A BOUND FOR $|G : O_p(G)|_p$ IN TERMS OF THE LARGEST IRREDUCIBLE CHARACTER DEGREE OF A FINITE $p$-SOLVABLE GROUP $G$

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Abstract. Let $b(G)$ denote the largest irreducible character degree of a finite group $G$, and let $p$ be a prime. Two results are obtained. First, we show that, if $G$ is a $p$-solvable group and if $b(G) < p$, then $p^2 \nmid |G : O_p(G)|$. Next, we restrict attention to solvable groups and show that, if $b(G) \leq p^\alpha$ and if $P$ is a Sylow $p$-subgroup of $G$, then $|P : O_p(G)| \leq p^{2\alpha}$.

1. Introduction

Suppose $G$ is a finite group. Let $cd(G)$ denote the set $\{\chi(1) \mid \chi \in \text{Irr}(G)\}$, and let $b(G)$ denote the largest irreducible character degree of a group $G$. Theorem 12.29 of [1] states: Let $p$ be a prime and let $b(G) < p$. Then $G$ has a normal abelian Sylow-$p$ subgroup. It is immediate from this result that if $b(G) < p$, then $p^2 \nmid |G : O_p(G)|$. Later, in the same chapter of [1], we have Theorem 12.32: Suppose $b(G) < p^{3/2}$ for some prime $p$. Then $p^2 \nmid |G : O_p(G)|$. These facts raise the question: if $b(G) < p^\alpha$ for a real number $\alpha$, what can be said about the $p$ part of $|G : O_p(G)|$? We address this question in the case when $G$ is a $p$-solvable group and obtain the following two results:

**Theorem A.** Let $G$ be a $p$-solvable group and let $p$ be a prime. If $b(G) < p^2$, then $p^2 \nmid |G : O_p(G)|$.

**Theorem B.** Let $G$ be a solvable group, let $p$ be a prime, and let $\alpha$ be a real number. If $b(G) \leq p^\alpha$ and if $P$ is a Sylow $p$-subgroup of $G$, then $|P : O_p(G)| \leq p^{2\alpha}$. In addition, if $|G|$ is odd, then $|P : O_p(G)| \leq p^\alpha$.

2. Preliminaries

In this section, we establish several facts regarding coprime actions that will be needed in the proofs of Theorem A and Theorem B.

**Theorem (2.1) (Brodkey).** Let $G$ be a finite group and assume that $S \in \text{Syl}_p(G)$ is abelian. Then there exists $T \in \text{Syl}_p(G)$ with $S \cap T = O_p(G)$.

**Proof.** This is Theorem 5.28 of [2].

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Lemma (2.2). Suppose that a $p$-group $P$ acts on a $p'$-group $H$. If $P$ fixes every character in $\text{Irr}(H)$, then the action of $P$ on $H$ is trivial.

Proof. If $P$ fixes every character of $H$, then, by Brauer’s Theorem (6.32 of [1]), $P$ fixes every conjugacy class of $H$. Since the size of a conjugacy class of $H$ is a $p'$-number, it follows that each conjugacy class contains a fixed point of $P$. Let $C = C_H(P)$ be the subgroup of fixed points. Since $C$ meets each class of $H$ nontrivially, it follows that $H$ is the (setwise) union of $H$-conjugates of $C$. This forces $H = C$, and thus the action of $P$ on $H$ is trivial. \qed

Last, we prove a special case of Theorem A.

Proposition (2.3). Suppose an abelian $p$-group $P$ acts faithfully on an abelian $p'$-group $H$, and let $G = H \rtimes P$. If $b(G) < p^2$, then $|P| \leq p$.

Proof. If $x \in C_P(h)$, then $h = h^x$; thus $x = x^h$ and, since $x \in P$, it follows that $x \in P \cap P^h$. Thus, in the action of $P$ on $H$, the stabilizer of a point $h$ in $H$, which is $C_P(h)$, is contained in $P \cap P^h$. Since the action of $P$ on $H$ is faithful and $H$ is abelian, Brauer’s Theorem (6.32 of [1]) implies that the action of $P$ on the abelian group $\text{Irr}(H)$ is faithful. Since $P$ is abelian, Bredley’s Theorem (2.1) together with the fact that point stabilizers are contained in Sylow intersections implies that there is a regular orbit of $P$ on $\text{Irr}(H)$. Thus, for some character $\lambda \in \text{Irr}(H)$, we have $\lambda^G(\lambda) = H$. It follows that $\lambda^G \in \text{Irr}(G)$, and therefore $|P| = |G : H| = \lambda^G(1) \leq b(G)$. Since $b(G) < p^2$, the conclusion holds. \qed

3. Proof of Theorem A

In this section we prove Theorem A. We begin with a technical lemma.

Lemma (3.1). Suppose that $G$ is a group with subgroups $H, P, A,$ and $B$ such that $G = HP$ and $P = AB$. If $B \leq N_G([H, A])$, then $[H, A] \triangleleft G$ and $[H, A] \cdot [H, B] = [H, P]$.

Proof. Assume that $B \leq N_G([H, A])$. Since $H$ and $A$ normalize $[H, A]$, we have that $[H, A] \triangleleft G$ and it follows that the product $[H, A] \cdot [H, B]$ is a subgroup.

Clearly $[H, A] \cdot [H, B] \leq [H, P]$. We will show that this is an equality. Let $[h, x]$ be a generator of $[H, P]$, with $h \in H$ and $x \in P$. Write $x = ab$, where $a \in A$ and $b \in B$. One can check that:

$$[h, x] = [h, ab] = [h, b][h, a]^b \in [H, B] \cdot [H, A]^b = [H, A] \cdot [H, B].$$

It follows that $[H, A] \cdot [H, B] = [H, P]$. \qed

Proof of Theorem A. Let $G$ be a counterexample of minimal order. We may assume that $O_p(G) = 1$. Set $H = O_{p'}(G)$. By the famous Lemma 1.2.3 of Hall and Higman, see Lemma 14.22 of [1], we have $C_G(H) \leq H$.

First, we will prove a fact that will be used repeatedly: If $K$ is a subgroup of $G$ with $H \leq K$, then $O_p(K) = 1$. Assume that $K \geq H$. Since $H$ is a $p'$-group, $H$ and $O_p(K)$ are disjoint normal subgroups of $K$; thus $O_p(K) \leq C_K(H) \leq H$. Since $H$ has $p'$ order, this forces $O_p(K) = 1$.

Now fix $P \in \text{Syl}_p(G)$. As we have seen, $O_p(HP) = 1$. Since $P \in \text{Syl}_p(HP)$ and since $G$ is a minimal counterexample, we have $G = HP$.

Next, we will show that $P$ is abelian of order $p^2$. Let $P_1 \leq P$ with $|P : P_1| = p$. Since $H \leq HP_1$, we have $O_p(HP_1) = 1$. Since $|HP_1| < |HP|$, the minimality of $G$ implies that $|P_1| \leq p$. It follows that $|P| = p^2$ and that $P$ is abelian.
Let \( \psi \in \text{Irr}(H) \) and \( \chi \in \text{Irr}(G|\psi) \). By Clifford’s Theorem (6.1 of [1]), we have \( \chi|_H = e \sum_{i=1}^{t} \psi_i \), where \( \{\psi_i\}_{i=1}^{t} \) is the complete orbit of \( \psi \) in the conjugation action of \( G \) on \( \text{Irr}(H) \), labeled so that \( \psi_1 = \psi \). Also \( et \) divides \( |G : H| = |P| = p^2 \), by Corollary 11.29 of [1]. Since \( \chi(1) = et\psi(1) \) and since \( et \) divides \( p^2 \), the hypothesis on character degrees of \( G \) implies that \( et \leq p \). It follows that \( et \) divides \( p \). We claim that \( e = 1 \). Suppose, for a contradiction, that \( e > 1 \). Then \( e = p \) and \( t = 1 \); consequently, \( \psi \) is \( G \)-invariant. Since \( H \) is a normal Hall subgroup of \( G \), it follows that \( \psi \) extends to \( G \) (see Gallagher’s Theorem 8.15 of [1]). Further, since \( P \) is abelian, every irreducible character of \( G \) that lies over \( \psi \) must be an extension (Corollary 6.17 of [1]); however, this contradicts \( \chi_H = p\psi \). Thus, as claimed, \( e = 1 \), and it follows that \( \chi|_H = \sum_{i=1}^{p} \psi_i \) or \( \chi|_H = \psi \).

Now let \( A \) be a subgroup of \( P \) that fixes every character in \( \text{Irr}(H) \). By Lemma (2.2), \( A \leq \text{C}_P(H) \leq H \), and, since \( A \) is a \( p \)-group, this implies that \( A = 1 \). Thus, the action of \( P \) on \( \text{Irr}(H) \) is faithful. Next we will deduce that \( P \) must be an elementary abelian \( p \)-group. Let \( I_P(\psi) \) denote the stabilizer in \( P \) of \( \psi \). As we have seen, for every character \( \psi \in \text{Irr}(H) \), either \( |P : I_P(\psi)| = 1 \) or \( p \); as a consequence, \( |I_P(\psi)| \geq 1 \). If \( P \) is cyclic, then \( P \) has a unique subgroup of order \( p \). This subgroup would have to be contained in \( I_P(\psi) \), for every character \( \psi \in \text{Irr}(H) \). This contradicts the fact that the action on \( \text{Irr}(H) \) is faithful. Thus \( P \) is not cyclic, and, therefore, is elementary abelian of order \( p^2 \).

Next we will show that \( H = [H,P] \). By properties of coprime actions, we have that \( H = [H,P] \cdot \text{C}_H(P) \). If \( X \leq P \), then \( [H,X] \triangleleft G \), since \( H \) normalizes \( [H,X] \) and \( P \) normalizes both \( H \) and \( X \). Also, if \( X \) is nontrivial, then \( [H,X] \) is nontrivial, since \( \text{C}_G(H) \leq H \). In particular, \( [H,P] \) is a nontrivial normal subgroup of \( G \). Now consider \( [H,P] \cdot P \). Observe that \( \text{O}_p([H,P] \cdot P) \) and \( [H,P] \) are disjoint normal subgroups of \( [H,P] \cdot P \), therefore they centralize each other. Of course, \( \text{O}_p([H,P] \cdot P) \) is contained in \( P \) and centralizes \( \text{C}_H(P) \), thus it follows that \( \text{O}_p([H,P] \cdot P) \) centralizes \( H = [H,P] \cdot \text{C}_H(P) \). Since the action of \( P \) on \( H \) is faithful, we have \( \text{O}_p([H,P] \cdot P) = 1 \). Further, \( P \in \text{Syl}_p([H,P] \cdot P) \); therefore, by the minimality of \( G \), we have \( G = [H,P] \cdot P \). Thus, we may conclude that \( H = [H,P] \).

Let \( M \leq H \) be a minimal normal subgroup of \( G \); we will show that \( M = [H,X] \) for some nontrivial subgroup \( X \leq P \). Consider the group \( G/M \). Since \( G \) is a minimal counterexample and since a Sylow \( p \)-subgroup of \( G/M \) is isomorphic to \( P \), we must have \( \text{O}_p(G/M) > 1 \). Let \( X \leq P \) such that \( \text{O}_p(G/M) = XM/M \). Then \( X > 1 \) and \( [H,X] \leq M \). As we have seen, \( 1 < [H,X] \triangleleft G \) for every nontrivial subgroup \( X \leq P \); thus \( M = [H,X] \).

We now make several observations about an arbitrary subgroup \( A \) of \( P \), with \( |A| = p \). We have seen that \( A \) must move some character in \( \text{Irr}(H) \). Also, we know that \( 1 < [H,A] \triangleleft G \).

Assume now that \( [H,A] < H \), and let \( B = \text{O}_p(P \cdot [H,A]) \). We will show that \( A \) and \( B \) have the following dual relationship:

(i) \( [H,B] < H \) and \( A = \text{O}_p(P \cdot [H,B]) \);
(ii) \( B = \text{C}_P([H,A]) \) and \( A = \text{C}_P([H,B]) \);
(iii) \( |B| = |A| = p \) and \( A \cap B = 1 \), thus \( P = AB \).

By properties of coprime actions, \( H = [H,A] \cdot \text{C}_H(A) \). Now consider the group \( P \cdot [H,A] \). This is a proper subgroup of \( G \), since \( [H,A] \) is proper in \( H \); also, \( P \in \text{Syl}_p(P \cdot [H,A]) \). Since \( G \) is a minimal counterexample, it follows that \( B > 1 \). Next observe that \( [H,A] \) and \( B \) are disjoint normal subgroups of \( P \cdot [H,A] \),
As we have seen, \( [H, A] A = [H, A] \), which is nontrivial. It follows that \( A \) does not centralize \([H, A]\), however, \( B \) does, and therefore \( A \not\leq B \). Since \( B \) is nontrivial and \( |P| = p^2 \), it follows that \( |B| = p \). Also, since \( A \) is nontrivial, we have that \( P = AB \) and \( A \cap B = 1 \); thus statement (iii) is proved. Further, since \( A \not\leq C_P([H, A]) \), we have \( C_P([H, A]) < P \). Thus \( 1 < B \leq C_P([H, A]) < P \), and it follows that \( B = C_P([H, A]) \). Thus the first statement in (ii) is proved.

Observe that, since \( B \) centralizes \([H, A]\) and since \( P \) is abelian, we have \([A, H], B] = 1 = [[B, A], H] \). By the Three Subgroups Theorem, it follows that \([H, B], A] = 1 \), and thus \( A \leq C_P([H, B]) \). Since \( A \) is not centralized by \( H \), we have that \([H, B] < H \), and thus the first statement in (i) holds. Further, since \( C_P([H, B]) \) is contained in the abelian group \( P \), we have \( C_P([H, B]) \leq O_p(P \cdot [H, B]) \), and, since \([H, B] < H \), the same reasoning that we used to show that \( O_p(P \cdot [H, A]) \) is proper in \( P \) yields that \( O_p(P \cdot [H, B]) \) is proper in \( P \). Since \( A \) is nontrivial, it follows that \( A = C_P([H, B]) = O_p(P \cdot [H, B]) \), and the rest of the assertion has been proved.

When the situation arises that \([H, A] < H \), we will call the group \( B \) thus identified the dual of \( A \).

Now suppose that \( A \) is a subgroup of \( P \) of order \( p \), and assume that \([H, A]\) is proper in \( H \). We claim that \([H, A]\) is a minimal normal subgroup of \( G \). Since \( 1 < [H, A] \triangleleft G \), we may fix a minimal normal subgroup \( M \) of \( G \) with \( M \leq [H, A] \). As we have seen, \( M = [H, X] \) for some nontrivial subgroup \( X \leq P \). If \( X = A \), then \( M = [H, A] \) and the claim holds. Otherwise, \( X \neq A \), in which case \( P = XA \), and Lemma (3.1) yields

\[
H = [H, X] \cdot [H, A] \leq M \cdot [H, A] \leq [H, A].
\]

This contradicts the fact that \([H, A]\) is proper in \( H \). Therefore \( X = A \), and hence \([H, A]\) is minimal normal.

Continue to assume that \( A \) is a subgroup of \( P \) of order \( p \) with \([H, A] \) proper in \( H \), and let \( B \) be the dual of \( A \). We will show that \( H = [H, A] \times [H, B] \), where \([H, A] \) and \([H, B] \) are minimal normal subgroups of \( G \). Since \([H, A] \) is proper in \( H \), the duality of \( A \) and \( B \) implies that \([H, B] \) is proper in \( H \), and thus both \([H, A] \) and \([H, B] \) are minimal normal subgroups of \( G \). Since \( P = AB \), Lemma (3.1) yields \([H, A] \cdot [H, B] = H \). If \([H, A] \cap [H, B] > 1 \), then, by the minimality of the factors, we have \([H, A] = [H, B], \) and hence \( H = [H, A] \), which is a contradiction. It follows that \([H, A] \cap [H, B] = 1 \), and thus \( H = [H, A] \times [H, B] \).

We will now show that, in fact, there are no subgroups \( A \leq P \) of order \( p \) with \([H, A] \) proper in \( H \). Assume, for a contradiction, that such a subgroup \( A \) does exist, and let \( B \) be the dual of \( A \). Then \( P = A \times B \), and \( H = [H, A] \times [H, B] \). We claim that

\[
G = ([H, A] \cdot A) \times ([H, B] \cdot B).
\]

Each of \([H, A] \cdot A \) and \([H, B] \cdot B \) is a subgroup of \( G \). By duality, \( A \) centralizes \([H, B] \), and thus \( A \) centralizes \([H, B] \cdot B \). Also \([H, A] \) centralizes \([H, B] \), since these are disjoint normal subgroups, and \([H, A] \) centralizes \( B \), by the duality of \( A \) and \( B \). Thus \([H, A] \cdot A \) centralizes \([H, B] \cdot B \). Since \( H = [H, A] \cdot [H, B] \) and \( P = AB \), we have that \( G = ([H, A] \cdot A) \cdot ([H, B] \cdot B) \). To see that this is a direct product, we consider the orders of the factors. Since \( H \) and \( P \) are direct products, we have
that \(|H| = |[H, A]| \cdot |[H, B]|\) and \(|P| = |A| \cdot |B|\). Since \(G = HP\), it follows that
\[|G| = |H| \cdot |P| = |A| \cdot |[H, A]| \cdot |B| \cdot |[H, B]|.\]

Finally, since \(G = ([H, A] \cdot A) \cdot ([H, B] \cdot B)\), we can deduce that \(([H, A] \cdot A) \cap ([H, B] \cdot B) = 1\), and thus \(G = ([H, A] \cdot A) \times ([H, B] \cdot B)\).

Next, observe that \(A\) must move some character in \(\text{Irr}([H, A])\). If not, then by Lemma (2.2), \(A\) acts trivially on \([H, A]\). However, by properties of coprime actions, \(([H, A], A) = [H, A]\), which contradicts the fact that \([H, A] > 1\). Thus, the direct factor \([H, A] \cdot A\) has some irreducible character of degree divisible by \(p\). The same is true for \([H, B] \cdot B\), hence \(G\) must have an irreducible character of degree at least \(p^2\) and this contradicts the hypothesis on \(b(G)\). Therefore, for every subgroup \(A\) of order \(p\), we must have \([H, A] = H\).

For our last observation before returning to character theory, we will show that \(H\) is the direct product of isomorphic nonabelian simple groups. First, we will show that \(H\) is a minimal normal subgroup of \(G\). Let \(M\) be minimal normal in \(G\) with \(M \leq H\). Then \(M = [H, X]\) for a nontrivial subgroup \(X \leq P\). Since \(X > 1\), we have \(H = [H, X]\), and therefore \(H = M\). It now follows that \(H\) is the direct product of isomorphic simple groups. Further, if one of these direct factors of \(H\) is abelian, then \(H\) is abelian and Proposition (2.3) leads to a contradiction. It follows that \(H\) has the claimed structure.

Let \(\psi \in \text{Irr}(H)\), and assume that \(\psi\) is moved by \(P\). Let \(A = I_P(\psi)\), and let \(\chi \in \text{Irr}(G|\psi)\). We have seen that \(1 < A < P\), and that \(\chi_H\) is the sum of \(p\) distinct conjugates of \(\psi\). Since \(\chi(1) < p^2\), it follows that \(\psi(1) \leq p - 1\). Now, let \(C = C_H(A)\); notice that \(H = [H, A] \cdot C\) and that \(P \leq N_G(C)\), since \(P\) centralizes \(A\) which uniquely determines \(C\). Also note that \(\ker(\psi) < H\), since the trivial character is, of course, invariant.

We now consider \(\psi_C\). By Theorem 13.14 of [1], which follows from the Glauberman correspondence, we have that \(\psi_C = aa + p\Phi\) where \(\alpha \in \text{Irr}(C)\), \(a \equiv \pm 1\) (mod \(p\)), and \(\Phi\) is a possibly zero, character of \(C\). Since \(\psi(1) \leq p - 1\), we have \(\Phi = 0\), and \(a = 1\) or \(a = p - 1\). Thus, either \(\psi_C = \alpha\) or \(\psi_C = (p - 1)\alpha\), where \(\alpha \in \text{Irr}(C)\), and if the latter case holds, then \(\alpha\) is linear. The character \(\alpha\) is the Glauberman correspondent of \(\psi\). Note that, since \(\psi\) is nontrivial, \(\alpha\) is nontrivial as well. Next we will see that, in fact, the case \(\psi_C = \alpha\) never occurs.

Suppose that \(\psi_C = \alpha\). Let \(\hat{\psi}\) be the canonical extension of \(\psi\) to the inertial subgroup \(I = I_G(\psi)\); thus \(\hat{\psi}\) is the unique extension of \(\psi\) to \(I\) with \(a(\hat{\psi})\) a \(p\)'-number. The existence of \(\hat{\psi}\) is guaranteed by Gallagher’s Theorem (Corollary 6.28 of [1]). Let \(\mathcal{R}\) be an irreducible representation that affords \(\hat{\psi}\). Then \([\mathcal{R}(C), \mathcal{R}(A)] = 1\), since \(C\) is centralized by \(A\). Also, \(\mathcal{R}_C\) is irreducible, since it affords the irreducible character \(\alpha\). By Schur’s Lemma, we have that \(\mathcal{R}_A\) is a scalar representation. Thus \([\mathcal{R}(H), \mathcal{R}(A)] = 1\), and it follows that \([H, A] \leq \ker(\hat{\psi})\). Further, since \([H, A] \leq H\), we have \([H, A] \leq \ker(\hat{\psi}) \cap H = \ker(\psi) < H\). However, we have shown that \([H, A] = H\) for every subgroup \(A \leq P\) with \(|A| = p\). This contradiction implies that, for every non-\(P\)-invariant character \(\psi \in \text{Irr}(H)\), we have \(\psi_C = (p - 1)\alpha\) for some nontrivial linear character \(\alpha \in \text{Irr}(C)\).

For the final contradiction, we consider what is known about \(\psi\). Since \(\psi\) is not \(P\)-invariant, we have \(\psi \neq 1_H\), and thus its Glauberman correspondent \(\alpha\) is a nontrivial linear character of \(C\). Since \(\alpha\) is nontrivial, \(C\) is not contained in \(\ker(\psi)\), and since \(\alpha\) is linear, \(Z(\psi)\) is contained in \(C\); thus \(Z(\psi) > \ker(\psi)\). Since each
of $\mathbb{Z}(\psi)$ and $\text{ker}(\psi)$ is normal in $H$, it follows that $H$ has a nontrivial abelian (in fact, cyclic) section $\mathbb{Z}(\psi)/\text{ker}(\psi)$. This contradicts the fact that $H$ is the direct product of nonabelian simple groups. Thus, our minimal counterexample $G$ cannot exist.

3. Proof of Theorem B

In this section, we consider what can be said if the character degree hypothesis of Theorem A is weakened to $b(G) \leq p^a$. If attention is restricted to solvable groups, then, as a direct consequence of a theorem of D. Passman, Theorem B is gained.

Proof of Theorem B. Without loss of generality, we may assume that $1 = O_p(G) < G$. For $P \in \text{Syl}_p(G)$, our aim is to show that $|P| \leq p^{2a}$. Set $H = O_{p'}(G)$, and note that $H$ is nontrivial, since $G$ is solvable with $O_p(G) = 1$. As in the proof of Theorem A, we may assume that $G = HP$.

Next we show that we may assume that $H$ is nilpotent, or equivalently, that $H = F(G)$. Since $O_p(G) = 1$, we have $F(G) \leq H$. Further, since $G$ is solvable, $C_G(F(G)) \leq F(G)$. It follows that $O_p(F(G) \cdot P) = 1$, and, since $b(F(G) \cdot P) \leq b(G) \leq p^a$, the hypotheses hold in the group $F(G) \cdot P$. Since $P \in \text{Syl}_p(F(G) \cdot P)$, we may hence assume that $H$ is nilpotent.

Now, we consider the coprime action of $P$ on the abelian group $\text{Irr}(H/\Phi(H))$. Since the action of $P$ on the nilpotent group $H$ is coprime and faithful, it follows that the action of $P$ on the abelian group $\text{Irr}(H/\Phi(H))$ is faithful. By Corollary 2.4 of [3], there exists $\lambda \in \text{Irr}(H/\Phi(H))$, such that the $P$-orbit of $\lambda$ has size at least $\sqrt{|P|}$, and thus for any character $\chi \in \text{Irr}(G|\lambda)$, we have $\chi(1) \geq \sqrt{|P|}$. Since $p^a \geq b(G) \geq \chi(1)$, it follows that $p^a \geq \sqrt{|P|}$, and therefore $|P| \leq p^{2a}$.

Finally, we observe that, if $|G|$ is odd, then Corollary 2.4 of [3] asserts that there exists $\lambda \in \text{Irr}(H/\Phi(H))$, such that the $P$-orbit of $\lambda$ has size at least $|P|$. In this case, it follows that $|P| \leq p^a$. □

References


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