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F_Δ TYPE FREE RESOLUTIONS OF MONOMIAL IDEALS

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ABSTRACT. Let $M = (m_1, \dots , m_r)$ be a monomial ideal of $S = k[x_1, \dots , x_n]$. Bayer-Peeva-Sturmfels studied a subcomplex F_Δ of the Taylor resolution, defined by a simplicial complex $\Delta \subset 2^r$. They proved that if M is *generic* (i.e., no variable x_i appears with the same non-zero exponent in two distinct monomials which are minimal generators), then F_{Δ_M} is the *minimal* free resolution of S/M, where Δ_M is the *Scarf complex* of M.

In this paper, we prove the following: for a generic (in the above sense) monomial ideal M and each integer $\operatorname{depth} S/M \le i < \dim S/M$, there is an *embedded* prime $P \in \operatorname{Ass}(S/M)$ of $\dim S/P = i$. Thus a generic monomial ideal with no embedded primes is Cohen-Macaulay (in this case, Δ_M is shellable). We also study a non-generic monomial ideal M whose *minimal* free resolution is F_Δ for some Δ. In particular, we prove that if all associated primes of M have the same height, then M is Cohen-Macaulay and Δ is pure and strongly connected.

1. PRELIMINARY RESULTS

Let $S = k[x_1, \dots , x_n]$ be a polynomial ring over a field k. S has a natural $\mathbb{N}^n$-grading such that each homogeneous component is a 1-dimensional k-vector space spanned by a single monomial. Let M be a monomial ideal generated by *monomials* $m_1, \dots , m_r$ ($m_1, \dots , m_r$ and M are used in this sense throughout this paper). For a subset $I \subset \{1, \dots , r\}$ we set $m_I = \operatorname{lcm}\{m_i \mid i \in I\}$. Let $a_I \in \mathbb{N}^n$ be the exponent vector of m_I and $S(-a_I)$ the free S-module with one generator in multidegree a_I. The *Taylor complex* of S/M is the $\mathbb{N}^n$-graded module $F = \bigoplus_{I\subset\{1,\dots,r\}} S(-a_I)$ with basis denoted by $\{e_I\}_{I\subset\{1,\dots,r\}}$ and equipped with the differential

$$d(e_I) = \sum_{i\in I} \operatorname{sign}(i, I) \cdot \frac{m_I}{m_{I\setminus i}} \cdot e_{I\setminus i},$$

where $\operatorname{sign}(i, I)$ is $(-1)^{j+1}$ if i is the j-th element in the ordering of I. This is an $\mathbb{N}^n$-graded free resolution of S/M over S having length r and 2^r terms. The minimal free resolution of S/M is always an $\mathbb{N}^n$-graded subcomplex of the Taylor complex F, but F is far from minimal when $r \gg n$.

We say that $\Delta \subset 2^{\{1,\dots,r\}}$ is a *simplicial complex*, if $I \in \Delta$ and $J \subset I$ always imply $J \in \Delta$. An element of Δ is called a *face*, and the dimension of a face I is defined by $\dim I = |I| - 1$. The dimension of the simplicial complex Δ is

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$\dim \Delta = \max\{\dim I \mid I \in \Delta\ \}$. Note that the empty set $\emptyset$ is a face (of dimension -1) of any non-empty simplicial complex. Faces of dimension 0 (resp. 1) are called *vertices* (resp. *edges*). Maximal faces under inclusion are called *facets*. A simplicial complex with only one facet is called a *simplex*.

For a simplicial complex $\Delta \subset 2^{\{1,\cdots,r\}}$, we define $F_\Delta := \bigoplus_{I \in \Delta} S(-a_I)$ to be a submodule of the Taylor complex F. Since F_Δ is closed under the differential d, F_Δ is a subcomplex of the Taylor complex.

Definition 1.1 (Bayer-Peeva-Sturmfels)**.** Let $M = (m_1, \dots, m_r)$ be a monomial ideal. We define a simplicial complex:

$$\Delta_M := \{I \subset \{1, \dots, r\} \mid m_I \neq m_J \text{ for all } J \subset \{1, \dots, r\} \text{ other than } I\}.$$

We call Δ_M the *Scarf complex* of M.

For each $1 \leq i \leq r$, $\{i\} \in \Delta_M$ if and only if m_i is a minimal generator of M. It is easy to see that F_{Δ_M} is always contained in the minimal free resolution of S/M as a subcomplex. But F_{Δ_M} is not acyclic in general. For example, if $M = (xy, yz, zx)$, then Δ_M is of the form • • • and F_{Δ_M} is of the form $0 \to S^3 \to S \to 0$. This is clearly non-acyclic. If $\Delta \subset 2^{\{1,2,3\}}$ is a simplicial complex whose facets are two edges (of course this Δ is not unique), then F_Δ is the minimal free resolution.

Definition 1.2 (Bayer-Peeva-Sturmfels)**.** A monomial ideal M is called *generic* if no variable x_i appears with the same non-zero exponent in two distinct monomials which are minimal generators of M.

$(x^2y^3, x^3z^2, xyz, y^2)$ is a generic monomial ideal, but (xy, xz) is not.

Theorem 1.3 (Bayer-Peeva-Sturmfels)**.** *If M is a generic monomial ideal, then the complex F_{Δ_M} defined by the Scarf complex Δ_M is acyclic and gives the minimal free resolution of S/M over S.*

Example 1.4. (1) Any monomial ideal $M \subset k[x, y]$ is always generic and can be written as

$$M = (x^{a_1}y^{b_1}, x^{a_2}y^{b_2}, \dots, x^{a_r}y^{b_r}),$$

where $a_1 > a_2 > \cdots > a_r$ and $b_1 < b_2 < \cdots < b_r$. The facets of Δ_M are edges $\{1, 2\}, \{2, 3\}, \dots, \{r-1, r\}$. So the combinatoric property of Δ_M depends only on r. The minimal free resolution F_{Δ_M} is of the form $0 \to S^{r-1} \to S^r \to S \to 0$. When $r \geq 2$, we have that

$$S/M \text{ is Cohen-Macaulay} \iff \dim S/M = 0 \iff a_r = b_1 = 0.$$

Even if M is generic, Δ_M does not determine fundamental properties of M such as $\operatorname{ht} M$, $\dim S/M$ and Cohen-Macaulayness.

(2) Set $M := (xy, xz) \subset k[x, y, z]$. M is not generic, but F_{Δ_M} is the minimal free resolution of S/M (in this case, Δ_M is a 1-simplex). A similar phenomenon occurs when $M = (x^3y^2, y^3z^2, z^3x^2, x^2y^2z^2)$. In this case, the Scarf complex Δ_M is three edges joined at one vertex, and M is Cohen-Macaulay.

If M is a monomial ideal which is not generic, then typically, the minimal free resolution of S/M cannot be written as F_Δ for any Δ, though Bayer-Peeva-Sturmfels [1] proved that there is a simplicial complex Δ of $\dim \Delta \leq n - 1$ such that F_Δ is acyclic.

Example 1.5 (Miyazaki)**.** Let $M := (x_1x_2, x_3x_4, x_5x_6, x_1x_7, x_2x_7, x_3x_7, x_5x_7)$ be an ideal of $S = k[x_1, \dots , x_7]$. Note that all minimal generators of M have (total) degree 2. The minimal free resolution of S/M over S is of the form

$$0 \to S^2 \to S^8 \to S^{12} \to S^7 \to S \to S/M \to 0.$$

By calculation, we see that any matrix representing the map $S^2 \to S^8$ always has an entry of degree 3. Hence F_Δ is not a minimal free resolution of S/M for any Δ.

In the next section, we see that there is a large class of monomial ideals whose minimal free resolutions are not of the form F_Δ for any Δ. For example, if M is a Gorenstein ideal which is not complete intersection, then there is no Δ such that F_Δ is the minimal free resolution (see Corollary 2.11).

2. Main results

We first show that a generic monomial ideal M has many *embedded* associated primes if M is not Cohen-Macaulay.

Definition 2.1. Let Δ be a simplicial complex. We say Δ is pure if all its facets are of the same dimension. A pure simplicial complex is *shellable* if the facets of Δ can be given a linear order $I_1, \dots , I_t$ satisfying the following condition: for all i, j, $1 \le j < i \le t$, there exist some $v \in I_i \setminus I_j$ and some $s \in \{1, 2, \dots , i-1\}$ with $I_i \setminus I_s = \{v\}$.

Let Δ be a pure simplicial complex. We say Δ is *strongly connected*, if Δ satisfies the following condition: for any two facets I and I', there is a sequence of facets $I_1, I_2, \dots , I_s$ such that $I = I_1$, $I' = I_s$ and $\dim(I_i \cap I_{i+1}) = \dim \Delta - 1$ for each $1 \le i \le s-1$. It is easy to see that a shellable simplicial complex is strongly connected.

Lemma 2.2. *Let $M = (m_1, \dots , m_r)$ be a generic monomial ideal, and let P, P' be associated primes of M such that $\operatorname{ht} P < \operatorname{ht} P'$. Then for each integer s such that $\operatorname{ht} P < s \le \operatorname{ht} P'$, there is an embedded associated prime $Q \in \operatorname{Ass}(S/M)$ with $\operatorname{ht} Q = s$.*

Proof. Choose an integer D larger than the (total) degree of any minimal generator of M. Consider an artinian monomial ideal

$$M^* := M + (x_1^D, x_2^D, \dots , x_n^D).$$

Here we consider $m_{r+i} = x_i^D$ for $1 \le i \le n$, and Δ_{M^*} is a simplicial complex on $\{1, \dots , r+n\}$. M^* is also generic and the Scarf complex Δ_{M^*} is pure $(n-1)$-dimensional (see [1], also Proposition 2.9 below). If $x_i^d \in M$ for some d, x_i^D is not a minimal generator of M^* and $\{r+i\} \notin \Delta_{M^*}$. Bayer-Peeva-Sturmfels [1] also showed that Δ_{M^*} is shellable (this is a consequence from convex geometry). In particular, Δ_{M^*} is strongly connected.

For a facet I of Δ_{M^*}, we set

$$P_I := (x_i \mid 1 \le i \le n \text{ such that } r+i \notin I).$$

By [1, Theorem 8.1], we have $\operatorname{Ass}(S/M) = \{P_I \mid I \text{ is a facet of } \Delta_{M^*}\}$. Since Δ_{M^*} is pure $(n-1)$-dimensional, we have $\dim(S/P_I) = |I \cap W| = n - |I \cap V|$ where $W = \{r+1, \dots , r+n\}$ and $V = \{1, \dots , r\}$. Hence $\operatorname{ht}(P_I) = |I \cap V|$. There are facets I and I' of Δ_{M^*} such that $P_I = P$ and $P_{I'} = P'$. Since Δ_{M^*} is strongly connected, there is a sequence of facets $I_1, I_2, \dots , I_s$ such that $I = I_1$, $I' = I_s$ and $\dim(I_i \cap I_{i+1}) = \dim \Delta_{M^*} - 1$ for each $1 \le i \le s-1$.

Let i be an integer such that $1 \leq i \leq s-1$. Set $\{c\} := I_i \setminus I_{i+1}$ and $\{d\} := I_{i+1} \setminus I_i$. If $\operatorname{ht} P_{I_i} > \operatorname{ht} P_{I_{i+1}}$, then $c \in V$ and $d \notin V$ (i.e., $c \notin W$ and $d \in W$). Hence we have $\operatorname{ht} P_{I_i} = \operatorname{ht} P_{I_{i+1}} + 1$ and $P_{I_i} \supset P_{I_{i+1}}$. If $\operatorname{ht} P_{I_i} < \operatorname{ht} P_{I_{i+1}}$, then $\operatorname{ht} P_{I_i} = \operatorname{ht} P_{I_{i+1}} - 1$ and $P_{I_i} \subset P_{I_{i+1}}$. So we can prove the assertion. □

Theorem 2.3. *Let $M = (m_1, \dots, m_r)$ be a generic monomial ideal. For each integer i such that $\operatorname{depth} S/M \leq i < \dim S/M$, there is an embedded associated prime $P \in \operatorname{Ass}(S/M)$ with $\dim S/P = i$.*

Proof. Let M^* be an artinian monomial ideal defined in the proof of Lemma 2.2, and let $J \in \Delta_M$ be a facet with $\dim J = \dim \Delta_M$. Since Δ_M is a subcomplex of Δ_{M^*}, there is a facet I of Δ_{M^*} such that $J = I \cap \{1, \dots, r\}$. Let $P_I \in \operatorname{Ass}(S/M)$ be an associated prime defined in the proof of Lemma 2.2. Since F_{Δ_M} is the minimal free resolution, we have

$$\dim(S/P_I) = n - |J| = n - (\dim \Delta_M + 1) = n - \text{proj-dim}(S/M) = \operatorname{depth}(S/M).$$

On the other hand, there clearly exists a prime ideal $P \in \operatorname{Ass}(S/M)$ with $\dim S/P = \dim S/M$. So the assertion follows from Lemma 2.2. □

Bayer-Peeva-Sturmfels [1] proved that a generic monomial ideal M is Cohen-Macaulay, if M is pure dimensional, i.e., all associated primes of M have the same height. But we can prove a stronger result.

Corollary 2.4. *Let $M = (m_1, \dots, m_r)$ be a generic monomial ideal. If M has no embedded associated primes, then M is Cohen-Macaulay. In this case, Δ_M is shellable.*

Proof. The former statement immediately follows from Theorem 2.3. So it suffices to prove the shellability of Δ_M. Let M^* be as in the proof of Lemma 2.2, and $I \in \Delta_{M^*}$ a facet. Let $P_I \in \operatorname{Ass}(S/M)$ be as in the proof of Lemma 2.2. Note that $\operatorname{ht}(P_I) = |I \cap \{1, \dots, r\}|$. Since M is Cohen-Macaulay, we have $|I \cap \{1, \dots, r\}| = \operatorname{ht} M$. In particular, the cardinality $|I \cap \{1, \dots, r\}|$ does not depend on the choice of a facet $I \in \Delta_{M^*}$. So Δ_M is shellable by [2, Theorem 11.13]. □

If M is not Cohen-Macaulay, Δ_M may be non-pure, and may be non-shellable even in the non-pure sense of [3].

The next result states that it is very difficult for a generic monomial ideal to be a (non-Cohen-Macaulay) Buchsbaum ideal.

Corollary 2.5. *Let M be a generic monomial ideal which is not Cohen-Macaulay. If $\dim S/M \geq 2$, then S/M is not Buchsbaum.*

Proof. If S/M is Buchsbaum and $\operatorname{depth}(S/M) \geq 1$, then it is well known that M is pure dimensional. Hence S/M is Cohen-Macaulay by Corollary 2.4. So we may assume that $\operatorname{depth}(S/M) = 0$ (i.e., $(x_1, \dots, x_n) \in \operatorname{Ass}(S/M)$). By Theorem 2.3, for each $0 < i < \dim S/M$, there is an associated prime $P \in \operatorname{Ass}(S/M)$ with $\dim S/P = i$. This is a contradiction (in fact, $\ell(H^i_{\mathbf{m}}(S/M)) = \infty$ for all $1 \leq i \leq \dim S/M - 1$, where $\mathbf{m} := (x_1, \dots, x_n)$). □

We now study a non-generic monomial ideal M whose minimal free resolution can be written as F_Δ for some Δ. It is easy to see that Δ always contains the Scarf complex Δ_M as a subcomplex. Some results on generic monomial ideals remain valid for M in somewhat weaker form.

Theorem 2.6. *Let $M = (m_1, \dots, m_r)$ be a (not necessarily generic) monomial ideal. Suppose that there is a simplicial complex Δ on $\{1, \dots, r\}$ such that F_Δ is the minimal free resolution of S/M. If Δ has a facet of dimension $i-1$, then there is an associated prime $P \in \operatorname{Ass}(S/M)$ with $\operatorname{ht} P = i$.*

Proof. It is well known that $\dim \operatorname{Ext}^i_S(S/M, S) \leq n - i$, and the equality holds iff there is an associated prime $P \in \operatorname{Ass}(S/M)$ with $\operatorname{ht} P = i$ (cf. [4, Theorem 8.1.1]).

On the other hand, $\operatorname{Ext}^i_S(S/M, S)$ is the i-th cohomology of the cochain complex $F^*_\Delta := \operatorname{Hom}(F_\Delta, S)$. Let $I \in \Delta$ be a facet of dimension $i-1$, and $e^*_I \in (F^*_\Delta)^i$ the dual base of $e_I \in (F_\Delta)_i$. Since I is a facet, e^*_I is a cocycle of F^*_Δ. So we can regard $e^*_I \in \operatorname{Ext}^i_S(S/M, S)$. For some ideal $L \subset S$, we have

$$S/L \simeq S \cdot e^*_I \subset \operatorname{Ext}^i_S(S/M, S).$$

Note that $|I| = i$ and

$$d(e_I) = \sum_{j \in I} m'_j \cdot e_{I \setminus j},$$

for some monomials $m'_1, \dots, m'_i$. These monomials are non-constant, since F_Δ is minimal. We have $L' := (m'_1, \dots, m'_i) \supset L$, and

$$\dim \operatorname{Ext}^i_S(S/M, S) \geq \dim S/L \geq \dim S/L' \geq n - i,$$

by Krull's theorem. By the remark above, $\dim \operatorname{Ext}^i_S(S/M, S) = n - i$ and there is an associated prime $P \in \operatorname{Ass}(S/M)$ with $\operatorname{ht} P = i$. □

Even if M is generic, there is the case that the Scarf complex Δ_M does not have a facet of dimension $i-1$, though there is an associated prime $P \in \operatorname{Ass}(S/M)$ with $\operatorname{ht} P = i$. In fact, there is a generic monomial ideal $M \subset S$ of $\operatorname{ht} M = 1$ such that Δ_M is an $(n-1)$-simplex.

Corollary 2.7. *Let $M = (m_1, \dots, m_r)$ be a monomial ideal. Suppose that there is a simplicial complex Δ on $\{1, \dots, r\}$ such that F_Δ is the minimal free resolution of S/M. Then there is an associated prime $P \in \operatorname{Ass}(S/M)$ of $\dim S/P = \operatorname{depth} S/M$.*

Proof. By the previous theorem, there is an associated prime $P \in \operatorname{Ass}(S/M)$ with $\operatorname{ht}(P) = \dim \Delta + 1$. Since F_Δ is minimal, we have $\dim S/P = \operatorname{depth} S/M$ by the same argument as the proof of Theorem 2.3. □

Example 2.8. Set $M := (xw, yw, zw) \subset k[x, y, z, w]$. Then the Scarf complex Δ_M of M is a 2-simplex, and F_{Δ_M} is the minimal free resolution of S/M. We have $M = (x, y, z) \cap (w)$ and $\operatorname{depth} S/M = 1$. Note that M has no embedded associated primes nor associated primes of height 2. So Theorem 2.3 does not hold for a non-generic monomial ideal M, even if F_{Δ_M} is acyclic.

Proposition 2.9. *Let $M = (m_1, \dots, m_r)$ be a monomial ideal. Suppose that there is a simplicial complex Δ on $\{1, \dots, r\}$ such that F_Δ is the minimal free resolution of S/M. If M is pure dimensional, then S/M is Cohen-Macaulay. In this case, Δ is pure and strongly connected.*

Proof. The former assertion immediately follows from Corollary 2.7. The purity of Δ easily follows from Theorem 2.6. We now prove the strong connectedness. Let $F^*_\Delta := \operatorname{Hom}(F_\Delta, S)$ be the S-dual of the complex F_Δ. By the local duality, F^*_Δ gives the minimal free resolution of the canonical module $\omega_{S/M}$ up to degree shifting (cf., [4]). If $I \in \Delta$ is a facet, then the dual base e^*_I corresponds to a minimal generator of $\omega_{S/M}$. If a face $L \in \Delta$ of $\dim L = \dim \Delta - 1$ is contained in facets $I_1, \dots, I_t \in \Delta$, then e_L^* corresponds to a relation among $e_{I_1}^*, \dots, e_{I_t}^*$. Conversely, if $G_1 \to G_0 \to \omega_{S/M} \to 0$ is the minimal representation, then G_1 is generated by

$$\{e_L^* \mid L \in \Delta \text{ of } \dim L = \dim \Delta - 1\}.$$

Hence, if Δ is not strongly connected, then $\omega_{S/M}$ is not indecomposable. This is a contradiction. □

Let Δ be a Cohen-Macaulay simplicial complex (i.e., the Stanley-Reisner ring $k[\Delta]$ is Cohen-Macaulay). Then it is well known that Δ is pure and strongly connected (cf. [2, Proposition 11.7]). But Δ is not shellable in general, though a shellable simplicial complex is always Cohen-Macaulay (cf., [4, 6]).

Problem 2.10. Let M be a Cohen-Macaulay monomial ideal. Suppose that there is a simplicial complex Δ such that F_Δ is the minimal free resolution. Is Δ shellable?

Corollary 2.11. *Let M be a monomial ideal. Suppose that there is a simplicial complex $\Delta \subset 2^{\{1,\dots,r\}}$ such that F_Δ is the minimal free resolution of S/M. If M is Gorenstein, then it is complete intersection. In particular, when M is generic, S/M is Gorenstein if and only if it is complete intersection.*

Proof. By the Proposition 2.9, Δ is pure. Since F_Δ is the minimal free resolution of S/M, the number of the facets of Δ is equal to the Cohen-Macaulay type of S/M. So Δ is a simplex in this case. Since S/M is Cohen-Macaulay, we have

$$\begin{aligned} \operatorname{ht}(M) = \text{proj-dim}\, S/M = \dim \Delta + 1 &= \text{the number of the vertices of } \Delta \\ &= \text{the minimal number of generators of } M. \end{aligned}$$

□

Even if M is generic, there is the case that the Scarf complex Δ_M is a simplex but M is not complete intersection. For example, $M = (x^2, xy)$ or $M = (x^2y, y^2z, z^2x)$. That is, there is a non-complete intersection (generic) monomial ideal whose minimal free resolution coincides with the Taylor complex. The next result follows from Proposition 2.9 immediately (this is maybe a well-known result).

Proposition 2.12. *Let $M = (m_1, \dots, m_r)$ be a (not necessarily generic) monomial ideal. Suppose that the Taylor complex of M gives the minimal free resolution of S/M. If M is pure dimensional, then M is complete intersection (of course, the Taylor complex coincides with the Koszul complex in this case).*

Remark 2.13. If F_Δ is acyclic, then it has a structure of a DG-algebra (skew commutative associative differential graded S-algebra) structure (see [1]). Hence the Taylor complex itself and the minimal free resolution of a generic monomial ideal have DG-algebra structures. But there is a monomial ideal M whose minimal free resolution does not have a DG-algebra structure. The minimal free resolution of this ideal cannot be written as F_Δ for any Δ.

Let M be a codimension 3 Gorenstein monomial ideal. It is well known that the minimal free resolution of S/M has a DG-algebra structure. But if M is not complete intersection, F_Δ is not the minimal free resolution of S/M for any Δ by Corollary 2.11. Thus a DG-algebra structure of the minimal free resolution $\mathbf{F}$ is not a sufficient condition for the existence of a simplicial complex Δ such that $\mathbf{F} \simeq \mathbf{F}_\Delta$.

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