

**A SEQUENTIAL PROPERTY OF $C_p(X)$
AND A COVERING PROPERTY OF HUREWICZ**

MARION SCHEEPERS

(Communicated by Andreas R. Blass)

ABSTRACT. $C_p(X)$ has the *monotonic sequence selection property* if there is for each f , and for every sequence $(\sigma_n : n < \omega)$ where for each n σ_n is a sequence converging pointwise monotonically to f , a sequence $(f_n : n < \omega)$ such that for each n f_n is a term of σ_n , and $(f_n : n < \omega)$ converges pointwise to f . We prove a theorem which implies for metric spaces X that $C_p(X)$ has the monotonic sequence selection property if, and only if, X has a covering property of Hurewicz.

Some nice duality results have been proved which relate properties of the closure operator of function spaces with open covering properties of the domain spaces. To list a few, let us first fix conventions and notations. Throughout X will be a topological space which has at least the Tychonoff separation property – this means that every one element subset of X is closed, and whenever C is a closed subset of X and x is a point from the complement of C , then there is a continuous function from X to the closed unit interval \mathbb{I} which maps x to 0 and maps each element of C to 1.

The set of all functions from X to the real line \mathbb{R} , denoted \mathbb{R}^X , is considered as a power of the real line and is endowed with the Tychonoff product topology. The set of continuous functions from X to \mathbb{R} is a subset of \mathbb{R}^X ; when endowed with the topology it inherits from \mathbb{R}^X , this space is denoted $C_p(X)$. The topology of $C_p(X)$ is known as the *topology of pointwise convergence*. All constant functions from X to \mathbb{R} are elements of $C_p(X)$. The function which is everywhere equal to zero is denoted \mathbf{o} . The closure operator of $C_p(X)$ can be quite complicated.

A space is said to have *countable tightness* if for every subset A , a point is in the closure of A if, and only if, it is in the closure of a countable subset of A . A well-known theorem of Arkhangel'skiĭ and Pytkeev implies that $C_p(X)$ has countable tightness if, and only if, every finite power of X has the Lindelöf property (every open cover has a countable subcover). Gerlits and Nagy found another interesting equivalent: An open cover of a space is an ω -cover if the space itself is not a member

Received by the editors December 15, 1995 and, in revised form, April 11, 1996.

1991 *Mathematics Subject Classification*. Primary 54E99.

Key words and phrases. Countable fan tightness, countable strong fan tightness, strong Fréchet property, γ -set, Hurewicz property, Lusin set, Menger property, Rothberger property, Sierpiński set, $S_1(\Gamma, \Gamma)$, \mathfrak{b} , $\text{cov}(\mathcal{M})$, \mathfrak{d} , \mathfrak{p} .

The author's research was supported in part by NSF grant DMS 95 - 05375.

of the cover, and there is for every finite subset of the space an element of the cover which contains that set. Gerlits and Nagy proved in [3] that $C_p(X)$ has countable tightness if, and only if, every ω -cover of X has a countable subset which is an ω -cover of X .

A second notion in this direction is as follows. A space has *countable fan tightness* if for each element x , if $(A_n : n < \omega)$ is a sequence of subsets of X such that for each n $x \in \overline{A_n}$, then there is a sequence $(F_n : n < \omega)$ of finite sets such that for each n $F_n \subseteq A_n$, and such that $x \in \overline{(\cup_{n < \omega} F_n)}$. This property may have been introduced by Arkhangel'skii, who proved in [1] that $C_p(X)$ has countable fan tightness if, and only if, every finite power of X has the *Menger property*: X has the Menger property if there is for every sequence $(\mathcal{U}_n : n = 1, 2, 3, \dots)$ of open covers of X a corresponding sequence $(\mathcal{V}_n : n = 1, 2, 3, \dots)$ of finite sets such that for each n $\mathcal{V}_n \subseteq \mathcal{U}_n$, and $\cup_{n=1}^{\infty} \mathcal{V}_n$ is a cover of X . Hurewicz proved in [3] that for metric spaces this covering property is equivalent to a property introduced by Menger in [6]. In [5] it was shown that every finite power of a space has Menger's property if, and only if, for every sequence $(\mathcal{U}_n : n < \omega)$ of ω -covers of X there is a sequence $(\mathcal{V}_n : n < \omega)$ of finite sets such that for each n , $\mathcal{V}_n \subseteq \mathcal{U}_n$, and $\cup_{n < \omega} \mathcal{V}_n$ is an ω -cover.

Sakai introduced a stronger form of countable fan tightness, called *countable strong fan tightness*. A space has countable strong fan tightness if for each element x , if $(A_n : n < \omega)$ is a sequence of subsets of the space such that for each n $x \in \overline{A_n}$, then there is a sequence $(x_n : n < \omega)$ such that for each n $x_n \in A_n$, and such that $x \in \overline{\{x_n : n < \omega\}}$. Sakai then proves – [10]: $C_p(X)$ has countable strong fan tightness if, and only if, each finite power of X has Rothberger's property C'' : A space has Rothberger's property C'' if there is for every sequence $(\mathcal{U}_n : n < \omega)$ of open covers, a corresponding sequence $(U_n : n < \omega)$ such that for each n $U_n \in \mathcal{U}_n$, and $\{U_n : n < \omega\}$ is a cover. Rothberger introduced this property in his 1938 paper [8]. Sakai further shows that every finite power of X has Rothberger's property C'' if, and only if, for every sequence $(\mathcal{U}_n : n < \omega)$ of ω -covers of the space, there is a sequence $(U_n : n < \omega)$ such that for each n we have $U_n \in \mathcal{U}_n$, and $\{U_n : n < \omega\}$ is an ω -cover.

A space has the *strong Fréchet property* if, whenever a point is in the closure of some set, then there is a sequence in that set converging to the point. Let us now say that an open cover of a space is a γ -cover if it is infinite and each element of the space is in all but finitely many of the sets in the cover. Gerlits and Nagy proved in [3] that $C_p(X)$ has the strong Fréchet property if, and only if, for every sequence $(\mathcal{U}_n : n < \omega)$ of ω -covers of X there is a sequence $(U_n : n < \omega)$ such that for each n $U_n \in \mathcal{U}_n$, and $\{U_n : n < \omega\}$ is a γ -cover of X . This property of X is nowadays called the γ -property.

Another property in the same spirit as these, and which is used often for numerical sequences, is as follows: A space has the *sequence selection property* if for each x , for every sequence $(\sigma_n : n < \omega)$ where each σ_n is a sequence converging to x , there is a sequence $(x_n : n < \omega)$ such that for each n x_n is a term of σ_n , and $(x_n : n < \omega)$ converges to x . As we shall show, this property is related to the following covering property which was introduced by Hurewicz in [3]: A space has the *Hurewicz property* if there is for every sequence $(\mathcal{U}_n : n < \omega)$ of open covers of the space a sequence $(\mathcal{V}_n : n < \omega)$ of finite sets such that for each n , $\mathcal{V}_n \subseteq \mathcal{U}_n$, and each element of the space is in all but finitely many of the sets $\cup \mathcal{V}_n$. It was pointed out in [11] that the Hurewicz property is equivalent to the covering property obtained by restricting the \mathcal{U}_n 's to being γ -covers.

We shall say that a sequence $(f_n : n < \omega)$ of elements of $C_p(X)$ converges *pointwise monotonically* to \mathfrak{o} if for each $x \in X$ the sequence $(f_n(x) : n < \omega)$ of real numbers is monotonic (*i.e.*, for each n $f_n(x) \geq f_{n+1}(x)$) and converges to 0. Let us say that $C_p(X)$ has the *monotonic sequence selection property* if: There is, for every sequence $(\sigma_n : n < \omega)$ such that each σ_n is a sequence in $C_p(X)$ which converges pointwise monotonically to \mathfrak{o} , a sequence $(f_n : n < \omega)$ such that for each n f_n is a term of σ_n , and such that $(f_n : n < \omega)$ converges pointwise to \mathfrak{o} .

Recall that a space X is normal if there is for every pair C and D of pairwise disjoint nonempty closed subsets of X a continuous function f from X to \mathbb{I} such that for each $c \in C$ $f(c) = 0$, and for each $d \in D$, $f(d) = 1$. It is *perfectly normal* if it is normal and every closed set is the intersection of countably many open sets. Every normal space in which each one-element subset is closed is also a Tychonoff space (while the converse is not true).

Theorem 1. *For a perfectly normal space X , the following are equivalent:*

- a. X has the Hurewicz property.
- b. $C_p(X)$ has the monotonic sequence selection property.

So as to clearly display what actually is being proved, we present the proof of Theorem 1 in two lemmas.

Lemma 2. *Let X be a Tychonoff space which has the Hurewicz property. Then $C_p(X)$ has the monotonic sequence selection property.*

Proof. For each m , let $(f_n^m : n < \omega)$ be a sequence in $C_p(X)$ which is pointwise monotonically converging to \mathfrak{o} . For each m and n , define

$$U_n^m = \{x \in X : |f_n^m(x)| < (\frac{1}{2})^m\}.$$

Since each f_n^m is continuous, each U_n^m is an open subset of X . Then define, for each m ,

$$\mathcal{U}_m = \{U_n^m : n < \omega\}.$$

Since the sequence $(f_n^m : n < \omega)$ converges to \mathfrak{o} , \mathcal{U}_m is an open cover of X . Since the sequence converges pointwise monotonically to \mathfrak{o} we see that $U_n^m \subseteq U_k^m$ whenever $n \leq k$.

It follows that, for each \mathcal{U}_m , either it is a finite cover of X , or else it is a γ -cover of X . For each m such that \mathcal{U}_m is a finite cover of X let n_m be the least n such that $U_n^m = X$. If there are only finitely many m for which \mathcal{U}_m is not a finite cover of X , then we choose the n_m 's for the remaining finitely many m 's arbitrarily. The sequence $(f_{n_m}^m : m < \omega)$ selected like this converges pointwise to \mathfrak{o} .

Thus, assume that there are infinitely many m such that \mathcal{U}_m is not a finite cover of X . Apply the Hurewicz property to these and find for each m an n_m such that the sequence $(U_{n_m}^m : \mathcal{U}_m \text{ not finite})$ is a γ -cover of X . Combining these n_m 's with the ones selected for the m 's for which \mathcal{U}_m is finite, we find a sequence $(f_{n_m}^m : m < \omega)$ which pointwise converges to \mathfrak{o} . □

Lemma 3. *If X is perfectly normal and if $C_p(X)$ has the monotonic sequence selection property, then X has the Hurewicz property.*

Proof. For each m let \mathcal{U}_m be a γ -cover of X . We may assume that each \mathcal{U}_m is countable. Enumerate each \mathcal{U}_m bijectively as $(U_n^m : n < \omega)$.

Since X is perfectly normal, each open set can be represented as an increasing union of closed sets. Thus, for each m and n let $(U_{n,k}^m : k < \omega)$ be an increasing sequence of closed sets such that U_n^m is a union of these sets.

By Urysohn's Lemma we choose for each m , n and k a continuous function

$$f_{n,k}^m : X \rightarrow \mathbb{I}$$

such that if $x \notin U_n^m$ then $f_{n,k}^m(x) = 1$, and if $x \in U_{n,k}^m$, then $f_{n,k}^m(x) = 0$. Then for each m and n define:

$$g_n^m(x) = \prod_{j \leq n} f_{j,n}^m(x).$$

Then each g_n^m is in $C_p(X)$, and $(g_n^m : n < \omega)$ converges to \mathbf{o} pointwise monotonically.

Apply the sequence selection property to the sequence $((g_n^m : n < \omega) : m < \omega)$ of sequences from $C_p(X)$. For each m we find an n_m such that the sequence $(g_{n_m}^m : m < \omega)$ converges to \mathbf{o} pointwise.

Consider any $x \in X$. Then $(g_{n_m}^m(x) : m < \omega)$ converges to 0, and so we may fix an M such that $|g_{n_m}^m(x)| < 1$ whenever $m \geq M$. This implies that for an $m \geq M$ there is a $j \leq n_m$ such that $|f_{j,n_m}^m(x)| < 1$, which in turn implies that for each $m \geq M$ there is a $j \leq n_m$ such that $x \in U_j^m$.

Thus, if for each m we let \mathcal{V}_m be the finite subset $\{U_j^m : j \leq n_m\}$ of \mathcal{U}_m , then the sequence $(\mathcal{V}_m : m < \omega)$ witnesses the Hurewicz property for the sequence $(\mathcal{U}_m : m < \omega)$. \square

The following property was introduced in [11]: A space X has property $S_1(\Gamma, \Gamma)$ if there is for every sequence $(\mathcal{U}_n : n < \omega)$ of γ -covers of X a sequence $(U_n : n < \omega)$ such that for each n $U_n \in \mathcal{U}_n$, and such that $\{U_n : n < \omega\}$ is a γ -cover of X . This property was further investigated in [5] where it was shown that no uncountable compact set of real numbers has this property. Thus, this property is strictly stronger than the Hurewicz property.

Theorem 4. *If the space X has property $S_1(\Gamma, \Gamma)$, then $C_p(X)$ has the sequence selection property.*

Proof. For each m , let $(f_n^m : n < \omega)$ be a sequence from $C_p(X)$ which converges pointwise to \mathbf{o} . Fix m and for each n define

$$U_n^m = \{x \in X : |f_n^m(x)| < (\frac{1}{2})^m\}.$$

Then for each m and for each x , for all but finitely many n $x \in U_n^m$. Thus, for each m , either $\mathcal{U}_m = \{U_n^m : n < \omega\}$ contains X as an element, or else it is a γ -cover of X .

For each m such that \mathcal{U}_m has X as a member, let n_m be the least n with $U_n^m = X$. Consider the remaining \mathcal{U}_m 's: If there are only finitely many left we choose n_m arbitrarily for each of those finitely many m 's; then the sequence $(f_{n_m}^m : m < \omega)$ is as desired. Thus, suppose that there are infinitely many m for which \mathcal{U}_m is a γ -cover of X . Apply property $S_1(\Gamma, \Gamma)$ to the sequence of those and choose for each of these m 's an n_m such that the corresponding $U_{n_m}^m$'s form a γ -cover of X . Then combining these n_m 's with the earlier selected ones we find a sequence $(f_{n_m}^m : m < \omega)$ which is pointwise convergent to \mathbf{o} . \square

COMPARISON WITH OTHER CLOSURE PROPERTIES

We give a short comparison of the closure properties mentioned before, and the sequential properties introduced here. There are several other sequential properties which have subtle differences with what we introduced here, but these will be discussed and compared elsewhere. The survey papers [7] and [12] contain information on many of the concepts discussed in this section.

A set of real numbers is a *Sierpiński set* if it is uncountable and yet its intersection with every set of Lebesgue measure zero is countable. A set of real numbers is a *Lusin set* if it is uncountable and its intersection with every set of first Baire category is countable.

It is easy to see that γ -sets have property $S_1(\Gamma, \Gamma)$. Sierpiński sets also have property $S_1(\Gamma, \Gamma)$. γ -sets have Rothberger's property C'' . Since sets with property C'' have (strong) measure zero, Sierpiński sets do not have Rothberger's property C'' , and *ipso facto* they do not have the γ -property. Sierpiński showed that the Continuum Hypothesis implies the existence of Sierpiński sets. Thus, the Continuum Hypothesis implies that there are sets X of real numbers for which $C_p(X)$ has the sequence selection property, but not the strong Fréchet property, and (by Sakai's theorem) also not countable strong fan tightness.

Using the Continuum Hypothesis one could also construct a Sierpiński set X such that the algebraic sum $X + X$ is the set of all irrational numbers. For such a Sierpiński set X , X^2 does not have Menger's covering property, and thus by Arkhangel'skii's theorem, $C_p(X)$ does not have countable fan tightness. It follows that the sequence selection property does not imply countable fan tightness.

For two functions f and g from ω to ω we write $f \prec g$ to denote that for all but finitely many n we have $f(n) < g(n)$. The set of these functions is denoted ${}^\omega\omega$, and \prec is a partial order on it, usually called the *eventual domination order*. A subset of ${}^\omega\omega$ is *unbounded* in the eventual domination order if there is no single element which eventually dominates each element of the subset. According to a theorem of Hurewicz in [4] a separable metric space X has the Hurewicz property if, and only if, every continuous image of X into ${}^\omega\omega$ is bounded. One can show that if X is a Lusin subset of ${}^\omega\omega$, then it is an unbounded subset in the eventual domination order. Thus, no Lusin set has the Hurewicz property. Lusin showed that the Continuum Hypothesis implies the existence of a Lusin set. One can even show that the Continuum Hypothesis implies the existence of a Lusin set, all of whose finite powers have Rothberger's property C'' – this is done in [5]. Let X be such a Lusin set. Then by Sakai's theorem $C_p(X)$ has countable strong fan tightness, and by Theorem 1 it does not have the monotonic sequence selection property – whence in particular it does not have the sequence selection property.

These examples suggest another property which we'll call the *weak sequence selection property*: A space has the weak sequence selection property at a point if there is for every sequence $(\sigma_n : n < \omega)$ of sequences from the space, each converging to that point, a corresponding sequence $(x_n : n < \omega)$ such that for each n x_n is a term of σ_n , and such that the point under consideration is in the closure of the set $\{x_n : n < \omega\}$. It is clear that the sequence selection property and also countable strong fan tightness both imply the weak sequence selection property. In light of the examples just discussed we see that even for the special situation where the space is $C_p(X)$ for some set X of real numbers, neither of these notions is provably equal to the weak sequence selection property. Moreover, Arkhangel'skii's theorem

and the Sierpiński set whose square does not have the Menger property also show that the weak sequence selection property for $C_p(X)$ does not imply countable fan tightness. The Lusin set all of whose finite powers have Rothberger's property C'' together with Sakai's theorem shows that the weak sequence selection property of $C_p(X)$ also doesn't imply the monotonic sequence selection property.

Some of these non-implications can also be arrived at by cardinality considerations: The minimal cardinality of an unbounded subset of ${}^\omega\omega$ is denoted \mathfrak{b} . It is well known that \mathfrak{b} is an uncountable cardinal number. A subset of ${}^\omega\omega$ is *cofinal* in the eventual domination order if there is for each function in ${}^\omega\omega$ an element of that subset which eventually dominates the given function. The minimal cardinality of a cofinal subset of ${}^\omega\omega$ is denoted \mathfrak{d} . In [5] it was shown that \mathfrak{b} is the least cardinality of a space which does not have the Hurewicz covering property, and \mathfrak{b} is the minimal cardinality of a space which does not have the property $S_1(\Gamma, \Gamma)$. Thus,

Lemma 5. *The minimal cardinality of a set of real numbers X for which $C_p(X)$ does not have the sequence selection property or the monotonic sequence selection property is \mathfrak{b} .*

The cardinal number \mathfrak{b} also arose in another sort of sequence selection property that was studied by Blass and Jech – [2]. For a set X the symbol $F(X)$ denotes the set of all real-valued functions with domain X . $F(X)$ is said to have the *Egoroff property* if for any sequence $(\sigma_n : n < \omega)$ where each σ_n is a sequence from $F(X)$ which pointwise monotonically increases to $f \in F(X)$, there is a sequence $(g_n : n < \omega)$ which pointwise monotonically increases to f and for each n and each m there is a term of σ_m which pointwise exceeds g_n . According to Theorem 2 of [2] $F(X)$ has the Egoroff property if, and only if, the cardinality of X is less than \mathfrak{b} . Thus, when considering the Egoroff property we may as well assume that X is a set of real numbers. For X a set of real numbers $C_p(X)$ is a proper subset of $F(X)$, and the Egoroff property for $F(X)$ implies fairly directly the monotonic sequence selection property for $C_p(X)$. But more is true: If $F(X)$ has the Egoroff property, then $C_p(X)$ has the sequence selection property. The reason is that by Theorem 2 of [2] X has cardinality less than \mathfrak{b} , whence X has property $S_1(\Gamma, \Gamma)$; Theorem 4 then implies that $C_p(X)$ has the sequence selection property.

It was also showed in [5] that the minimal cardinality of a set of real numbers not all of whose finite powers have the Menger property is \mathfrak{d} . In light of Arkhangel'skii's theorem, the minimal cardinality for a set X of real numbers such that $C_p(X)$ doesn't have countable tightness is \mathfrak{d} .

Let \mathcal{M} denote the collection of first category subsets of the real line. Then $\text{cov}(\mathcal{M})$ is the least cardinal number κ such that the real line is the union of κ first category subsets. According to Baire's category theorem $\text{cov}(\mathcal{M})$ is uncountable. Rothberger (essentially) showed in the third section of [9] that the minimal cardinality of a set of real numbers which doesn't have property C'' is $\text{cov}(\mathcal{M})$. In [5] it was shown that this is also the critical number for sets of real numbers not all of whose finite powers have property C'' . Thus, the minimal cardinality of a set X of real numbers for which $C_p(X)$ doesn't have countable strong fan tightness is $\text{cov}(\mathcal{M})$.

A family \mathcal{F} of subsets of ω has a *pseudo-intersection* if there is an infinite set A such that for each $F \in \mathcal{F}$ the set $A \setminus F$ is finite. The symbol \mathfrak{p} denotes the minimal cardinality of a family \mathcal{F} which has no pseudo-intersection, and yet any finite subset of the family has an infinite intersection. It is also well-known that the

minimal cardinality of a set of real numbers which does not have the γ -property is \mathfrak{p} . In light of Gerlits and Nagy's theorem the minimal cardinality of a set X of real numbers such that $C_p(X)$ doesn't have the strong Fréchet property is \mathfrak{p} .

It is known that $\mathfrak{p} \leq \min\{\text{cov}(\mathcal{M}), \mathfrak{b}\}$ and that $\mathfrak{d} \geq \max\{\mathfrak{b}, \text{cov}(\mathcal{M})\}$. It is also known that no equations between any two of these cardinal numbers is provable, and that neither $\text{cov}(\mathcal{M}) \leq \mathfrak{b}$ nor $\mathfrak{b} \leq \text{cov}(\mathcal{M})$ is provable. This gives another explanation for the non-relatedness facts pointed out earlier with the Continuum Hypothesis.

Problem 1. Does countable fan tightness of $C_p(X)$ imply the weak sequence selection property?

Problem 2. Does the monotonic sequence selection property of $C_p(X)$ imply the weak sequence selection property?

REFERENCES

- [1] A.V. Arkhangel'skiĭ, *Hurewicz spaces, analytic sets and fan tightness of function spaces*, **Soviet Mathematical Doklady** 33 (1986), 396 – 399. MR **87i**:54079
- [2] A. Blass and T. Jech, *On the Egoroff property of pointwise convergent sequences of functions*, **Proceedings of the American Mathematical Society** 98 (1986), 524 – 526. MR **87i**:54013
- [3] J. Gerlits and Zs. Nagy, *Some properties of $C_p(X)$ I*, **Topology and its Applications** 14 (1982), 151 – 161. MR **84f**:54021
- [3] W. Hurewicz, *Über die Verallgemeinerung des Borelschen Theorems*, **Mathematische Zeitschrift** 24 (1925), 401 – 421.
- [4] W. Hurewicz, *Über Folgen stetiger Funktionen*, **Fundamenta Mathematicae** 9 (1927), 193 – 204.
- [5] W. Just, A.W. Miller, M. Scheepers, P.J. Szeptycki, *Combinatorics of open covers (II)*, **Topology and its Applications** 73 (1996), 241–266.
- [6] K. Menger, *Einige Überdeckungssätze der Punktmengenlehre*, **Sitzungsberichte. Abt. 2a, Mathematik, Astronomie, Physik, Meteorologie und Mechanik** (Wiener Akademie) 133 (1924), 421 - 444.
- [7] A.W. Miller, *Special subsets of the real line*, **Handbook of Set-Theoretic Topology** (K. Kunen and J.E. Vaughan, eds.), Elsevier Science Publishers (1984), 143 – 159. MR **86i**:54037
- [8] F. Rothberger, *Eine Verschärfung der Eigenschaft C''* , **Fundamenta Mathematicae** 30 (1938), 50 – 55.
- [9] F. Rothberger, *Sur les familles indénombrables de suites de nombres naturels et les problèmes concernant la propriété C* , **Proceedings of the Cambridge Philosophical Society** 37 (1941), 109 – 126. MR **2**:352a
- [10] M. Sakai, *Property C'' and function spaces*, **Proceedings of the American Mathematical Society** 104 (1988), 917 – 919. MR **89m**:54004
- [11] M. Scheepers, *Combinatorics of open covers I: Ramsey theory*, **Topology and its Applications**, 69 (1996), no. 1, 31–62. CMP 96:09
- [12] E.K. van Douwen, *The integers and Topology*, **Handbook of Set-Theoretic Topology** (K. Kunen and J.E. Vaughan, eds.), Elsevier Science Publishers (1984), 111 – 167. MR **87f**:54008

DEPARTMENT OF MATHEMATICS, BOISE STATE UNIVERSITY, BOISE, IDAHO 83725
E-mail address: marion@math.idbsu.edu