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THE MAIN INVOLUTIONS OF THE METAPLECTIC GROUP

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ABSTRACT. We determine the set of automorphisms of a metaplectic group which lift the main involution of the general linear group over an infinite field. Some basic properties of these automorphisms are also established.

1. Introduction

Metaplectic central extensions of $\mathrm{GL}(n)$ (see section 3 for the precise sense of the word "metaplectic" as used in this paper) are of interest in several fields, notably representation theory and the theory of automorphic forms. When $n \geq 3$, the lack of a simple concrete model of these covers makes them awkward to work with and means that many basic questions about them are as yet not satisfactorily answered. In this note we shall determine the lifts of the *main involution*, $g \mapsto w_0{}^t g^{-1} w_0^{-1}$, of $\mathrm{GL}(n)$ to the metaplectic groups and show that the lifts are again involutions. Here w_0 denotes the permutation matrix whose $(i,j)-$entry is 1 if $i+j=n+1$ and 0 otherwise. It should be noted that it has been shown in [3] (see Proposition 3.1 and the discussion following it) that the main involution of $\mathrm{GL}(n)$ has at least one lift to the metaplectic group. However, contrary to a claim made in that reference, we shall show that in general there is no $2-$cocycle in the metaplectic class (again, see section 3 for the definition of this term) which is itself stabilized by the main involution. Recently there has been some interest in finding $2-$cocycles having as many convenient properties (for computational purposes) as possible to represent metaplectic central extensions ([1], which should soon appear, is an example). This result answers in the negative a question which has been raised in this context.

The author's interest in the metaplectic groups stems from their application to number theory. In that setting, F is generally a non-Archimedean local field and $\mathrm{GL}(n)$ and $\widetilde{\mathrm{GL}}(n)$ are topological groups under a topology derived from that of F. In the last section we shall prove a general result which implies that the lifts of the main involution are homeomorphisms under suitable hypotheses. In fact, this result will imply a similar conclusion for the lifts of any automorphism of the F-points of an affine algebraic group defined over F to a topological central extension.

2. Preliminaries on lifting automorphisms

Let

$$1 \longrightarrow A \longrightarrow \widetilde{G} \xrightarrow{\ p\ } G \longrightarrow 1 \tag{1}$$

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be a central extension of a group G by an abelian group A, $\boldsymbol{s} : G \to \widetilde{G}$ be a section of p and τ be the 2−cocycle representing the class of (1) in $\mathrm{H}^2(G, A)$ with respect to $\boldsymbol{s}$. If $f : G \to G$ is an automorphism, then a *lift* of f is an automorphism $\tilde{f} : \widetilde{G} \to \widetilde{G}$ making the diagram

$$\begin{array}{ccccccccc} 1 & \longrightarrow & A & \longrightarrow & \widetilde{G} & \xrightarrow{p} & G & \longrightarrow & 1 \\ & & \| & & \downarrow{\scriptstyle \tilde{f}} & & \downarrow{\scriptstyle f} & & \\ 1 & \longrightarrow & A & \longrightarrow & \widetilde{G} & \xrightarrow{p} & G & \longrightarrow & 1 \end{array}$$

commute. Note that, by the five-lemma, any homomorphism $\tilde{f}$ making this diagram commute is in fact a lift of f. We shall denote by $\mathcal{L}(f)$ the set of all lifts of f.

The group $\mathrm{Aut}(G)$ acts on $\mathrm{H}^2(G, A)$ by $f[\sigma] = [\sigma \circ (f^{-1} \times f^{-1})]$ for any 2−cocycle σ. The set $\mathcal{L}(f)$ is precisely described in terms of this action by the following, which is a combination of Propositions 5–1–1 and 5–1–4 of [7].

Lemma 1. 1. *The set $\mathcal{L}(f)$ is non-empty if and only if $f[\tau] = [\tau]$.*
2. *If $\mathcal{L}(f) \neq \emptyset$, then $\mathcal{L}(f)$ is a principal homogeneous space for the group* $\mathrm{Hom}(G, A)$ *under the action*

$$(\varphi \cdot \tilde{f})(\tilde{g}) = \varphi(p(\tilde{g}))\tilde{f}(\tilde{g}) .$$

3. Preliminaries on the metaplectic groups

We fix an infinite field F; all linear groups appearing below will be those associated with this field. Let A be an abelian group and $c : F^\times \times F^\times \to A$ a Steinberg symbol. By a theorem of H. Matsumoto there is a central extension

$$1 \longrightarrow A \longrightarrow \widetilde{\mathrm{SL}}(n) \xrightarrow{\phi} \mathrm{SL}(n) \longrightarrow 1 \tag{2}$$

corresponding to this data. We will use [5] as our reference for this theorem and the associated definitions since Milnor works in exactly the degree of generality we require. It is shown there that it is possible to choose a section $\boldsymbol{s} : \mathrm{SL}(n) \to \widetilde{\mathrm{SL}}(n)$ of ϕ in such a way that the corresponding two-cocycle τ satisfies

$$\tau(d, d') = \prod_{i \geq j} c(u_i, v_j) \tag{3}$$

where $d = \mathrm{diag}(u_1, \ldots, u_n)$ and $d' = \mathrm{diag}(v_1, \ldots, v_n)$ lie in $\mathrm{SL}(n)$.

Let $\eta : \mathrm{GL}(n) \to \mathrm{SL}(n+1)$ be the embedding $\eta(g) = \mathrm{diag}(g, \det(g)^{-1})$. We shall call the classes in $\mathrm{H}^2(\mathrm{GL}(n), A)$ containing any of the cocycles

$$\sigma(g, g') = c(\det(g), \det(g'))^{m+1} \tau^{-1}(\eta(g), \eta(g')) , \tag{4}$$

where m is any integer, *metaplectic classes*. The inverse has been inserted for consistency with the choices made in [2]; the integer m is equal to what is called c in that paper. It is by now well-known that the formulæ for the metaplectic cocycles given in [2] are not precisely correct in all cases, but this will have no effect on our work here. We call any covering group of $\mathrm{GL}(n)$ which corresponds to a metaplectic cocycle a *metaplectic group*. The symbol $\widetilde{\mathrm{GL}}(n)$ will stand for any metaplectic group, with the group A, the symbol c and the integer m being understood.

4. Algebraic results

We shall use the notation defined in the Introduction and the previous section. In addition, the main involution will be written as $g \mapsto {}^{\iota}g$ and we shall assume that $n \geq 2$, since there is nothing to prove when $n = 1$.

Let $w_1 = \mathrm{diag}(w_0, 1) \in \mathrm{GL}(n+1)$ and for $g \in \mathrm{GL}(n+1)$ put $\hat{g} = w_1 {}^t g^{-1} w_1^{-1}$. The map $g \mapsto \hat{g}$ is an involution of $\mathrm{GL}(n+1)$ which stabilizes $\mathrm{SL}(n+1)$. Its pull-back under η is the main involution on $\mathrm{GL}(n)$.

Lemma 2. *We have* $[\tau]^{\widehat{}} = [\tau]$ *in* $\mathrm{H}^2(\mathrm{SL}(n+1), A)$.

Proof. It follows from the remark on page 95 of [5] that the natural map

$$\mathrm{H}^2(\mathrm{SL}(n+1), A) \longrightarrow \mathrm{H}^2(SH_{n+1}, A) \tag{5}$$

is injective, where SH_{n+1} denotes the subgroup of diagonal matrices in $\mathrm{SL}(n+1)$. It is therefore sufficient to show that the image of $[\tau]$ under this map is stable under the action of the automorphism $g \mapsto \hat{g}$.

Let us define $\kappa : SH_{n+1} \to A$ by

$$\kappa(d) = \prod_{k>\ell} c(u_k, u_\ell) \tag{6}$$

for $d = \mathrm{diag}(u_1, \ldots, u_{n+1})$. A routine computation, using the bilinearity and skew-symmetry of the symbol c, verifies the equation

$$\tau(d, d')\hat{\tau}^{-1}(d, d') = \kappa(dd')\kappa(d)^{-1}\kappa(d')^{-1}\,. \tag{7}$$

Since the expression on the right of this equation is a two-coboundary, the lemma follows. □

In the following $\boldsymbol{s} : \mathrm{GL}(n) \to \widetilde{\mathrm{GL}}(n)$ is the section corresponding to the $2-$cocycle (4) and H_n denotes the subgroup of $\mathrm{GL}(n)$ consisting of diagonal matrices.

Proposition 1. *The set of lifts of the main involution of* $\mathrm{GL}(n)$ *to* $\widetilde{\mathrm{GL}}(n)$ *is in one-to-one correspondence with the set* $\mathrm{Hom}(F^\times, A)$. *The correspondence may be chosen so that if* $\chi : F^\times \to A$ *lies in this set, then the corresponding lift,* $\iota(\chi)$, *satisfies*

$${}^{\iota(\chi)}\boldsymbol{s}(h) = \chi(\det(h)) \prod_{i>j} c(h_i, h_j) \cdot \boldsymbol{s}({}^{\iota}h) \tag{8}$$

for all $h \in H_n$. *All of the lifts are themselves involutions.*

Proof. It follows from Lemma 2 that there is a map $\kappa : \mathrm{SL}(n+1) \to A$ extending (6) and satisfying

$$\tau(g, g')\hat{\tau}^{-1}(g, g') = \kappa(gg')\kappa(g)^{-1}\kappa(g')^{-1} \tag{9}$$

for all $g, g' \in \mathrm{SL}(n+1)$. If we set $\lambda(g) = \kappa(\eta(g))$ for $g \in \mathrm{GL}(n)$, then from (4) we have

$$\sigma^{-1}(g, g'){}^{\iota}\sigma(g, g') = \lambda(gg')\lambda(g)^{-1}\lambda(g')^{-1} \tag{10}$$

for all $g, g' \in \mathrm{GL}(n)$. It follows from this that the set of lifts of the main involution is non-empty. The proof of Lemma 1 shows that one of these lifts satisfies

${}^{\iota_0}\boldsymbol{s}(g) = \lambda(g)\boldsymbol{s}({}^{\iota}g)$. Since the commutator subgroup of $\mathrm{GL}(n)$ is $\mathrm{SL}(n)$, we have $\mathrm{Hom}(F^\times, A) \cong \mathrm{Hom}(\mathrm{GL}(n), A)$ via

$$\chi \mapsto \left(g \mapsto \chi(\det(g))\right). \tag{11}$$

The first claim now follows from Lemma 1.

A computation using the identity $c(\alpha,\alpha) = c(-1,\alpha)$, valid for all Steinberg symbols, shows that if $h = \mathrm{diag}(h_1,\dots,h_n) \in H_n$, then

$$\lambda(h) = c(-1, \det(h)) \prod_{i>j} c(h_i, h_j). \tag{12}$$

Consequently, the various lifts of the main involution satisfy

$$\boldsymbol{s}(h) \mapsto \chi(\det(h))\, c(-1,\det(h)) \prod_{i>j} c(h_i,h_j) \cdot \boldsymbol{s}({}^{\iota}h) \tag{13}$$

for the various choices of $\chi \in \mathrm{Hom}(F^\times, A)$. Since $\alpha \mapsto (-1,\alpha)$ itself lies in this group, we may "shift the base point" to obtain a correspondence satisfying (8).

If $\tilde{g} \mapsto {}^{\iota}\tilde{g}$ is any lift of the main involution to $\widetilde{\mathrm{GL}}(n)$, then $\tilde{g} \mapsto {}^{\iota}({}^{\iota}\tilde{g})$ is a lift of the identity map and hence ${}^{\iota}({}^{\iota}\tilde{g}) = \psi(\det(p(\tilde{g})))\tilde{g}$ for some $\psi \in \mathrm{Hom}(F^\times, A)$, by Lemma 1, where $p : \widetilde{\mathrm{GL}}(n) \to \mathrm{GL}(n)$ is the projection map. Taking $\tilde{g} = \boldsymbol{s}(\mathrm{diag}(\alpha, 1, \dots, 1))$ in this equation and using (8) twice gives $\psi(\alpha) = 1$ for all $\alpha \in F^\times$. Thus $\tilde{g} \mapsto {}^{\iota}\tilde{g}$ is an involution. □

Proposition 2. *Suppose that there is a* $2-$*cocycle in the metaplectic class which is fixed by the main involution. Then* $c(-1,\alpha) = 1$ *for all* $\alpha \in F^\times$.

Proof. Since the $2-$cocycle lies in the same class as σ, it has the form

$$(g, g') \mapsto \sigma(g,g')\nu(gg')\nu(g)^{-1}\nu(g')^{-1} \tag{14}$$

for some function $\nu : \mathrm{GL}(n) \to A$. We are supposing that

$$\sigma({}^{\iota}g, {}^{\iota}g')\nu({}^{\iota}(gg'))\nu({}^{\iota}g)^{-1}\nu({}^{\iota}g')^{-1} = \sigma(g,g')\nu(gg')\nu(g)^{-1}\nu(g')^{-1} \tag{15}$$

for all $g, g' \in \mathrm{GL}(n)$. Substituting the value of $\sigma({}^{\iota}g, {}^{\iota}g')$ given by (10) into this equation reveals that the function $g \mapsto \lambda(g)\nu({}^{\iota}g)\nu(g)^{-1}$ is an element of $\mathrm{Hom}(\mathrm{GL}(n), A)$. The elements of this group are trivial on $\mathrm{SL}(n)$ and so

$$\lambda(g) = \nu(g)\nu({}^{\iota}g)^{-1} \tag{16}$$

for all $g \in \mathrm{SL}(n)$. Let $g = \mathrm{diag}(\alpha, 1, \dots, 1, \alpha^{-1})$ with $\alpha \in F^\times$. Then $g \in \mathrm{SL}(n)$ and ${}^{\iota}g = g$ and so (16) implies that $\lambda(g) = 1$. From (12), $\lambda(g) = c(\alpha^{-1}, \alpha) = c(-1, \alpha)$ and the claim follows. □

Remark 1. In the case of greatest interest for applications, F is a local field and c is the n^{th} order Hilbert symbol on F for some $n \geq 2$. In that case, $c(-1,\alpha) = 1$ for all $\alpha \in F^\times$ if and only if -1 is an n^{th} power in F. Thus, even in this restricted situation, $2-$cocycles fixed by the main involution do not generally exist.

Remark 2. It is well-known that it is possible to choose $\boldsymbol{s} : \mathrm{GL}(n) \to \widetilde{\mathrm{GL}}(n)$ in such a way that its restriction to the subgroup N of unipotent upper-triangular matrices is a homomorphism. If $\tilde{g} \mapsto {}^{\iota}\tilde{g}$ is any lift of the main involution, then, with $\boldsymbol{s}$ chosen in this way, the map $n \mapsto {}^{\iota}\boldsymbol{s}(n)\boldsymbol{s}({}^{\iota}n)^{-1}$ is an element of the group $\mathrm{Hom}(N, A)$. Since N is a vector group, a non-trivial element of $\mathrm{Hom}(N, A)$ gives rise to a non-trivial element of $\mathrm{Hom}(F^+, A)$. Thus if A has finite exponent, b say, and $b \neq 0$ in F, then $\mathrm{Hom}(N, A)$ consists solely of the trivial homomorphism and ${}^{\iota}\boldsymbol{s}(n) = \boldsymbol{s}({}^{\iota}n)$.

5. Topological results

Up to this point we have worked in a purely algebraic context. However, in most applications F is a topological field and c is topologically well-behaved and thus $\widetilde{\mathrm{GL}}(n)$ is a topological group. One can then ask whether the lifts of the main involution are necessarily continuous. The work required to answer this question under unrealistically general topological hypotheses seems disproportionate to the interest of the result. We shall instead prove a result which has a more useful kind of generality and then deduce the continuity from it in the case of most frequent interest.

We first require some terminology. Recall that an exact sequence

$$1 \longrightarrow A \longrightarrow \widetilde{G} \xrightarrow{\ p\ } G \longrightarrow 1 \tag{17}$$

is called *topological* if A, $\widetilde{G}$ and G are Hausdorff topological groups, the inclusion of A in $\widetilde{G}$ is continuous and closed and the projection p is continuous and open. Also, a topological group is called an *$\ell-$group* if it is Hausdorff and has a neighborhood base at the identity consisting of compact open subgroups.

Lemma 3. *Suppose that*

$$1 \longrightarrow A \longrightarrow \widetilde{G} \xrightarrow{\ p\ } G \longrightarrow 1 \tag{18}$$

is a topological exact sequence. If A and G are $\ell-$groups, then $\widetilde{G}$ is an $\ell-group$. If, in addition, A is discrete, then there are a compact open subgroup of G over which the sequence is split and a continuous, open section $\boldsymbol{s} : G \to \widetilde{G}$ which is a homomorphism on that subgroup.

Proof. Suppose that A and G are $\ell-$groups. Let V be an open neighborhood of the identity in $\widetilde{G}$. Then we may choose an open neighborhood of the identity, V' say, such that $V' \cdot V' \cdot V' \subseteq V$. Since A is an $\ell-$group, we may find a compact, open subgroup, K, of A such that $K \subseteq V' \cap A$. There is then an open set $W' \subseteq V'$ such that $W' \cap A = K$. Let

$$m : K \times \widetilde{G} \times K \times \widetilde{G} \times K \times \widetilde{G} \times K \longrightarrow \widetilde{G} \tag{19}$$

be the multiplication map. Then

$$K \times \{e\} \times K \times \{e\} \times K \times \{e\} \times K \subseteq m^{-1}(W') \tag{20}$$

and hence, by the Tube Lemma, there is an open neighborhood of the identity in $\widetilde{G}$, W'' say, such that

$$K \times W'' \times K \times W'' \times K \times W'' \times K \subseteq m^{-1}(W')\,. \tag{21}$$

Replacing W'' by $W'' \cap (W'')^{-1}$ we may assume that W'' is symmetric. Let us define $W = K \cdot W'' \cdot K$. Then W is a symmetric, open neighborhood of the identity in $\widetilde{G}$ and

$$W = K \cdot W'' \cdot K \subseteq V' \cdot V' \cdot V' \subseteq V\,. \tag{22}$$

Moreover, $K \cdot W \cdot K = W$ and $A \cap (W \cdot W \cdot W) \subseteq A \cap W' = K$, by construction.

Since G is an $\ell-$group, the open neighborhood $p(W)$ of the identity in G contains a compact, open subgroup, J say. We claim that $p^{-1}(J) \cap W \subseteq V$ is a subgroup of $\widetilde{G}$. It is symmetric and contains the identity, so we only have to show that it is closed under multiplication. Suppose that w_1 and w_2 are in $p^{-1}(J) \cap W$. Then $p(w_1 w_2)$

lies in J and since $J \subseteq p(W)$ there is some $w_3 \in W$ such that $p(w_1 w_2) = p(w_3)$. This equation implies that

$$w_1 w_2 w_3^{-1} \in A \cap (W \cdot W \cdot W) \subseteq K \tag{23}$$

and so $w_1 w_2 \in K w_3 \subseteq KW \subseteq W$. This establishes our claim. We have a topological short exact sequence

$$1 \longrightarrow K \longrightarrow p^{-1}(J) \cap W \xrightarrow{\ p\ } J \longrightarrow 1 \tag{24}$$

and it follows (see, for example, I, section 19, Chapter 3 of [6]) that $p^{-1}(J) \cap W$ is compact. Since it is also open, we have shown that $\widetilde{G}$ is an $\ell-$group.

If A is discrete, then there is an open set U' in $\widetilde{G}$ such that $U' \cap A = \{e\}$. Since $\widetilde{G}$ is an $\ell-$group, we may find a compact, open subgroup $\widetilde{U}$ of $\widetilde{G}$ such that $\widetilde{U} \subseteq U'$. Then $\widetilde{U} \cap A = \{e\}$ and so if we set $U = p(\widetilde{U})$, then $p|_{\widetilde{U}} : \widetilde{U} \to U$ is an isomorphism of topological groups. Its inverse splits the sequence over U. Let S be a left transversal for U in G and, for each $s \in S$, choose any $\widetilde{s} \in \widetilde{G}$ with $p(\widetilde{s}) = s$. If we define $\boldsymbol{s} : G \to \widetilde{G}$ by $\boldsymbol{s}(su) = \widetilde{s}(p|_{\widetilde{U}})^{-1}(u)$ for $s \in S$ and $u \in U$, then $\boldsymbol{s}$ is the required section. □

Lemma 4. *Let*

$$1 \longrightarrow A \longrightarrow \widetilde{G} \xrightarrow{\ p\ } G \longrightarrow 1 \tag{25}$$

be a topological central extension of $\ell-$groups with A discrete and of finite exponent b. Suppose that G has a neighborhood base at the identity consisting of compact, open subgroups U such that U^b is open, where U^b denotes the group generated by $\{u^b \mid u \in U\}$. If f is a topological automorphism of G, then any lift of f to $\widetilde{G}$ is also a topological automorphism.

Proof. Lemma 3 implies that we may find a compact, open subgroup K of G and a continuous, open section $\boldsymbol{s}$ such that $\boldsymbol{s} : K \to \boldsymbol{s}(K)$ is a homomorphism. Let $f \in \mathrm{Aut}(G)$ be continuous. Then, by hypothesis, we may find a compact, open subgroup L of $f^{-1}(K)$ such that L^b is open. Let $\widetilde{f}$ be a lift of f and define $\zeta : p^{-1}(L) \to p^{-1}(L)$ by $\zeta = \widetilde{f}^{-1} \circ \boldsymbol{s} \circ f \circ p$. Then $p \circ \zeta = p$ and so ζ is a lift of the identity map on L. From Lemma 1, it follows that ζ differs from the identity map by the action of some element of $\mathrm{Hom}(L, A)$. But any such homomorphism is trivial on L^b and so ζ is the identity on $p^{-1}(L^b)$. Composing on the left with $\widetilde{f}$ we obtain $\widetilde{f} = \boldsymbol{s} \circ f \circ p$ on $p^{-1}(L^b)$ and hence on $\boldsymbol{s}(L^b)$. Now both $\boldsymbol{s}(L^b)$ and $\boldsymbol{s}(f(L^b))$ are neighborhoods of the identity in $\widetilde{G}$ and it follows from what we have just done that $\widetilde{f} : \boldsymbol{s}(L^b) \to \boldsymbol{s}(f(L^b))$ is a homeomorphism. Since $\widetilde{f}$ is an automorphism and the topology of $\widetilde{G}$ is homogeneous, the claim follows. □

Proposition 3. *Let F be a non-Archimedean local field and A a discrete group of finite exponent b. Suppose that $b \neq 0$ in F. Let G be the $F-$points of an affine algebraic group defined over F with the 'classical' topology (that is, the weakest topology which makes all polynomial functions from G to F continuous). Suppose that*

$$1 \longrightarrow A \longrightarrow \widetilde{G} \xrightarrow{\ p\ } G \longrightarrow 1 \tag{26}$$

is a topological central extension of G. If f is a topological automorphism of G, then any lift of f to $\widetilde{G}$ is also a topological automorphism.

Proof. By considering the action of G on its affine algebra it is possible to embed G as an $F-$Zariski closed algebraic subgroup of some $\mathrm{GL}(n, F)$ in such a way that the classical topology on $\mathrm{GL}(n, F)$ induces the classical topology on G. Since $\mathrm{GL}(n, F)$ with its classical topology is an $\ell-$group, it follows that G is an $\ell-$group. It is also an analytic group over F and the differential at the identity of the map from G to itself given by $u \mapsto u^b$ is multiplication by b, which is an isomorphism by hypothesis. It follows that the image under this map of any neighborhood of the identity is also a neighborhood of the identity. Consequently, if U is any open subgroup of G, then U^b is also open. We have thus verified the hypotheses of Lemma 4 and the conclusion follows. ☐

Example 1. We sketch an example to show that the hypothesis that $b \neq 0$ in F is necessary. Let F be the local field $\mathbb{F}_2((x))$ of formal Laurent series with coefficients from $\mathbb{F}_2$ and let $\mathbb{G}_{\mathrm{a}}$ be the additive group. It is well-known that $G = \mathbb{G}_{\mathrm{a}}(F)$ with the classical topology is isomorphic as a topological group to

$$\left(\bigoplus_{n=-\infty}^{-1} \mathbb{Z}/2\mathbb{Z}\right) \times \left(\prod_{n=0}^{\infty} \mathbb{Z}/2\mathbb{Z}\right) \tag{27}$$

where $\mathbb{Z}/2\mathbb{Z}$ has the discrete topology, the second factor has the resulting product topology and the first factor has the discrete topology. Let $A = \mathbb{Z}/2\mathbb{Z}$. There are numerous discontinuous homomorphisms from G to A; let χ be one of them. Then $(a, g) \mapsto (a\chi(g), g)$ is a discontinuous lift of the identity automorphism of G to $\widetilde{G} = A \times G$.

Corollary 1. *Let F be a non-Archimedean local field and suppose that the group of n^{th} roots of unity in F has order n. Let c be the n^{th} order Hilbert symbol on F and $\widetilde{\mathrm{GL}}(n)$ the corresponding metaplectic group. Then the lift of any topological automorphism of $\mathrm{GL}(n)$ to $\widetilde{\mathrm{GL}}(n)$ is also a topological automorphism.*

Proof. The statement of the corollary requires a little explanation, after which it will be seen to be an immediate consequence of Proposition 3. The kernel of the metaplectic extension referred to in the statement is $A = \mu_n(F)$, the group of n^{th} roots of unity in F. By assumption this is a cyclic group of order n and hence the polynomial $T^n - 1 \in F[T]$ is separable. Thus n, which is the exponent of A, is not zero in F.

It remains to explain why

$$1 \longrightarrow A \longrightarrow \widetilde{\mathrm{GL}}(n) \xrightarrow{\ p\ } \mathrm{GL}(n) \longrightarrow 1 \tag{28}$$

is a topological central extension. It has been shown by Matsumoto that, with F and c as chosen here and giving $\mathrm{SL}(n)$ its classical topology, $\widetilde{\mathrm{SL}}(n)$ can be made into a topological group in such a way that (2) is a topological central extension (see [4], Théorème 8.2 and the discussion which follows it). The same conclusion then holds for the central extension of $\mathrm{GL}(n)$ which corresponds with the $2-$cocycle $(g, g') \mapsto \tau(\eta(g), \eta(g'))$, in the notation of section 3, since η is closed and continuous. The $2-$cocycle $(g, g') \mapsto c(\det(g), \det(g'))^{m+1}$ is trivial on the open subgroup $\{g \in \mathrm{GL}(n) \mid \det(g) \in (F^\times)^n\}$ and hence also corresponds to a topological central extension. The group operations on the set of central extensions of $\mathrm{GL}(n)$ by A may be performed on the group level using Baer's recipes and these preserve the subset of topological extensions. Thus (28), which corresponds to the cocycle σ

defined by (4), may be made into a topological central extension by giving $\widetilde{\mathrm{GL}}(n)$ an appropriate topology. $\square$

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