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DECOMPOSITION THEOREMS IN ORTHOMODULAR POSETS: THE PROBLEM OF UNIQUENESS

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Abstract. We provide some conditions which are equivalent to the uniqueness of Hewitt-Yosida-type decomposition and Lebesgue-type decomposition for real valued additive functions defined on orthomodular posets.

1. Introduction

In the last years many classical decomposition theorems have been proved in the setting of additive functions defined on orthomodular posets ([12], [4], [10], [3], [2], [5], [6], [13]). However it is well known that while in some cases it is possible to prove the uniqueness of the decomposition ([4], [3], [2], [13]), it fails in many others (see [7, n.11] and [11] introduction).

Recently G.T. Rüttimann ([11]) has given some conditions that ensure the uniqueness of decomposition for a Hewitt-Yosida-type theorem. This result is obtained through the concept of hereditary orthomodular posets and filtering functions.

In this paper we prove a uniqueness theorem for an abstract decomposition of the set $J^+(L)$ of non-negative additive functions defined in an orthomodular poset L by means of a solid subset.

We obtain this result by using an appropriate generalization of the notion of hereditary orthomodular poset and filtering function. This procedure also provides some conditions that ensure the uniqueness of the Lebesgue-type and Hewitt-Yosida-type decompositions.

2. Definitions

We will work in orthomodular lattices or in orthocomplete orthomodular posets; for the sake of completeness we give some basic definitions (we refer to [1], [8] and [9] for all the results about the structures we use):

Consider $(L, \leq, 0, 1, ')$ where L is a set, $\leq$ is a binary relation on L, 0 and 1 are two distinct elements of L and $'$ is a function from L to L. We say that $(L, \leq, 0, 1, ')$ is an **orthomodular poset** (OMP) if the following conditions are satisfied:

i) $(L, \leq)$ is a partially ordered set,

ii) 0 is the least element of L and 1 is the greatest one,

iii) $' : L \to L$ is a decreasing function such that $p'' = p$ and $p \wedge p' = 0$ for all $p \in L$,

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iv) if $p, q \in L$ with $p \leq q'$ then $p \vee q$ exists in L,

v) if $p, q \in L$ with $p \leq q$ then $q = p \vee (p' \wedge q)$(orthomodular law).

If in addition $(L, \leq)$ is a lattice, then $(L, \leq, 0, 1, ')$ is called an **orthomodular lattice** (OML)

A distributive orthomodular lattice is called a **Boolean algebra**.

If p and q in L satisfy Condition iv) above, we call them **orthogonal** and we write $p \perp q$. A subset M of L is said to be orthogonal if every element x in M is orthogonal to every element in $M \setminus \{x\}$.

If in an OMP L the join of any system of mutually orthogonal elements exists, we say that L is an **orthocomplete orthomodular poset**.

From now on, by $(L, \leq, 0, 1, ')$ we will denote an OML or an orthocomplete OMP. For brevity we will indicate it by L.

We denote by $J^+(L)$ (respectively $J_c^+(L)$) the set of all the finitely additive (resp. completely additive [1]) nonnegative elements of $\mathbb{R}^L$.

$J^+(L) = \{\alpha : L \to \mathbb{R}^+ : \alpha \text{ is finitely additive}\}$;

$J_c^+(L) = \{\alpha : L \to \mathbb{R}^+ : \alpha \text{ is completely additive}\}$.

We will refer to the usual (pointwise) order relation on $J^+(L)$: if α and β are elements of this set, we will write $\alpha \leq \beta$ to mean that $\alpha(p) \leq \beta(p) \quad \forall p \in L$.

If $\alpha \leq \beta$, α is said to be **dominated** by β.

If $p \in L$, we denote by L_p the set of all the elements of L which are less than or equal to p; so $L_p = \{x \in L : x \leq p\}$.

L_p inherits in a natural way the structure of L in the sense that if $\leq_p$ is the restriction of $\leq$ to L_p and $'^p$ is defined by $'^p : x \in L_p \to x' \wedge p \in L_p$, then $\leq_p$ is still an order relation and $'^p$ an orthocomplementation on L_p, so $(L_p, \leq_p, 0, p, '^p)$ is an OML or an orthocomplete OMP itself.

If we denote by μ_p the restriction to L_p of an element μ of $J^+(L)$, it is obvious that

$\mu \in J^+(L) \Rightarrow \mu_p \in J^+(L_p)$,

$\mu \in J_c^+(L) \Rightarrow \mu_p \in J_c^+(L_p)$.

Let C be a subset of $J_c^+(L)$ such that

1) $\alpha \in J^+(L),\ \beta \in C,\ \alpha \leq \beta \ \Rightarrow \alpha \in C$,

2) if $\alpha \in C$ and $r \in \mathbb{R}^+$ then $r\alpha \in C$.

Given an element p of L, denote by C_p the set

$$C_p = \{\alpha \in J^+(L_p) : \exists \beta \in C : \alpha \leq \beta_p\}.$$

Let us note some facts:

- $\alpha \in C \Leftrightarrow r\alpha \in C \quad \forall r \in \mathbb{R}^+$,
- C_p is a subset of $J_c^+(L_p)$ which verifies Properties 1) and 2),
- $C_1 = C$.

In order to study the decomposition of the elements of $J^+(L)$ in two parts, one in C and the other "far from C", we need to make precise the concept of "to be far from a set".

We mean, by saying that a function λ is "far from C", that the null function is the only element of C which is dominated by λ.

[1] Recall that μ is completely additive (c.a.) if the equality $\sum_{x \in M} \mu(x) = \mu(\bigvee M)$ holds whenever M is an orthogonal part of L such that the supremum of M exists in L.

If e.g. we think C to be the set of all the functions which are absolutely continuous with respect to a fixed one, this is a way to define the orthogonality with respect to that function. So we define $C^{\#} = \{\lambda \in J^+(L) : \mu \in C,\ \mu \leq \lambda \Rightarrow \mu \equiv 0\}$ and analogously, $\forall p \in L$,

$$C_p^{\#} = \{\lambda \in J^+(L_p) : \mu \in C_p,\ \mu \leq \lambda \Rightarrow \mu \equiv 0\}.$$

From now on, C will denote a subset of $J_c^+(L)$ verifying properties 1) and 2).

3. Heredity

While it is obvious (by definition of C_P) that μ_p belongs to C_p when μ is in C, it is not true in general that $\mu \in C^{\#}$ implies $\mu_p \in C_p^{\#}$ as we can see in the following example.

Example 1. Let us consider an OML L with four atoms (see Figure 1). In this case $J^+(L) = J_c^+(L)$ since L is finite.

Let us take the set $C = \{\mu \in J^+(L) : \mu(q) = \mu(1)\}$. It is easy to check that
- C verifies conditions 1) and 2) of Section 2;
- $C^{\#} = \{\mu \in J^+(L) : \mu(q) = 0\}$;
- $C_p = J^+(L_p)$;
- the only element in $C_p^{\#}$ is the null function.

If we take a function in $C^{\#}$ which has positive value in p, its restriction to L_p does not belong to $C_p^{\#}$.

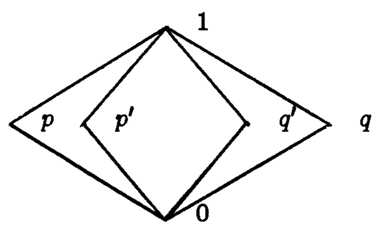

FIGURE 1

If L is such that the restrictions to L_p of the elements of $C^{\#}$ belong to $C_p^{\#}$ for every element p in L, then L is called C-hereditary:

Definition 1. L is C-**hereditary** if $\mu \in C^{\#} \Rightarrow \mu_p \in C_p^{\#} \quad \forall p \in L$.

Proposition 1. *Every Boolean algebra is C-hereditary for every $C \subseteq J_c^+(L)$ which verifies 1) and 2) of Section 2.*

Proof. Let $\lambda \in C^{\#}$, $p \in L$ and $\nu \in C_p$ be such that ν is dominated by λ_p. Define $\tilde{\nu} : x \in L \to \nu(x \wedge p)$; obviously $\tilde{\nu}$ is an element of $J^+(L)$. Moreover we have

$$\tilde{\nu}(x) = \nu(x \wedge p) \leq \lambda_p(x \wedge p) = \lambda(x \wedge p) \leq \lambda(x) \ ,$$

so $\tilde{\nu}$ is dominated by λ.

If we show that $\tilde{\nu} \in C$, it will follow that $\tilde{\nu} \equiv 0$, and hence $\nu \equiv 0$.

Now, since ν belongs to C_p, there exists an element α in C such that ν is dominated by α_p and therefore

$$\tilde{\nu}(x) = \nu(x \wedge p) \leq \alpha_p(x \wedge p) = \alpha(x \wedge p) \leq \alpha(x).$$

This means that $\tilde{\nu}$ is dominated by α so it belongs to C.

Now, the desired conclusion holds.

We will use the following useful lemma (due to Rüttimann [11, Lemma 5.3]):

Lemma 1. *If $\lambda \in J^+(L)$, $\nu \in J_c^+(L)$, and for an element $p \in L$ it is $\lambda(p) < \nu(p)$, then there exists $q \in L_p \setminus \{0\}$ such that $\lambda(x) < \nu(x)$ for every $x \in L_q \setminus \{0\}$.*

In hereditary structures the existence of a decomposition ensures its uniqueness, in the sense that if we suppose that it is possible to decompose an element of $J^+(L)$ into the sum of two functions, one in C and the other in $C^{\#}$, then those two functions are uniquely determined. The precise meaning of this statement is explained by the following theorem.

Theorem 1. *Let L be C-hereditary and $\lambda \in J^+(L)$. If $\lambda = \alpha + \beta$ with $\alpha \in C$ and $\beta \in C^{\#}$, then this decomposition is unique.*

Proof. Let $\lambda = \alpha + \beta = \alpha' + \beta'$ where $\alpha, \alpha' \in C$; $\beta, \beta' \in C^{\#}$ and suppose that for some (nonzero) p in L it is $\alpha(p) \neq \alpha'(p)$. We may suppose that $\alpha(p) < \alpha'(p)$.

By the lemma above there exists an element q in $L_p \setminus \{0\}$ such that $\alpha(x) < \alpha'(x)$ for every x in $L_q \setminus \{0\}$.

Since the function $\nu = \alpha'_q - \alpha_q = \beta_q - \beta'_q$ is nonnegative and dominated by α'_q, it belongs to C_q.

On the other side ν is dominated by β_q which is an element of $C_q^{\#}$, hence ν is the null function. This implies that $\alpha'_q \equiv \alpha_q$ and so we have $\alpha(q) = \alpha'_q(q)$.

This is a contradiction, so the theorem is proved.

4. Filtering functions

The concept of C-filtering functions is useful to characterize the C-hereditary structures. Let us give the definition:

Definition 2. A subset I of L is said **filtering** if $\forall p \in L \setminus \{0\} \quad \exists q \in L_p \setminus \{0\}$ s.t. $q \in I$.

From the definition it easily follows that a set which contains a filtering set is a filtering set itself. Further, a filtering set contains all the atoms of L.

We will use the following useful characterization of filtering sets:

Proposition 2. *A subset I of L is a filtering set if and only if for any element p of L there exists a subset M of I such that*

1) *M is orthogonal,*
2) *$p = \bigvee M$.*

Proof. It can be proved as [11, Corollary 3.4], observing that according to Rüttimann a function ν is filtering if and only if $\ker \nu$ [2] is a filtering set.

Definition 3. A function $\nu \in J^+(L)$ is said C**-filtering** if $\ker \nu \cup (\bigcap_{\mu \in C} \ker \mu)$ is a filtering set.

We denote by $J^+_{f,C}(L)$ and $J^+_{f,C}(L_p)$ respectively the sets of C-filtering and C_p filtering functions.

Note that an element ν of C belongs to $J^+_{f,C}(L)$ if and only if $\ker \nu$ is a filtering set.

[2] For $\alpha \in J^+(L)$, $\ker \alpha = \{x \in L \ : \ \alpha(x) = 0\}$.

It is quite obvious that

Proposition 3. $\nu \in J^+_{f,C}(L) \Rightarrow \nu_p \in J^+_{f,C}(L_p) \quad \forall p \in L.$

Proof. If $x \in L_p \setminus \{0\}$, by hypothesis there exists $q \in L_p \setminus \{0\}$ such that $q \in \ker\nu \cup (\bigcap_{\mu\in C} \ker\mu)$.

If $q \in \ker\nu$, then $q \in \ker\nu_p$, because $q \leq p$.

If $q \in \bigcap_{\mu\in C} \ker\mu$, then $q \in \bigcap_{\mu\in C_p} \ker\mu$ because $\bigcap_{\mu\in C_p} \ker\mu = L_p \cap (\bigcap_{\mu\in C} \ker\mu)$.

In any case $q \in \ker\nu_p \cup (\bigcap_{\mu\in C_p} \ker\mu)$, so ν_p is filtering.

All the C-filtering functions are "far from C" in the sense that they are elements of $C^\#$:

Proposition 4. $J^+_{f,C}(L) \subseteq C^\#$.

Proof. If λ is in $J^+_{f,C}(L)$ and ν is an element of C which is dominated by λ, then from $\ker\lambda \subseteq \ker\nu$ we conclude that $\ker\lambda \cup (\bigcap_{\mu\in C} \ker\mu) \subseteq \ker\nu \cup (\bigcap_{\mu\in C} \ker\mu) = \ker\nu$.

So $\ker\nu$ is a filtering set; this is sufficient to say that $\nu \equiv 0$ because, from Proposition 2, it follows that a completely additive function is determined by the values it attains on a filtering set. $\square$

We are ready to prove the main theorem which says that, if we suppose that every element of $J^+(L)$ is decomposable into two parts, one in C and the other "far from C", then L is C-hereditary if and only if the set of all C-filtering functions coincides with the set of functions "far from C". More properly we have:

Theorem 2. *If $J^+(L) = C + C^\#$, then the following are equivalent:*

1) $J^+(L) = C \oplus J^+_{f,C}(L)$,
2) $J^+(L) = C + J^+_{f,C}(L)$,
3) $C^\# = J^+_{f,C}(L)$,
4) $C^\# \subseteq J^+_{f,C}(L)$,
5) *L is C-hereditary.*

Proof. We will show that 4) $\Leftrightarrow$ 5) and that 1) $\Rightarrow$ 2) $\Rightarrow$ 3) $\Rightarrow$ 1); this is enough for our aim because Proposition 4 ensures that 3) $\Leftrightarrow$ 4).

4) $\Rightarrow$ 5) Let ν be an element of $C^\#$ ($\subseteq J^+_{f,C}(L)$). From Proposition 3 we get $\nu_p \in J^+_{f,C}(L_p)$. Proposition 4 ensures that $\nu_p \in C^\#_p$, so we are done.

5) $\Rightarrow$ 4) Take λ in $C^\#$ and p in $L \setminus \{0\}$; we have to find an element $q \in L_p \setminus \{0\}$ such that $q \in \ker\lambda \cup (\bigcap_{\mu\in C} \ker\mu)$.

If $p \in \bigcap_{\mu\in C} \ker\mu$, then we can take $q = p$. If not, let us take a function $\nu \in C$ such that $\nu(p) > 0$ and $r \in \mathbb{R}^+$ such that $\lambda(p) < r\nu(p)$. By Lemma 1 there exists an element q in $L_p \setminus \{0\}$ such that $\lambda(x) < r\nu(x)$ for every x in $L_q \setminus \{0\}$. Moreover since ν belongs to C, $r\nu$ belongs to C and therefore $\lambda_q (\leq r\nu_q)$ is an element of C_q.

On the other side λ belongs to $C^\#$ and L is C-hereditary, so λ_q is an element of $C^\#_q$. Then $\lambda_q \equiv 0$ and hence q belongs to $\ker\lambda$.

Since $q \in L_p \setminus \{0\}$ is such that $q \in \ker\lambda \cup (\bigcap_{\mu\in C} \ker\mu)$, we have $\lambda \in J^+_{f,C}(L)$.

1) $\Rightarrow$ 2) is obvious.

2) $\Rightarrow$ 3) by Proposition 4 we only have to show that $C^\# \subseteq J^+_{f,C}(L)$.

If λ is an element of $C^\#$, by the hypothesis we have $\lambda = \lambda_1 + \lambda_2$, with $\lambda_1 \in C$ and $\lambda_2 \in J^+_{f,C}(L)$. But $\lambda_1 \leq \lambda$ and $\lambda_1 \in C$ imply that $\lambda_1 = 0$, so $\lambda = \lambda_2 \in J^+_{f,C}(L)$.

3) $\Rightarrow$ 1) $J^+(L) = C + C^\# = C + J^+_{f,C}(L)$, hence the uniqueness follows from Theorem 1.

Our definition of filtering functions is different from Rüttimann's one. It depends on C in the sense that we only require $\ker\lambda \cup (\bigcap_{\mu\in C} \ker\mu)$, not $\ker\lambda$, to be a filtering set (it is a weaker request). If for any p in $L\setminus\{0\}$ there exists in C an element μ such that $\mu(p) > 0$, then the two definitions coincide, because in this case it is $\bigcap_{\mu\in C}\ker\mu = \{0\}$.

The analogue in Rüttimann of the theorem above is Corollary 6.2, which we can obtain by taking $C = J^+_c(L)$; we do not need to suppose L to be c-positive (for Rüttimann L is c-positive iff $\bigcap_{\mu\in J^+_c(L)}\ker\mu = \{0\}$) just because of the different definition of filtering functions.

5. Applications

With suitable choices of C it is possible to obtain some results about uniqueness of Lebesgue-type or Hewitt-Yosida-type decomposition theorems. Before showing them, we recall some definitions:

Let μ and λ be elements of $J^+(L)$.

Definition 4. We say that μ is λ-**continuous** and we write $\mu \ll \lambda$ if $\forall\varepsilon > 0\ \exists\delta > 0$ such that $\lambda(p) < \delta \Rightarrow \mu(p) < \varepsilon$.

Definition 5. We say that μ is λ-**singular** and we write $\mu \perp \lambda$ if $\alpha \in J^+(L),\ \alpha \ll \lambda,\ \alpha \le \mu\ \Rightarrow \alpha \equiv 0$.

In other words, μ is λ-singular if the null function is the only additive function which is simultaneously dominated by μ and λ-continuous.

Note that these two definitions are more general with respect to the ones used for example by Rüttimann and Schindler in [12].

Example 2 (Lebesgue-type decomposition)**.** $C = \{\nu \in J^+(L) : \nu \ll \mu\}$ where μ is an element of $J^+_c(L)$.

It is easy to show that C verifies conditions 1) and 2) of Section 2; let us prove that $C \subseteq J^+_c(L)$:

Let $\{x_i : i \in I\}$ be an orthogonal subset of L such that $\bigvee_{i\in I}\ x_i$ exists in L. Since μ belongs to $J^+_c(L)$, hence

$$\mu\left(\bigvee_{i\in I}\ x_i\right) = \sum_{i\in I}\mu(x_i) = \lim_{F\in\mathcal{F}}\mu\left(\bigvee_{i\in F}\ x_i\right)$$

where $\mathcal{F} = \{F \subseteq I : F \text{ is finite}\}$.

This ensures that for any positive δ there exists F_0 in $\mathcal{F}$ such that

$$F\in\mathcal{F},\quad F \supseteq F_0 \Rightarrow \mu\left(\bigvee_{i\in I}\ x_i \setminus \bigvee_{i\in F}\ x_i\right) < \delta.$$

Now, for any positive ε there exists a positive δ such that $\mu(x) < \delta$ implies $\nu(x) < \varepsilon$. So, if F is an element in $\mathcal{F}$ which contains F_0, we have that $\nu(\bigvee_{i\in I}\ x_i \setminus \bigvee_{i\in F}\ x_i) < \varepsilon$. This means that $\nu(\bigvee_{i\in I}\ x_i) = \sum_{i\in I}\nu(x_i)$.

In this case $C^{\#} = \{\lambda \in J^+(L) : \alpha \in C,\ \alpha \leq \lambda \ \Rightarrow \alpha \equiv 0\} = \{\lambda \in J^+(L) : \lambda \perp \mu\}$ and

$$J^+_{f,C}(L) = \{\lambda \in J^+(L) : \ker \lambda \cup (\bigcap_{\nu \in C} \ker \nu) \text{ is a filtering set}\}$$
$$= \{\lambda \in J^+(L) : \ker \lambda \cup \ker \mu \text{ is a filtering set}\}.$$

The latter equality holds because every element of C is μ-continuous; then

$$\ker \mu \subseteq \bigcap_{\nu \in C} \ker \nu.$$

On the other side μ belongs to C, hence $\ker \mu \supseteq \bigcap_{\nu \in C} \ker \nu$.

Let us observe that if $\ker \mu = \{0\}$, then $J^+_{f,C}(L) = \{\lambda : \ker \lambda \text{ is filtering}\}$, hence all the functions whose kernel is a filtering set are μ-singular. In this case the uniqueness of the decomposition is equivalent to the fact that these are the only μ-singular functions.

Example 3 (Lebesgue decomposition is not unique in general)**.** Let L be the OML considered in Example 1. A function μ of $J^+(L)$ is determined when we assign its values on the elements p, q and 1. For brevity we will denote it by $\mu^{\mu(1)}_{\mu(p),\mu(q)}$.

Let us consider the set $C = \{\alpha \in J^+(L) : \alpha \ll \mu^1_{0,\frac{1}{3}}\}$. Let us observe that

1) $C = \{\alpha \in J^+(L) : \alpha(p) = 0\}$;
2) $C^{\#} = \{\alpha \in J^+(L) : \alpha(p) = \alpha(1)\} = \{\alpha \in J^+(L) : \alpha(p') = 0\}$;
3) $J^+(L) = C + C^{\#}$.

L is not hereditary (by Example 1), so the uniqueness fails to hold. In fact for example

$$\mu^2_{\frac{1}{2},\frac{1}{3}} = \mu^{\frac{3}{2}}_{0,\frac{1}{6}} + \mu^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{6}} = \mu^{\frac{3}{2}}_{0,0} + \mu^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{3}}.$$

Note that in this case it is $J^+_{f,C}(L) = \{0\}$.

Example 4. Let L be the OML pictured in Figure 2.

Let us consider $C = \{\mu \in J^+(L) : \mu(c) = \mu(1)\}$. It is easy to verify that

1) $C^{\#} = \{\lambda \in J^+(L) : \lambda(c) = 0\}$;
2) $J^+(L) = C + C^{\#}$.

In this case there is uniqueness in the decomposition; in fact

$\lambda \in J^+_{f,C}(L) \ \Leftrightarrow\ \ker \lambda \cup (\bigcap_{\mu \in C} \ker \mu)$ is a filtering set $\Leftrightarrow \lambda(c) = 0 \ \Leftrightarrow\ \lambda \in C^{\#}$.

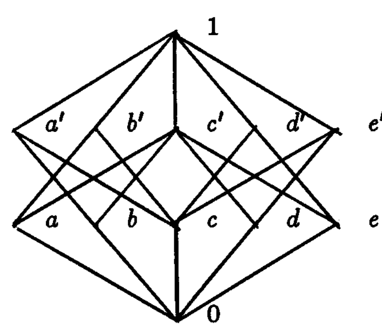

FIGURE 2

Example 5 (Hewitt-Yosida-type decomposition)**.** If we take $C = J_c^+(L)$, we obtain the result of Rüttimann [11, Corollary 6.2]. In this case the uniqueness of decomposition is equivalent to the fact that the restriction of every completely additive function to an arbitrary element p of L, is still completely additive on L_p.

Note that in examples 2 and 5 we do not need to suppose the existence of the decomposition $J^+(L) = C + C^{\#}$. This follows from [5] and [10] respectively.

Also note that in the Boolean case the existence of the decomposition it is enough to have its uniqueness.

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