD(a))(Xa)^{k-i-1}, (Xa)^k\right]_n$$

is an identity with derivation which is satisfied by R. If D is an outer derivation, a direct application of [17, Theorem 2, p. 65] or [6, Main Theorem, p. 251], together with [5, Theorem 2, p. 725] show that $[\sum_{i=0}^{k-1} (Xa)^i (Ya + XD(a))(Xa)^{k-i-1}, (Xa)^k]_n$ is an identity for Q, which yields easily that $[\sum_{i=0}^{k-1} (Xa)^i (Ya)(Xa)^{k-i-1}, (Xa)^k]_n$ is an identity for Q by first setting $Y = 0$. Since $a \notin C$, this identity is a nonzero generalized polynomial identity for R, so by Martindale's theorem [22, Theorem 3, p. 579] $H = \operatorname{soc} RC \neq 0$. Clearly the identity holds on $H \subseteq Q$. If H is commutative, then so is R and we are finished. Otherwise, since $Ha \subseteq H$ [18, Lemma 7, p. 779], there is a minimal left ideal $He \subseteq Ha$ with $e^2 = e \in H$ and $Hta = He$ for some $t \in H$. Consequently, He satisfies $[\sum_{i=0}^{k-1} X^i Y X^{k-i-1}, X^k]_n = 0$. Evaluating this expression with $X = he$ and $Y = (1-e)ye$ for arbitrary $h, y \in H$, and using $he(1-e)ye = 0$ results in $(1-e)ye(he)^{k(n+1)-1} = 0$. Because He is minimal, if $(1-e)ye \neq 0$, it follows that $He = H(1-e)ye$, so $(he)^{k(n+1)} = 0$ results. This means that He is a nil left ideal of bounded index and Levitzki's theorem [13, Lemma 1.1, p. 1] forces R to contain a nonzero nilpotent ideal. This contradiction shows that R must be commutative when D is outer.

We may now take $D(q) = [q, A]$ with $A \in Q - C$, since $D \neq 0$. As above, if we choose $a \in L - C$, then our assumption yields the identity $[A, (ra)^k]_{n+1} = 0$ for R. This is a nonzero generalized polynomial identity because $A \notin C$, so Martindale's theorem [22, Theorem 3, p. 579] shows that $H = \operatorname{soc} RC \neq 0$ and eHe is finite dimensional over C for $e^2 = e$ a minimal idempotent in H. Now the identity $[A, (Xa)^k]_{n+1}$ is also satisfied by Q [5, Theorem 2, p. 725] and hence by H. As in the case above, R is commutative if H is, so we proceed with the assumption that H is not commutative to get the contradiction $D = 0$.

We want to replace R with H and be able to assume that for any minimal idempotent $e \in H, Ce = eHe$. We note that $C = C(H), CH = H$ and $D(H) \subseteq H$ [18, Lemma 7, p. 59], and C centralizes H, so it is clear that $Ce \subseteq Z(eHe)$ for any idempotent $e \in H$. Assume first that C is a finite field. From the finite dimensionality of eHe over Ce it follows that eHe is a finite field, so for $z \in eHe$ and any $h \in H$, $zehe = ehez$, which forces $ze = ce$ for $c \in C(H) = C$ [22, Theorem 1, p. 577]. Therefore $Ce = Z(eHe) = eHe$ when C is a finite field. If C is infinite, then a Vandermonde determinant argument, for example that in [20, Lemma 2, p. 732], shows that $[A, (Xa)^k]_{n+1}$ is satisfied by any extension $H \otimes_C F$ of H, for F a field extension of C. In particular we can take F to be an algebraic closure of C. Now $C(H \otimes_C F) = F$ [10, Theorem 3.5, p. 59], $\operatorname{soc}(RC \otimes_C F) = H \otimes_C F$, and for any minimal idempotent $e \in H \otimes_C F$, $e(H \otimes_C F)e$ is finite dimensional over eF, again by [22], so $e(H \otimes_C F)e = eF$ because F is algebraically closed. Consequently, regardless of $\operatorname{card} C$, we may assume that $H = R$ and $eC = eHe$ for any minimal idempotent $e \in H$.

Since H satisfies the identity $[A, (Xa)^k]_{n+1}$, as for the case above when D was assumed to be outer, for some minimal idempotent $e \in H$ and some $t \in H$, $He = Hta$ satisfies the identity $[A, X^k]_{n+1}$. In particular if $X = e$ we obtain $[A, e]_{n+1} = 0$ and also $[A, e]_{n+2} = 0$. Since one of $n+1$ or $n+2$ is odd and $[A, e] = [A, e]_3$, it follows immediately that $[A, e] = 0$, and we may write $A = eAe + (1-e)A(1-e)$. But $eAe = e(Ae)e \in eHe = Ce$, so $A = ce + (1-e)A(1-e)$. For any $h \in H$ we evaluate $[A, (he)^k]_{n+1} = 0$ using the identities $[y, x]_{n+1} = \sum_{i=0}^{n+1} (-1)^i \binom{n+1}{i} x^i y x^{n+1-i}$ and $[x+y, z]_s = [x, z]_s + [y, z]_s$ to obtain

$$\begin{aligned}
0 &= (1-e)A(1-e)(he)^{k(n+1)} + \sum_{i=0}^{n+1} (-1)^i \binom{n+1}{i} (he)^{ki} ec (he)^{k(n+1-i)} \\
&= (1-e)A(1-e)(he)^{k(n+1)} + ec(he)^{k(n+1)} \\
&\quad + c \sum_{i=1}^{n+1} (-1)^i \binom{n+1}{i} (he)^{ki} e (he)^{k(n+1-i)}
\end{aligned}$$

$$= (1-e)A(1-e)(he)^{k(n+1)} + ec(he)^{k(n+1)} - c(he)^{k(n+1)}$$
$$= (1-e)A(1-e)(he)^{k(n+1)} - (1-e)c(he)^{k(n+1)}$$
$$= (1-e)(A-c)(1-e)(he)^{k(n+1)}.$$

Hence $(A-c)(he)^{k(n+1)} = (A-c)(e+(1-e))(he)^{k(n+1)} = (A-c)(1-e)(he)^{k(n+1)} = 0$ since $Ae = ce$ and $(A-c)(1-e) = (1-e)(A-c)(1-e)$. A result of B. Felzenszwalb [11, Theorem 2, p. 242] shows that in a ring with no nonzero nil left ideal, if $yt^s = 0$ for all $t \in L$, a nonzero left ideal, then $yL = 0$. Therefore, we have $(A-c)He = 0$, forcing $A = c \in C$ and $D = 0$, a contradiction. Consequently, R must be commutative, completing the proof of the theorem. □

The special case of Theorem 1 when $n = 0$ gives [12, Theorem 2, p. 19], since $kD(x)x^{k-1} = D(x^k) = 0$ forces $\operatorname{char} R = p|k$. Also, the theorem is a version of [8, Corollary 1, p. 36] for left ideals where we must assume that the exponents k are fixed but need not assume that R has no nil ideal. Before proving our Main Theorem, it will be helpful to collect a few observations together into a lemma.

Lemma. *Let R be a semiprime ring and M the maximal central ideal of R.*

(1) $M = \operatorname{ann}([R,R])$ *is a semiprime ideal of* R;
(2) *if $a \in R$ and Ra is central, then $a \in M$; and*
(3) *if D is a derivation of R, then $D(M) \subseteq M$.*

Proof. Since any annihilator ideal in a semiprime ring is a semiprime ideal, it suffices to show that $M = \operatorname{ann}([R,R])$ to prove (1). Let $A = \operatorname{ann}([R,R])$ and note that $0 = [MR,R] = M[R,R]$, so $M \subseteq A$. But $[A,R] \subseteq A \cap ([R,R]) = 0$ since R is semiprime, and $A = M$. Next observe that R/M has no nonzero central ideal. If $M \subseteq I$ is an ideal of R with I/M central in R/M, then $[I,R] \subseteq M$ implies that $[[I,R],R] = 0$, so $[I,[R,R]] = 0$ and I is central by [14, Lemma 1.1.8, p. 8] forcing $I = M$. Consequently, if $Ra+M$ is central in R/M, then $Ra \subseteq M$, which results in $a \in M$ by (1). Finally, for any derivation D it is easy to see that $D(Z(R)) \subseteq Z(R)$, the center of R, and then that $M+D(M)$ is an ideal of R in $Z(R)$. Thus $D(M) \subseteq M$ by the maximality of M. □

Proof of Main Theorem. Our assumption that $[[\dots[[D(x^{t_0}), x^{t_1}], x^{t_2}], \dots], x^{t_n}] = 0$ for all $x \in L$ implies that $[D(x^k), x^k]_n = 0$ for $k = t_0 t_1 \cdots t_n$ since powers of x commute, so we may as well assume that all $t_j = k$. We claim that $RD(R)L$ is a central ideal of R, and is not zero unless $D(L) = 0$. Should $D(R)L = 0$, then $L \subseteq \operatorname{ann}(D(R))$, the left or right annihilator of $(D(R))$, the ideal $D(R)$ generates. It is easy to see that $D(L) \subseteq D(\operatorname{ann}(D(R))) \subseteq D(R) \cap \operatorname{ann}(D(R)) = 0$, since R is semiprime. Consequently, to prove the existence of a nonzero central ideal, it suffices to assume that $D(L) \neq 0$ and show that $RD(R)L$ is central. Equivalently, we need to prove that for each prime ideal P of R, the image of $RD(R)L$ is central in R/P. This is clear if $D(R)L \subseteq P$, so we need only consider those prime ideals with $D(R)L \not\subseteq P$.

Let P be a prime ideal of R so that $D(R)L \not\subseteq P$, and suppose that $D(P) \subseteq P$. In this case, D induces a derivation E on R/P via $E(r+P) = E(r) + P$ and our hypothesis carries over from R to R/P using E and the left ideal $L + P \subseteq R/P$. Applying Theorem 1 gives either $E = 0$, $L + P \subseteq P$, or R/P commutative. Since the first two possibilities each force $D(R)L \subseteq P$, we must conclude that R/P is commutative, so $RD(R)L + P$ is central in R/P.

We may assume now that $D(R)L \not\subseteq P$ and $D(P) \not\subseteq P$. It is straightforward to check that $D(P) + P = B \subseteq R/P$ is a nonzero ideal. For any $t \in P$ and $y \in L$ our assumption that $[D((ty+y)^k), (ty+y)^k]_n = 0$, taken modulo P becomes $[\sum_{i=0}^{k-1} y^i(D(t)y + D(y))y^{k-i-1}, y^k]_n = 0$ in R/P. But

$$\left[\sum_{i=0}^{k-1} y^i D(y) y^{k-i-1}, y^k\right]_n = [D(y^k), y^k]_n = 0,$$

so $[\sum_{i=0}^{k-1} y^i D(t) y^{k-i}, y^k]_n = 0$ in R/P, which means that the expression $f(X,Y) = [\sum_{i=0}^{k-1} Y^i X Y^{k-i}, Y^k]_n$ yields $O_{R/P}$ when elements of B replace X and elements of $L+P$ replace Y. If for some $y \in (L+P) - P, yw = 0$ in R/P for $w \in R/P - O_{R/P}$, then for any $b \in B$ and $r \in R$, $O_{R/P} = f(wb, ry) = wb(ry)^{k(n+1)}$. Thus $wB(ry)^{k(n+1)} = 0$ in R/P, and since B is a nonzero ideal and R/P is prime, we must conclude that $Ry + P$ is a nil left ideal of bounded index in R/P, forcing the contradiction $y \in P$ by Levitzki's theorem [13, Lemma 1.1, p. 1]. Therefore, we may assume that each nonzero $y \in L + P$ has no right annihilator in R/P.

To simplify notation, we assume that R is a prime ring with a nonzero ideal B and nonzero left ideal L whose nonzero elements are left regular, that $f(X, y)$ is an identity for B for each $y \in L$, and show that R is commutative. Expanding $f(X, y)$ for $y \in L - 0$, yields the identity $\sum_{j=0}^{v} n_j y^j X y^{v-j}$ for B, where n_j are integers, $n_0 = 1$, and $v = k(n+1)$. This is a generalized linear identity for B, so by [18, Lemma 1, p. 766], $\{1, y, \ldots, y^v\}$ must be $C(R)$ dependent. Let $m(y) = y^s + \cdots + c_1 y + c_0 = 0$ with $c_i \in C(R)$ and s minimal. The definition of Q allows us to choose a nonzero ideal I of R so that all $c_i I \subseteq R$. Thus if $c_0 = 0$ and $m(y) = yg(y)$, then $g(y)I \subseteq R$, so $g(y)I$ is in the right annihilator of y, and $g(y)I = 0$ forces $g(y) = 0$, contradicting the minimality of s. Therefore $c_0 \neq 0$ and $J = c_0 I = I c_0 \subseteq Ry$. Now $f(X, Y)$ is a polynomial identity for $B \cap J \subseteq L$, and so for its central localization, a finite dimensional central simple algebra [16, Theorem 2, p. 57]. Applying [20, Lemma 2, p. 732] shows that $B \cap J$ is commutative or that $f(X, Y)$ is an identity for some $M_d(F)$ for F a field and $d > 1$. But $f(e_{12}, e_{22}) = e_{12} \neq 0$, for e_{12} and e_{22} matrix units in $M_d(F)$, so $B \cap J$ is commutative, forcing R to be commutative [14, Corollary, p. 7], and showing that our original semiprime ring must contain the nonzero central ideal $RD(R)L$.

Finally, we must show that $D(L), D(R)L \subseteq M$, the maximal central ideal of our semiprime ring R. We have just proven that $RD(R)L \subseteq M$, so by the Lemma $D(R)L \subseteq M$ and $D(R)D(L) \subseteq D(D(R)L) + D^2(R)L \subseteq D(M) + M = M$. Hence

$$D(L)RD(L) \subseteq D(LR)D(L) + M \subseteq M,$$

and the semiprimeness of M by the Lemma forces $D(L) \subseteq M$. Therefore, the proof of the Main Theorem is complete. $\square$

It is clear that the Main Theorem generalizes both [9] and [20], and in the way we mentioned after Theorem 1, [8] as well. We end the paper with another consequence of the Main Theorem by giving an extension to one-sided ideals of [2, Theorem 3, p. 385] and [7, Theorem 2, p. 120].

Theorem 2. *Let R be a semiprime ring and L a nonzero left ideal of R. If for integers $n, k \geq 1$, and some $a \in R, [a, x^k]_n = 0$ for all $x \in L$, then $[a, L] = 0$. When R is a prime ring, then $a \in Z(R)$, the center of R.*

Proof. Define a derivation D of R by $D(r) = [r, a]$. Then for all $x \in L$,

$$-[D(x^k), x^k]_{n-1} = [-D(x^k), x^k]_{n-1} = [a, x^k]_n = 0.$$

By the Main Theorem, either $D = 0$ or $D(L) \subseteq Z(R)$. When $D = 0$, $a \in Z(R)$ is immediate, and when $D(L) \subseteq Z(R)$, $[[a, L], R] = 0$. In particular, if $y \in L$ and $r \in R$, then

$$0 = [[a, ay], r] = [a[a, y], r] = [a, r][a, y],$$

so letting $r = ys$ for $s \in R$ shows that $[a, y]R[a, y] = 0$. Since R is semiprime we are forced to conclude that $[a, L] = 0$. When R is prime, $0 = [a, RL] = [a, R]L$, so $a \in Z(R)$, proving the theorem. $\square$

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Department of Mathematics, University of Southern California, Los Angeles, California 90089-1113
E-mail address: `clanski@math.usc.edu`