PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 127, Number 4, April 1999, Pages 1251–1256
S 0002-9939(99)04623-7

$\mathfrak{n}$-HOMOLOGY OF GENERIC REPRESENTATIONS FOR $GL(N)$

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(Communicated by Roe Gooodman)

ABSTRACT. We compute the $\mathfrak{n}$-homology for a class of representations of $GL(N,\mathbb{R})$ and $GL(N,\mathbb{C})$ which admit a Whittaker model. They are all completely reducible.

1. INTRODUCTION

1.1. Let $G = GL(N)$ over an archimedean local field F and let P be an upper triangular cuspidal parabolic subgroup. (All the parabolic subgroups considered in this note will be in standard upper triangular form.) For $F = \mathbb{R}$, P is a parabolic subgroup whose Levi part has simple factors given by either $GL(1)$ or $GL(2)$. For $F = \mathbb{C}$, P is the parabolic subgroup whose Levi part has simple factors given by $GL(1)$. Let Q be another parabolic subgroup of G and let $\mathfrak{n}_Q$ be the complexification of the Lie algebra of the uniradical of Q; this notational convention will be used for other groups as well. In this note we compute the $\mathfrak{n}_Q$-homology of Harish-Chandra modules for $GL(N)$ induced from P by sufficiently general characters on the $GL(1)$ factors and sufficiently general discrete series on the $GL(2)$ factors. By a result of Vogan (Theorem 6.2 of [5]) these modules, which will be irreducible, are generic in the sense that they possess a Whittaker model.

The main results of this paper, the computation of the $\mathfrak{n}_Q$-homology of these representations, are contained in Theorem 4.2 and Theorem 5.3. As a consequence, these homologies are all completely reducible. These results are archimedean analogues of Propositions 3.4 and 3.5 of [3] (which were easier to obtain due to previous work of Bernstein and Zelevinsky) and should play a similar role in the computation of archimedean L-factors for $GL(N)$ for both the Rankin–Selberg convolutions and the exterior square.

To establish the Theorems, using intertwining operators and the Frobenius reciprocity, we exhibit the obvious quotients of the $\mathfrak{n}_Q$-homology. We then show that these account for the whole homology using a vanishing result for $\mathfrak{n}$-homology due to Casselman (see [4]).

We would like to thank Nolan Wallach for helpful conversations during the early stages of this project.

Received by the editors April 30, 1996 and, in revised form, August 20, 1997.
1991 *Mathematics Subject Classification.* Primary 22E46.
The second author was partially supported by a grant from the NSA.

1.2. We fix some notation. In sections 1–4 we consider the case $F = \mathbb{R}$. Write $L_P = GL(1)^p \times GL(2)^q$ with $p+2q = N$ for the Levi component of P, and $C_2 = \{\pm 1\}$ for the cyclic group of order two. We have

$$GL(1) \simeq \mathbb{R}_{>0} \times C_2, \qquad GL(2) \simeq \mathbb{R}_{>0} \times SL(2)^{\pm},$$

where $\mathbb{R}_{>0}$ is the identity component of the center in either case. Therefore

$$L_P = A_P \cdot M_P, \text{ with } A_P \simeq (\mathbb{R}_{>0})^{p+q} \text{ and } M_P \simeq (C_2)^p \times (SL(2)^{\pm})^q. \tag{1.2.1}$$

We use the following notations: $\mathbb{C}_s$ denotes the character on $\mathbb{R}_{>0}$ sending a to a^s for $s \in \mathbb{C}$, ϵ for a character on C_2, and T_m for the discrete series of $SL(2)^{\pm}$ with (infinitesimal character) parameter $m \in \mathbb{Z}_{>0}$. We denote $\mathbb{C}_{(s_1,\cdots,s_p,2t_1,\cdots,2t_q)}$ for the character on A_P given by the tensor product of the characters determined by parameters $s_1, \cdots, s_p$ and $2t_1, \cdots, 2t_q$ in order. We fix a module for $L_P = A_P \cdot M_P$:

$$\Pi = \mathbb{C}_{(s_1,\cdots,s_p,2t_1,\cdots,2t_q)} \otimes (\epsilon_1 \otimes \cdots \otimes \epsilon_p \otimes T_{m_1} \otimes \cdots \otimes T_{m_q}). \tag{1.2.2}$$

1.3. Let P_m be the minimal parabolic subgroup with Levi part $L_m = A_m \cdot M_m$ where $A_m \simeq (\mathbb{R}_{>0})^N$ and $M_m \simeq (C_2)^N$. For $\mu = (\mu_1, \cdots, \mu_N)$ and $\delta = \delta_1 \otimes \cdots \otimes \delta_N$, characters on A_m and M_m, respectively, write $\pi(\mu;\delta)$ for their tensor product as a character on L_m. We fix a positive root system Φ^+ so that the nilpotent radical $\mathfrak{n}_m$ of $\mathfrak{p}_m$ is the span of root spaces corresponding to Φ^+. The Weyl group $W = W(G, A_m) = S_N$ acts on the characters of L_m via $w \cdot \pi(\mu;\delta) := \pi(w \cdot \mu; w \cdot \delta)$.

1.4. In terms of the notation in 1.2, the infinitesimal character for the (normalized) induced module $I_P^G(\Pi)$ is given by the Weyl group orbit of

$$\lambda = (s_1, \cdots, s_p, t_1 + \frac{m_1}{2}, t_1 - \frac{m_1}{2}, \cdots, t_q + \frac{m_q}{2}, t_q - \frac{m_q}{2}) \in \mathfrak{a}_m^*. \tag{1.4.1}$$

From now on we assume that Π and λ are *sufficiently general* in the following sense:

$$s_i,\ t_j,\ s_i - s_j (i \neq j)\ \notin \mathbb{Z} \text{ and } t_i - t_j (i \neq j),\ s_i - t_j \notin \frac{1}{2}\mathbb{Z}. \tag{1.4.2}$$

If we give the set of representations Π in (1.2.2) the structure of a countable number of copies of $\mathbb{C}^{p+q}$ in the natural way, then the sufficiently general Π forms the complement of a countable number of hyperplanes in each $\mathbb{C}^{p+q}$.

2. General facts

2.1. For clarity of presentation, we collect some standard results from general representation theory that will be needed for our computation. The first one is for representations of $SL(2)^{\pm}$. Let T_m be the discrete series module of $SL(2)^{\pm}$ with parameter $m \in \mathbb{Z}_{>0}$. Then there are two non-split exact sequences:

$$0 \to T_m \to I_{P_m}^{SL(2)^{\pm}}(\pi(\frac{m}{2}, \frac{-m}{2}; \delta_1, \delta_2)) \to F_{m,(\delta_1,\delta_2)} \to 0,$$

where P_m stands for the minimal parabolic subgroup of $SL(2)^{\pm}$, the character π is given by

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mapsto |a|^m \cdot \delta_1(\mathrm{sign}(a)) \cdot \delta_2(\mathrm{sign}(b)),$$

and $F_{m,(\delta_1,\delta_2)}$ is an irreducible finite-dimensional module of $SL(2)^{\pm}$ with $\delta_1(-1) \cdot \delta_2(-1)$ equal to the parity of m; there are two such modules and they are not equivalent. Similarly there is a dual version of this non-split exact sequence. For a reference see Section 1.4 of [6].

2.2. The following facts concern intertwining operators and the setting will be for $G = GL(N)$ in Section 1.

2.2.1. Proposition. *Suppose $\mu \in \mathfrak{a}_m^*$ is regular.*

(1) *If α is a simple root in Φ^+ and $\langle \mu, \check{\alpha} \rangle \notin \mathbb{Z}$, then*

$$I_{P_m}^G(\pi(\mu;\delta)) \simeq I_{P_m}^G(s_\alpha \cdot \pi(\mu;\delta)).$$

(2) *If $\langle \mu, \check{\alpha} \rangle \notin \mathbb{Z}$ for all $\alpha \in \Phi - \Phi(L_P)$, then the induction functor I_P^G gives an equivalence of the category of Harish-Chandra modules for L_P with infinitesimal character $\mu - \rho_P$ and the category of Harish-Chandra modules for G with infinitesimal character μ.*

Note that this guarantees that for Π sufficiently general the module $I_P^G(\Pi)$ is irreducible.

2.3. Proposition 2.2.1 is standard, but for lack of a direct reference we give a sketch of a proof using $\mathcal{D}$-module theory. We will use the notation and refer to results in [2]. Using the identification given in Theorem 9.4, the first statement is simply Proposition 5.9 (1). As for the second statement, let K be the complexification of a fixed maximal compact subgroup of G (modulo center). Since the K-orbit of $P_{\mathbb{C}}$, the complexification of P, in the generalized flag variety $Y = G_{\mathbb{C}}/P_{\mathbb{C}}$ is open, the assumption on μ implies that there are no $\tilde{\mathcal{D}}_\mu$ modules supported on the complement of this open orbit in Y. Thus the $(\tilde{\mathcal{D}}_\mu, K)$-modules on the open orbit account for all the $(\tilde{\mathcal{D}}_\mu, K)$-modules on Y. The global section functor $\tilde{\Gamma}$ in this case gives an equivalence of categories by Theorem 4.15.

3. $\mathfrak{n}_m$-HOMOLOGY OF $I_P^G(\Pi)$

3.1. We go back to the setting in Section 1. Let $\delta = \epsilon_1 \otimes \cdots \otimes \epsilon_p \otimes \delta_{p+1} \otimes \cdots \otimes \delta_{p+2q}$ be a character on A_m, with ϵ_j given in (1.2.2) and δ_j arbitrary. Recalling that λ is the infinitesimal character of $I_P^G(\Pi)$ given by (1.4.1), set

$$\mathcal{P} := W \cdot \{\pi(\lambda;\delta) : \delta_{p+j}(-1) \cdot \delta_{p+j+1}(-1) = \text{parity of } m_j, \quad \text{for } j = 1, 3, 5, \cdots, 2q-1\}. \tag{3.1.1}$$

For each subset (possibly empty) I of $\{1, 2, \cdots, q\}$, set

$$\mathcal{P}_I := \{\pi(\mu;\nu) \in \mathcal{P} : t_j - \frac{m_j}{2} \text{ precedes } t_j + \frac{m_j}{2} \text{ as components of } \mu \quad \text{for exactly those } j \in I\}. \tag{3.1.2}$$

Clearly $\mathcal{P}$ is the disjoint union of $\mathcal{P}_I$ for all subsets I. We make the following observation.

3.1.3. *For eligible ν,*

(1) $\pi(\lambda;\nu) \in \mathcal{P}_\emptyset$.

(2) $\pi(\mu;\nu) \in \mathcal{P}_\emptyset$ *if and only if as components of μ, $t_j + \frac{m_j}{2}$ precedes $t_j - \frac{m_j}{2}$ for all $j = 1, \cdots, q$.*

3.2. Lemma. (1) *For each $\pi(\mu;\nu) \in \mathcal{P}$, the induced module $I_{P_m}^G(\pi(\mu;\nu))$ has a unique irreducible submodule, which we denote by $J^G(\mu;\nu)$. All irreducible constituents of $I_{P_m}^G(\pi(\mu;\nu))$ appear with multiplicity one.*

(2) *$J^G(\mu;\nu) \simeq J^G(\mu';\nu')$ if and only if $\pi(\mu;\nu)$ and $\pi(\mu';\nu')$ lie in the same $\mathcal{P}_I$.*

Proof. The assumption (1.4.2) for λ ensures that any $\mu \in W \cdot \lambda$ satisfies $\langle \mu, \check{\alpha} \rangle \notin \mathbb{Z}$ for any simple root α not in $\Phi(L_P)$. By Proposition 2.2.1 (1), for $\pi(\mu;\nu)$ and $\pi(\mu';\nu')$ belonging to the same $\mathcal{P}_I$, $I_{P_m}^G(\pi(\mu;\nu)) \simeq I_{P_m}^G(\pi(\mu';\nu'))$. Moreover, we may assume that

$$\pi(\mu;\nu) = (\prod_{j\in I} s_{p+2j-1}) \cdot \pi(\lambda;\delta),$$

for some δ; where s_{p+2j-1} is the transposition $(p+2j-1, p+2j)$ in $W = S_N$, the reflection about a simple root. Let $GL(2)_j$ be the $GL(2)$ factor of L_P positioned at the $p+2j-1,\ p+2j$ rows in G. By 2.1, we have the following non-split exact sequences:

$$0 \to T_{m_j} \to I_{P_m\cap GL(2)_j}^{GL(2)_j}(\pi(\mu;\nu)|_{P_m\cap GL(2)_j}) \to F_{m_j,(\delta_{p+2j-1},\delta_{p+2j})} \to 0, \text{ if } j \notin I.$$

Dually, if $j \in I$, there is a non-split exact sequence:

$$0 \to F_{m_j,(\delta_{p+2j-1},\delta_{p+2j})} \to I_{P_m\cap GL(2)_j}^{GL(2)_j}(\pi(\mu;\nu)|_{P_m\cap GL(2)_j}) \to T_{m_j} \to 0.$$

Therefore by Proposition 2.2.1(2), $I_{P_m}^G(\pi(\mu;\nu)) = I_P^G(I_{L_P\cap P_m}^{L_P}(\pi(\mu;\nu)))$ contains a unique irreducible submodule given by:

$$J^G(\mu;\nu) = I_P^G(\mathbb{C}_\mu|_{A_P} \otimes L_1 \otimes \cdots \otimes L_q), \tag{3.2.1}$$

where $L_j = T_{m_j}$ if $j \notin I$ and $L_j = F_{m_j,(\delta_{p+2j-1},\delta_{p+2j})}$ if $j \in I$. Also irreducible constituents all appear with multiplicity one. This completes the proof of Lemma 3.2. □

3.3. Proposition. *If $\pi(\mu;\nu) \in \mathcal{P}_I$, then*

$$H_0(\mathfrak{n}_m, J^G(\mu;\nu)) = \bigoplus_{\pi(\mu';\nu')\in\mathcal{P}_I} \pi(\mu';\nu').$$

Proof. Since $\mathrm{H}_0(\mathfrak{n}_m, J^G(\mu;\nu))$ is completely reducible because μ is regular ($\mathfrak{n}_m$ is the nilradical of $\mathfrak{p}_m$, see Lemma 6.6 of [4] or Lemma 12.2.4 of [7]), by the Frobenius reciprocity it suffices to determine those π so that $J^G(\pi(\mu;\nu)) \hookrightarrow I_{P_m}^G(\pi)$. The proposition follows from Lemma 3.2. □

3.4. Corollary. $H_0(\mathfrak{n}_m, I_P^G(\Pi)) = \bigoplus_{\pi(\mu;\nu)\in\mathcal{P}_\emptyset} \pi(\mu;\nu)$.

4. $\mathfrak{n}_Q$-HOMOLOGY OF $I_P^G(\Pi)$

4.1. Recall that Q is an arbitrary parabolic containing P_m and L_Q is its Levi component. For the computation of the $\mathfrak{n}_Q$-homology, we need to partition the parameter set $\mathcal{P}_\emptyset$ according to the simple factors of L_Q.

4.1.1. Definition. *For each μ appearing in the first components of elements in $\mathcal{P}_\emptyset$, set*

$$I_\mu = \{l : \mu_l = t_j + \frac{m_j}{2} \text{ or } t_j - \frac{m_j}{2} \text{ for some } j \text{ with both } t_j + \frac{m_j}{2} \text{ and } t_j - \frac{m_j}{2} \text{ appearing as components of } \mu, \text{ in a common simple factor of } L_Q\}.$$

Let $\sim_Q$ be a relation on $\mathcal{P}_\emptyset$ defined by: $\pi(\mu;\nu) \sim_Q \pi(\mu';\nu')$ if

(1) *In the case $\mu = \mu'$, then $\nu_l = \nu'_l$ whenever $l \notin I_\mu$.*

(2) *In the case* $\mu \neq \mu'$, *there exist simple roots* $\beta_1, \cdots, \beta_l \in \Phi^+(L_Q)$ *so that* $s_{\beta_l} s_{\beta_{l-1}} \cdots s_{\beta_1} \cdot \mu = \mu'$, $s_{\beta_l} s_{\beta_{l-1}} \cdots s_{\beta_1} \pi(\mu;\nu) \sim_Q \pi(\mu';\nu')$, *in the sense of (1), and for all* $j = 0, \cdots, l-1$, $s_{\beta_{j+1}} s_{\beta_j} \cdots s_{\beta_1} \cdot \pi(\mu;\nu) \in \mathcal{P}_\emptyset$.

It is clear that $\sim_Q$ defines an equivalence relation on $\mathcal{P}_\emptyset$. Note that in the definition of I_μ, the component $t_j + \frac{m_j}{2}$ must precede $t_j - \frac{m_j}{2}$ (in a common simple factor of L_Q), say $\mu_l = t_j + \frac{m_j}{2}$ and $\mu_{l+r} = t_j - \frac{m_j}{2}$. Then $\nu_l(-1)\nu_{l+r}(-1) = \nu'_l(-1)\nu'_{l+r}(-1)$ (equal to the parity of m_j). We can now apply the results in Lemma 3.2 to L_Q (being a product of $GL(m)$). We have

4.1.2. Proposition. *For each* $\pi(\mu;\nu) \in \mathcal{P}_\emptyset$, *the induced module* $I^{L_Q}_{L_Q \cap P_m}(\pi(\mu;\nu))$ *has a unique irreducible submodule* $J^{L_Q}(\mu;\nu)$, *and*

$$J^{L_Q}(\mu;\nu) \simeq J^{L_Q}(\mu';\nu') \iff \pi(\mu;\nu) \sim_Q \pi(\mu';\nu').$$

4.2. Theorem. $H_0(\mathfrak{n}_Q, I^G_P(\Pi)) = \bigoplus_{\pi(\mu;\nu)\in\mathcal{P}_\emptyset/\sim_Q} J^{L_Q}(\mu;\nu)$.

Proof. By Corollary 3.4, Proposition 4.1.2, and Frobenius reciprocity there is a surjection:

$$\mathrm{H}_0(\mathfrak{n}_Q, I^G_P(\Pi)) \twoheadrightarrow \bigoplus_{\pi(\mu,\nu)\in\mathcal{P}_\emptyset/\sim_Q} J^{L_Q}(\mu;\nu). \tag{4.2.1}$$

Call the kernel U_1 and rewrite this as the following exact sequence of L_Q modules

$$0 \to U_1 \to U \to U_2 \to 0. \tag{4.2.2}$$

Applying Proposition 3.3 to $L_Q \cap P_m \subset L_Q$, in light of Corollary 3.4, we have

$$\mathrm{H}_0(\mathfrak{n}_m \cap \mathfrak{l}_Q, U) = \mathrm{H}_0(\mathfrak{n}_m, I^G_P(\Pi)) = \mathrm{H}_0(\mathfrak{n}_m \cap \mathfrak{l}_Q, U_2). \tag{4.2.3}$$

To complete the proof of Theorem 4.2, we must show that $U_1 = 0$.

Assume the contrary, i.e., that $U_1 \neq 0$. Then by a theorem of Casselman [1] the homology $\mathrm{H}_0(\mathfrak{n}_m \cap \mathfrak{l}_Q, U_1) \neq 0$. Then there exists $\mu \in \mathfrak{a}^*_m$ such that the μ-exponent of the homology, i.e., the $\mu + \rho(\mathfrak{l}_Q)$ component (as an $\mathfrak{a}_Q$-module), is

$$\mathrm{H}_0(\mathfrak{n}_m \cap \mathfrak{l}_Q, U_1)_\mu \neq 0.$$

Taking the $\mathfrak{n}_m \cap \mathfrak{l}_Q$-homology of the sequence (4.2.2), in light of (4.2.3), we have $\mathrm{H}_1(\mathfrak{n}_m \cap \mathfrak{l}_Q, U_2)_\mu \neq 0$. By the vanishing result of $\mathfrak{n}_m$-homology of Casselman (see Proposition 2.32 in [4]), there exists $\mu' < \mu$ with respect to a partial ordering of $\Phi^+(L_Q)$, i.e., $\mu' \neq \mu$ and $\mu \in \mu' + \sum_{\alpha\in\Phi^+(L_Q)} \mathbb{Z}_{\geq 0} \cdot \alpha$, such that

$$\mathrm{H}_0(\mathfrak{n}_m \cap \mathfrak{l}_Q, U_2)_{\mu'} \neq 0. \tag{4.2.4}$$

By (4.2.3) and Corollary 3.4, all the exponents for $\mathrm{H}_0(\mathfrak{n}_m \cap \mathfrak{l}_Q, U_2)$ must lie in $\mathcal{P}_\emptyset$, i.e., satisfy the condition in 3.1.3(2). Since λ is sufficiently general, they are therefore maximal with respect to the above partial ordering with respect to Φ^+ and hence with respect to $\Phi^+(L_Q)$. This contradicts (4.2.4) and completes the proof of Theorem 4.2. □

4.3 Remarks. 1. The above proof shows that $\mathrm{H}_i(\mathfrak{n}_m, I^G_P(\Pi)) = 0$ for $i > 0$.

2. The summands $J^{L_Q}(\mu;\nu)$ which occur on the right hand side of 4.2 are again irreducible L_Q-modules which admit a Whittaker model, and are sufficiently general in the sense of Section 1.
3. The argument for the proof of 4.2 can be used to show Corollary 3.4 directly without using Proposition 2.2.1(2).

5. The case $GL(N,\mathbb{C})$

5.1. In this section we consider the case when the ground field is $\mathbb{C}$. Generic representations of $G = GL(N,\mathbb{C})$ are those induced from a minimal parabolic subgroup P by generic characters. Write $L_P = (GL(1,\mathbb{C}))^N$, and $\Pi = \otimes_{i=1}^N \chi_i$ a character on L_P where each χ_i is a character of $GL(1,\mathbb{C})$. We assume that Π is *sufficiently general*, i.e., on $\mathbb{C} = \mathfrak{gl}(1,\mathbb{C})^*$,

$$d\chi_i - d\chi_j \notin \mathbb{Z} \quad \text{for} \quad i \neq j. \tag{5.1.1}$$

In light of Proposition 2.2.1, similar to Proposition 3.3, we have

5.1.2. Proposition. (1) *$I_P^G(\Pi)$ is irreducible and for any $w \in W = S_N$, $I_P^G(\Pi) \simeq I_P^G(w \cdot \Pi)$.*

(2) *$H_0(\mathfrak{n}_P, I_P^G(\Pi)) = \bigoplus_{w\in W} w \cdot \Pi$.*

5.2. As before let Q be an arbitrary parabolic subgroup of G containing P and $\mathfrak{n}_Q$ its Lie algebra. Write $W(L_Q)$ for the Weyl group of L_Q, the Levi component of Q. From Proposition 2.2.1, we have for $w \in W$,

$$I_{P\cap L_Q}^{L_Q}(\Pi) \simeq I_{P\cap L_Q}^{L_Q}(w\cdot\Pi) \iff w \in W(L_Q). \tag{5.2.1}$$

5.3. Theorem. $H_0(\mathfrak{n}_Q, I_P^G(\Pi)) = \bigoplus_{w\in W/W(L_Q)} I_{P\cap L_Q}^{L_Q}(w\cdot\Pi)$.

Note that the summands on the right hand side are all generic irreducible L_Q-modules which admit a Whittaker model.

Proof. Similar to the proof of Theorem 4.2, in view of (5.2.1), we have a natural surjection:

$$\mathrm{H}_0(\mathfrak{n}_Q, I_P^G(\Pi)) \twoheadrightarrow \bigoplus_{w\in W/W(L_Q)} I_{P\cap L_Q}^{L_Q}(w\cdot\Pi).$$

The argument in the proof of Theorem 4.2 carries over for this case since the character Π is sufficiently general (cf. (5.1.1)). □

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