PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 126, Number 8, August 1998, Pages 2511–2512
S 0002-9939(98)04844-8

ERRATA TO "NORMALIZERS OF NEST ALGEBRAS"

KEITH J. COATES

(Communicated by David R. Larson)

In [1], Proposition 5 has an error. The proposition purports to show that $\mathcal{S}_{\mathcal{N}}$, the normalizing semigroup for the nest algebra $\mathcal{N}$, is closed in the strong operator topology. The proof begins with the net T_λ in $\mathcal{S}_{\mathcal{N}}$ converging strongly to $T \in B(H)$. It then proceeds to (correctly) establish the first of the two criteria set forth in Theorem 2 needed to conclude that $T \in \mathcal{S}_{\mathcal{N}}$. The last sentence of the proof then asserts that the second criterion can be established in a similar manner by just replacing all the operators with their adjoints. This is the problem: since the adjoint operation is not strongly continuous, we cannot assume that T_λ^* converges strongly to T^*, which is needed to make the analogous argument work to establish the second criterion.

The following replacement for Proposition 5 fixes this, and strengthens the proposition. Since the adjoint operation is weakly continuous, we really do need only establish the first criterion, the second one following by replacing the operators involved with their adjoints.

Proposition 5. *$\mathcal{S}_{\mathcal{N}}$ is weakly closed.*

Proof. Let $\{T_\lambda\}_{\lambda\in\Lambda}$ be a net in $\mathcal{S}_{\mathcal{N}}$ converging weakly to $T \in B(H)$. Suppose there is $N \in \mathcal{N}$ with $\Phi_T(N)T(I-N) \neq 0$. Let $x \in (I-N)$ be such that $\Phi_T(N)Tx = y \neq 0$. Since $\langle T_\lambda x, y\rangle \to \langle Tx, y\rangle$, there is $\lambda_0 \in \Lambda$ such that $\lambda \geq \lambda_0$ implies that $\Phi_T(N)T_\lambda x = y_\lambda \neq 0$. But, $\Phi_{T_\lambda}(N)T_\lambda x = 0$ for all $\lambda \in \Lambda$, so $\Phi_{T_\lambda}(N) < \Phi_T(N)$ for all $\lambda \geq \lambda_0$.

If $P \in \mathcal{N}$, $P < \Phi_T(N)$, there is $z \in N$ such that $Tz \in \Phi_T(N)$, $Tz \notin P$. Let $w = (\Phi_T(N) - P)Tz$. Since $\langle T_\lambda z, w\rangle \to \langle Tz, w\rangle \neq 0$, there is $\lambda_1 \geq \lambda_0$ such that $\lambda \geq \lambda_1$ implies $T_\lambda z \notin P$, so that $\Phi_{T_\lambda}(N) > P$. We thus conclude that $\Phi_{T_\lambda}(N) \to \Phi_T(N)$ in the order topology, which is equivalent to convergence in the strong operator topology.

Now suppose there is $P \in \mathcal{N}$, $P < \Phi_T(N)$ such that $PTx \neq 0$. Since $\langle PT_\lambda x, PTx\rangle$ converges to the nonzero number $\|PTx\|^2$, there is $\lambda_2 \in \Lambda$ such that $\lambda \geq \lambda_2$ implies that $PT_\lambda x \neq 0$. But then there is $\lambda_3 \geq \lambda_2$ such that $\lambda \geq \lambda_3$ implies that $\Phi_{T_\lambda}(N)T_\lambda x \neq 0$, a contradiction since $T_\lambda \in \mathcal{S}_{\mathcal{N}}$ for all $\lambda \in \Lambda$. Thus, $PTx = 0$ for all $P \in \mathcal{N}$ with $P < \Phi_T(N)$. So for $\lambda \geq \lambda_0$, $\Phi_{T_\lambda}(N)Tx = 0$. But $\Phi_{T_\lambda}(N)T(I-N)$ converges strongly to $\Phi_T(N)T(I-N)$, so that $\Phi_T(N)Tx = 0$, contradicting the hypothesis.

Received by the editors November 30, 1997.
1991 *Mathematics Subject Classification.* Primary 47D25; Secondary 47D03.

Thus, $\Phi_T(N)T(I-N)=0$ so that $TN=\Phi_T(N)T$ for every $N\in\mathcal{N}$. A similar argument shows that $T^*N=\Phi'_T(N)T^*$ for every $N\in\mathcal{N}$, so $T\in\mathcal{S}_\mathcal{N}$ and $\mathcal{S}_\mathcal{N}$ is weakly closed. $\square$

References

[1] Keith J. Coates, *Normalizers of nest algebras*, Proc. Amer. Math. Soc. **126** (1998), 159–165. CMP 98:02

Department of Mathematics, Illinois Wesleyan University, Bloomington, Illinois 61702

E-mail address: kcoates@sun.iwu.edu