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NONCOMPLETE LINEAR SYSTEMS ON ELLIPTIC CURVES AND ABELIAN VARIETIES: ADDENDUM TO A PAPER BY CH. BIRKENHAKE

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Abstract. Here we give a result on the postulation (i.e. the 2-normality) of nonlinearly normal embeddings of Abelian varieties. This result improves some of the results proved in a recent paper by Ch. Birkenhake.

In [Bi1], [Bi2] and [Bi3], Ch. II, Ch. Birkenhake considered the postulation and the minimal free resolution of noncomplete embeddings into $\mathbf{P}^N$, respectively of Abelian varieties, projective spaces and curves. In [Bi1], §1 and §2, a general set-up for the study of the minimal free resolution of noncomplete embeddings of algebraic varieties into $\mathbf{P}^N$ was given. A cursory reading of [Bi1] shows that a key point of the proofs was the reduction to the case of a product of elliptic curves and then to the case (via Künneth formula) to the case of an embedding of an elliptic curve. In [Bi1] only the good properties of general projections into $\mathbf{P}^{m-1}$ of a complete embedding into $\mathbf{P}^m$ of an elliptic curve were used. However, there are similar results for general projections into $\mathbf{P}^s$ with s much smaller than $m-1$ (see [BE1], [BE2], [BE3] and the statement here of 1.1). Here we want to show how to use these results to improve some of the results of [Bi1]. Our result is the following theorem which improves [Bi1], Cor. 4.3.

Theorem 0.1. *Suppose (X, L) is a general complex Abelian variety of type $(d_1, \ldots, d_g)$. Fix an integer $n \geq 3$ such that $nd_g \geq 6$. Let w be the largest integer such that $(nd_g - w)(nd_g - w + 1) \geq 4nd_g$. Fix an integer c with $1 \leq c \leq wn^{g-1}$. Then the general vector subspace $V \subseteq H^0(X, L^{\otimes n})$ with $\operatorname{codim}(V) = c$ is 2-normal, i.e. the restriction map $S^2(V) \to H^0(X, L^{\otimes 2n})$ is surjective.*

For the postulation of general projections of Hirzebruch surfaces and Veronese embeddings of $\mathbf{P}^2$, see [BE4].

1. The proofs

As in [Bi1] we work in characteristic 0. Let $k \geq 1$ be an integer. Recall that with the terminology of [Bi1] a closed subscheme Z of $\mathbf{P}(V^*)$ is k-normal if the canonical map $S^k(V) \to H^0(Z, \boldsymbol{O}_Z(k))$ is surjective. The following result was proved in [BE1] and it is a very particular case of the particular case "$g = 1$" of [BE2] (projections into $\mathbf{P}^3$) and [BE3], Th. I (projections into $\mathbf{P}^N$, $N \geq 4$).

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Theorem 1.1. *Fix integers d, N with $d > N \geq 3$ and $(N+2)(N+1)/2 \geq 2d$. Fix an elliptic curve Y and $L \in \mathrm{Pic}^d(Y)$ and let $X \subset \mathbf{P}^{d-1}$ be the linearly normal embedding of Y determined by $H^0(Y, L)$. Then the general projection of X into $\mathbf{P}^N$ is k-normal for all integers $k \geq 2$.*

Remark 1.2. Note that since $h^0(\mathbf{P}^N, \boldsymbol{O}_{\mathbf{P}^N}(2)) = (N+2)(N+1)/2$ and $h^0(Y, L^{\otimes 2}) = 2d$, the statement of Theorem 1.1 is sharp.

Remark 1.3. Note that in the statement of Theorem 1.1 we do not need the assumption that L is a square, i.e. that d is even, made in [Bi1], Cor. 4.2. Hence in our improvement 0.1 of [Bi1], Cor. 4.3, we do not need the assumption that nd_g is even.

Proof of Theorem 0.1. Theorem 0.1 follows from the proof of [Bi1], Cor. 4.3, using 1.1 instead of [Bi1], Cor. 4.2. □

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