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A CORRECTION TO "SMALL REPRESENTATIONS OF FINITE DISTRIBUTIVE LATTICES AS CONGRUENCE LATTICES"

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(Communicated by Lance W. Small)

We would like to thank Ralph Freese for pointing out that Lemma 1 of this paper (these Proceedings **123** (1995), 1959–1961) is stated and proved under the assumption "$\Phi_1 \prec \Phi_2$ in $\operatorname{Con} L$", but it is used under the assumption "$\Phi_1 \prec \Phi_2$ in the poset of join-irreducible congruences of L".

We now correctly state and prove Lemma 1:

Lemma 1. *Let L be a finite lattice, and let v_i, $u_i \in L$ satisfy $v_i \prec u_i$, for $i = 1$, 2. Let $\Phi_i = \Theta(v_i, u_i)$, for $i = 1$, 2. If $\Phi_1 \prec \Phi_2$ in the poset of join-irreducible congruences of L, then there is a three-element chain $\{e_1, h, e_2\}$ in L such that $\Phi_i = \Theta(h, e_i)$, for $i = 1$, 2, and $e_1 < h < e_2$ or $e_2 < h < e_1$.*

Proof. We assume that the reader is familiar with the basic concepts and notations of projectivity in lattices. We follow the notation on pages 129–130 of [1]. Since $v_1 \equiv u_1\,(\Theta(v_2, u_2))$ and $v_1 \prec u_1$, by Theorem III.1.2 *ibid*, there is a sequence of projectivities

$$u_2/v_2 = y_1/x_1 \nearrow y_2/x_2 \searrow y_3/x_3 \nearrow \dots \searrow y_n/x_n = u_1/v_1,$$

for some natural number $n > 1$. Obviously,

$$\Phi_2 = \Theta(x_1, y_1) = \Theta(x_2, y_2) \geq \Theta(x_3, y_3) \geq \cdots \geq \Theta(x_n, y_n) = \Phi_1.$$

Since $\Phi_2 > \Phi_1$, there is a smallest i satisfying $\Theta(x_i, y_i) < \Phi_2$; obviously, $3 \leq i$ and $i \leq n$.

Let i be odd, and let $z = x_{i-1} \vee y_i$. Then $\Theta(x_{i-1}, z) = \Theta(x_i, y_i) < \Phi_2$, but $\Theta(x_{i-1}, y_{i-1}) = \Phi_2$, so $\Theta(z, y_{i-1}) = \Phi_2$. Since $u_1 \equiv v_1(\Theta(x_{i-1}, z))$ and $u_1 \prec v_1$, there are $u, v \in [x_{i-1}, z]$, $v \prec u$, such that $\Theta(u_1, v_1) \leq \Theta(u, v)$. So $\Phi_1 = \Theta(u_1, v_1) \leq \Theta(u, v) < \Phi_2$, and $\Theta(u, v)$ is a join-irreducible congruence, hence by the assumption on Φ_1 and Φ_2, it follows that $\Phi_1 = \Theta(u, v)$. Hence we can choose $e_2 = x_{i-1}$, $h = z$, and $e_1 = y_{i-1}$.

If i is even, then we proceed dually. □

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