

**A CHARACTERIZATION OF σ -COMPACTNESS
OF A COSMIC SPACE X
BY MEANS OF SUBSPACES OF R^X**

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(Communicated by Alan Dow)

ABSTRACT. This work is devoted to the relationship between topological properties of a space X and those of $C_p(X)$ (= the space of continuous real-valued functions on X , with the topology of pointwise convergence). The emphasis is on σ -compactness of X and on location of $C_p(X)$ in R^X . In particular, σ -compact cosmic spaces are characterized in this way.

1. INTRODUCTION

In 1974 J.P.R. Christensen [11] proved that a separable metrizable space X is σ -compact if and only if $C_p(X)$ is analytic, that is, $C_p(X)$ is a continuous image of the space of irrational numbers, and posed the following question: Is a completely regular space X σ -compact provided $C_p(X)$ is analytic? In 1980 this question was posed again in [16], Problem 37. In 1985 the second author answered Christensen's question positively [7]. The converse is false in general: the first author constructed a countable Fréchet-Uryshon \aleph_0 -space (in the sense of E. Michael, see [17]) such that $C_p(X)$ is not analytic (see [8], [9]). Recently O.G. Okunev [18] established that if a Tychonoff space X is σ -bounded, then there exists a $K_{\sigma\delta}$ -subspace A of R^X such that $C_p(X) \subset A$ (in this case, we say that $C_p(X)$ is $K_{\sigma\delta}$ -framed in R^X).

The main goal of this article is to generalize the two first results in the light of Okunev's one. In particular, we obtain that if X is a cosmic space, or a perfectly normal K -analytic space, then X is σ -compact if and only if $C_p(X)$ is K -analytic-framed in R^X (Theorems 2.4 and 2.7). To do this, we blend the suitably adapted technique used by Christensen in the original argument (see the proof that 4) \Rightarrow 5) in Theorem 2.3) with some other technical tools and concepts.

It follows from Theorem 2.4 that if X is a separable metrizable space such that $C_p(X)$ is K -analytic-framed in R^X , then X is σ -compact.

We also give an application of Theorem 2.4 to the problem of embedding a cosmic space in an analytic space (Theorem 2.8, Corollaries 2.9 and 2.10).

By a space we mean a Hausdorff topological space, R^X is the space of real-valued functions on X with the product topology, $C(X)$ is the set of continuous

Received by the editors December 8, 1996 and, in revised form, November 1, 1997 and November 12, 1997.

1991 *Mathematics Subject Classification.* Primary 54C35; Secondary 54D45, 28A05.

Key words and phrases. Function spaces, topology of pointwise convergence, σ -compactness, K -analytic spaces.

real-valued functions on X , and $C_p(X)$ is $C(X)$ with the topology generated by R^X . Our terminology is the same as in [3] and [12].

A mapping φ of a space X into the set $\mathcal{K}(Y)$ of compact subsets of a space Y is called *usco-compact* if $Y = \bigcup\{\varphi(x) : x \in X\}$ and if for every $x \in X$ and every open subset $G \supset \varphi(x)$ of Y , there exists an open neighbourhood H of x such that $\varphi(x') \subset G$, for each $x' \in H$. For a subset A of X , we put $\varphi(A) = \bigcup\{\varphi(x) : x \in A\}$. Recall that if A is compact, then $\varphi(A)$ is compact.

A space X is called a $K_{\sigma\delta}$ -space if it is a $K_{\sigma\delta}$ -subset of a space Z , that is, if X is the intersection of a countable family of σ -compact subsets of Z . A space X is *K -analytic* (G. Choquet [10]), if it is a continuous image of a $K_{\sigma\delta}$ -space; X is *K -analytic* if and only if it is an usco-compact image of the space ω^ω of irrational numbers, where ω is the discrete space of integers [14], [13].

A subspace X of Y is *\mathcal{P} -framed in Y* (where \mathcal{P} is a topological property) if there exists a subspace Z of Y with the property \mathcal{P} such that $X \subset Z \subset Y$ [4].

A *network* in a topological space X is a collection γ of subsets of X such that, for every $x \in X$ and every open subset G with $x \in G$, there exists $N \in \gamma$ such that $x \in N \subset G$. Spaces with a countable network are often called *cosmic* spaces; these are precisely continuous images of separable metrizable spaces (see [1]).

Recall that a subset Y of a space X is said to be *bounded* (in X) if for every $f \in C_p(X)$, the restriction $f|_Y$ is bounded. A space X is called *σ -bounded* if it is a countable union of bounded subspaces. A space X has the property \mathcal{P} projectively if every separable metrizable space Y , which is a continuous image of X , has the property \mathcal{P} . Every σ -bounded space is projectively σ -compact, since every σ -bounded paracompact space is σ -compact.

Note that R^X is projectively analytic (it suffices to apply an appropriate factorization theorem for continuous real-valued functions on R^X (see [12])).

2. σ -COMPACTNESS AND K -ANALYTIC-FRAMENESS

Let X and Y be two sets, A a subset of X , and f a mapping of X onto Y . Then p_A the *projection mapping* of R^X to R^A , that is, $p_A(f) = f|_A$, and \tilde{f} is the *dual mapping* of R^Y into R^X defined by $\tilde{f}(g) = g \circ f$. We need the following two lemmas, the first of which is obvious:

Lemma 2.1. *Let X be a normal space, A a closed subspace of X , \mathcal{P} a projective property, and let $C_p(X)$ be \mathcal{P} -framed in R^X . Then $C_p(A)$ is \mathcal{P} -framed in R^A .*

Lemma 2.2. *Let X and Y be topological spaces and f a continuous mapping from X onto Y . Then \tilde{f} is an embedding of R^Y onto $\tilde{f}(R^Y)$; $\tilde{f}(C(Y)) \subset C(X)$; and $\tilde{f}(R^Y)$ is closed in R^X . Moreover, if \mathcal{P} is a closed-hereditary property and $C_p(X)$ is \mathcal{P} -framed in R^X , then $C_p(Y)$ is \mathcal{P} -framed in R^Y .*

Proof. Clearly, \tilde{f} is a homeomorphism of R^Y onto $\tilde{f}(R^Y)$, $\tilde{f}(C(Y)) \subset C(X)$, and $\tilde{f}(R^Y)$ is closed in R^X . Now, let A be a subspace of R^X with the property \mathcal{P} such that $C(X) \subset A$. Note that $\tilde{f}(C(Y)) \subset A$. Since \mathcal{P} is closed-hereditary, the closed subspace $A \cap \tilde{f}(R^Y)$ of A has the property \mathcal{P} . From $\tilde{f}(C(Y)) \subset A \cap \tilde{f}(R^Y) \subset \tilde{f}(R^Y)$ we deduce that $C_p(Y)$ is \mathcal{P} -framed in R^Y .

Theorem 2.3. *Let us consider the following restrictions on a Tychonoff space X :*

- 1) X is σ -compact,
- 2) X is σ -bounded,
- 3) $C_p(X)$ is $K_{\sigma\delta}$ -framed in R^X ,
- 4) $C_p(X)$ is K -analytic-framed in R^X ,
- 5) X is projectively σ -compact.

Then we have: 1) \Rightarrow 2) \Rightarrow 3) \Rightarrow 4) \Rightarrow 5).

Proof. 1) \Rightarrow 2). Evident.

2) \Rightarrow 3). It is a result of O. Okunev [18]. For the sake of completeness, we give the proof. Let \bar{R} be the natural two-point compactification of R . Let (X_n) be a countable covering of X such that for every n and every $f \in C(X)$ the restriction of f to X_n is bounded. For each n , $p_{X_n}(C(X)) \subset \bigcup_{k \in \omega} [-k, k]^{X_n}$, and $(\bigcup_{k \in \omega} [-k, k]^{X_n}) \times \bar{R}^{X \setminus X_n}$ is a σ -compact subspace of \bar{R}^X . Now one easily verifies that $C(X) \subset \bigcap_n ((\bigcup_{k \in \omega} [-k, k]^{X_n}) \times \bar{R}^{X \setminus X_n}) \subset R^X$. Hence, $C_p(X)$ is $K_{\sigma\delta}$ -framed in R^X .

3) \Rightarrow 4). Evident.

4) \Rightarrow 5). Let X be a Tychonoff space such that $C_p(X)$ is K -analytic-framed in R^X . Let Y be a separable metrizable space, and let f be a continuous mapping of X onto Y . We must show that Y is σ -compact. Applying Lemma 2.2, we see that $C_p(Y)$ is K -analytic-framed in R^Y . Put $R_+ = \{x \in R : x > 0\}$ and $C^+(Y) = C(Y) \cap R_+^Y$. Using the exponential function, we see that $C_p^+(Y)$ is K -analytic-framed in R_+^Y . Let A be a K -analytic subspace of R_+^Y such that $C_p^+(Y) \subset A$, and let φ be an usco-compact mapping from ω^ω into $\mathcal{K}(A)$. For every $\sigma \in \omega^\omega$, put $\lambda(\sigma) = \inf \varphi(\{\tau \in \omega^\omega : \tau \leq \sigma\})$, where $\tau \leq \sigma$ means $\tau(n) \leq \sigma(n)$ for all $n \in \omega$, and \inf is taken with respect to the usual order on R^Y , that is $f \leq g$ if, for every $y \in Y$, $f(y) \leq g(y)$. Since $\{\tau \in \omega^\omega : \tau \leq \sigma\}$ is compact, $\varphi(\{\tau \in \omega^\omega : \tau \leq \sigma\})$ is compact. Hence, λ maps ω^ω into R_+^Y . Furthermore, the mapping λ has the properties:

a) $\sigma \leq \tau \Rightarrow \lambda(\tau) \leq \lambda(\sigma)$,

b) for every $f \in C_p^+(Y)$, there exists $\sigma \in \omega^\omega$ such that $\lambda(\sigma) \leq f$.

To see b), fix $f \in C_p^+(Y)$. As $f \in A$ and $\varphi(\omega^\omega) = A$; there is a $\sigma \in \omega^\omega$ such that $f \in \varphi(\sigma)$. Hence, $\lambda(\sigma) \leq f$.

Now, embed Y in a metric compactification \bar{Y} and take a metric d on \bar{Y} which induces the topology of \bar{Y} . For every $\sigma \in \omega^\omega$, put

$$\theta(\sigma) = \bigcap_{y \in Y} (\bar{Y} \setminus B(y, \lambda(\sigma)(y)))$$

where $B(y, \rho)$ is the open ball with center y and with radius ρ . Clearly, the mapping θ goes from ω^ω into $\mathcal{K}(\bar{Y} \setminus Y)$, and, applying a), for every $\sigma \leq \tau$ we obtain that $\theta(\sigma) \leq \theta(\tau)$. Let K be a compact subset of $\bar{Y} \setminus Y$. The restriction of the function $d(K, \dots)$ to Y belongs to $C_p^+(Y)$. Hence, applying b), we obtain that $K \subset \theta(\sigma)$ for some σ .

For every non-void compact subset K of ω^ω , put $\sigma(K) = \sup\{\tau \in \omega^\omega : \tau \in K\}$ (sup is taken with respect to the usual order on ω^ω). Define $\psi(K) = \theta(\sigma(K))$ for every non-void compact subset K of ω^ω and $\psi(\emptyset) = \emptyset$. We obtain a mapping of

$\mathcal{K}(\omega^\omega)$ into $\mathcal{K}(\overline{Y} \setminus Y)$ with the properties:

a') For every pair of compact subsets K, K' of ω^ω such that $K \subset K'$, we have $\psi(K) \subset \psi(K')$,

b') For each compact subset K' of $\overline{Y} \setminus Y$, there exists a compact subset K of ω^ω such that $K' \subset \psi(K)$.

Hence, the space $\overline{Y} \setminus Y$ is Polish by virtue of a result of Christensen [11] (see also J. Saint-Raymond [21]); therefore Y is σ -compact.

Remarks. It is not possible to replace property 4) by the next more general property:

6) $C_p(X)$ is (projectively analytic)-framed in R^X , since, as we have already mentioned, R^X is projectively analytic for every X .

Note also that in general 5) does not imply 2). Indeed, the one-point Lindelöfication of an uncountable discrete space is projectively σ -compact (even projectively countable) while obviously it is not σ -bounded (see [3] and [21] for more on that).

Now we can easily prove the result referred to in the title.

Theorem 2.4. *Let X be a regular cosmic space (in particular, a separable metrizable space). Then the following properties are equivalent:*

- 1) X is σ -compact,
- 2) $C_p(X)$ is K -analytic-framed in R^X ,
- 3) $C_p(X)$ is analytic-framed in R^X .

Proof. 1) \Leftrightarrow 2) results immediately from Theorem 2.3 and the following result of O.G. Okunev [20]: If a cosmic space is projectively σ -compact, then it is σ -compact.

3) \Rightarrow 2) is evident.

1) \Rightarrow 3). Let X be a σ -compact cosmic space and let (K_n) be a sequence of compact subsets of X covering X . The topological sum $Y = \sum K_n$ is separable, metrizable and σ -compact, hence $C_p(Y)$ is analytic by Christensen's theorem. Let f be the canonical mapping of Y onto X . In virtue of Lemma 2.2, the space R^X is embedded by the dual mapping \tilde{f} into R^Y , and the closed subset $\tilde{f}(R^X) \cap C_p(Y)$ of $C_p(Y)$ is analytic and contains $\tilde{f}(C_p(X))$.

Remarks. In the preceding proof the analytic space $\tilde{f}(R^X) \cap C_p(Y)$ is a linear topological space, and its fan-tightness is countable (see [3], chap. 2, Sec. 2). Note that 1) and 2) are not equivalent for Tychonoff spaces in general, since for every pseudocompact space X (which need not be σ -compact) the space $C_p(X)$ is obviously K -analytic-framed in R^X .

Corollary 2.5. *If a separable metrizable space X is such that $C_p(X)$ is K -analytic-framed in R^X , then $C_p(X)$ is analytic.*

Corollary 2.6. *Let \mathcal{J} ($\equiv \omega^\omega$) be the space of irrational numbers, then $C_p(\mathcal{J})$ is not K -analytic-framed in $R^{\mathcal{J}}$.*

Theorem 2.7. *Let X be a perfectly normal K -analytic space. Then the following properties are equivalent:*

- 1) X is σ -compact,
- 2) $C_p(X)$ is K -analytic-framed in R^X .

Proof. 1) \Rightarrow 2) follows from Theorem 2.3.

2) \Rightarrow 1). By contradiction. Suppose that X is not σ -compact. Then, by a result of Jayne and Rogers [15], we have a perfect mapping of a closed subset F of X onto ω^ω . Applying Lemmas 2.1 and 2.2, we obtain that $C_p(\omega^\omega)$ is K -analytic-framed in R^{ω^ω} , which it is not.

By an *analytic uniform space* we mean a uniform space which is analytic as a topological space. We call a regular cosmic space Y *regularly subanalytic*, if there exists a regular analytic space Z containing Y as a subspace. Similarly, a uniform space Y will be called *uniformly subanalytic*, if there exists an analytic uniform space Z containing Y as a uniform subspace. The next application of Theorem 2.4 seems to be of special interest.

Theorem 2.8. *Let X be a regular cosmic space. Then $C_p(X)$, with its natural uniformity, is uniformly subanalytic if (and only if) X is σ -compact.*

Proof. For the “only if” part it suffices to refer to Theorem 2.4 and to note that $C_p(X)$ is not only a topological subspace of R^X but also a uniform subspace of it.

Now, let $C_p(X)$, endowed with the natural uniformity, be a uniform subspace of an analytic uniform space Z . Obviously, we may assume that $C_p(X)$ is dense in Z . Since R^X is a complete uniform space, we can extend the identity mapping of $C_p(X)$ to a (uniformly) continuous mapping f of Z into R^X . Then the image $f(Z)$ is an analytic subspace of R^X containing $C_p(X)$. Thus, $C_p(X)$ is analytic-framed in R^X , and, according to Theorem 2.4, X is σ -compact.

Corollary 2.9. *The uniform space $C_p(\omega^\omega)$ is not uniformly subanalytic.*

Taking into account the remark after the proof of Theorem 2.4, we get the following result:

Corollary 2.10. *A regular cosmic space X is σ -compact if and only if $C_p(X)$ can be represented as a linear topological subspace of an analytic linear topological space.*

3. $C_p C_p(X)$ AND K -ANALYTIC-FRAMENESS

In this section, all spaces are Tychonoff.

Proposition 3.1. *If a space X is not pseudocompact, then $C_p C_p(X)$ is not K -analytic-framed in $R^{C(X)}$.*

Proof. Indeed, there is an infinite discrete family of non-empty open sets in X . Therefore, the discrete space ω is l -embedded into X , that is, there exists a continuous linear mapping

$$\varphi : C_p(\omega) = R^\omega \longrightarrow C_p(X)$$

such that $g = \varphi(g)|_\omega$ for every $g \in R^\omega$ (see [5]). Hence, R^ω is a retract of $C_p(X)$ (see [4]). Therefore, for the dual mapping $\tilde{\varphi} : R^{C(X)} \longrightarrow R^{R^\omega}$, we have

$$\tilde{\varphi}(C(C_p(X))) = C(R^\omega).$$

Now, ω^ω is closed in the separable metrizable space R^ω ; hence, using Lemma 2.1, for the projection p_{ω^ω} of R^{R^ω} into R^{ω^ω} we have $p_{\omega^\omega}(C(R^\omega)) = C(\omega^\omega)$. Hence,

$$p_{\omega^\omega} \circ \tilde{\varphi}(C(C_p(X))) = C(\omega^\omega).$$

Since $C_p(\omega^\omega)$ is not K -analytic-framed in R^{ω^ω} (by Corollary 2.6), it follows that $C_p C_p(X)$ is not K -analytic-framed in $R^{C(X)}$.

A space X is called a P -space, if every G_δ -subset of X is open in X . If M is a subspace of X , we denote by $C_p(M|X)$ the subspace of $C_p(M)$, which is the image of $C_p(X)$ under the restriction mapping.

In the proof of Theorem 3.3, we need the next theorem which is slightly more general than Theorem 1.2.2 in [3] and is proved by the same argument. (See also assertions 9.2, 9.6 and 9.12 in [2].)

Theorem 3.2. *If $C_p(M|X)$ is σ -countably compact, then M is a P -space (in itself), and M is bounded in X .*

Theorem 3.3. *If $C_p C_p(X)$ is K -analytic-framed in $R^{C(X)}$, then every countable subspace M of X is discrete.*

Proof. The restriction mapping of $C_p(X)$ onto $C_p(M|X)$ is surjective and continuous. Therefore, by the dual mapping, $R^{C(M|X)}$ topologically embeds as a closed subspace into $R^{C(X)}$ in such a way, that $C_p C_p(M|X)$ embeds as a subspace into $C_p C_p(X)$.

Let Z be a K -analytic subspace of $R^{C(X)}$ such that $C_p C_p(X) \subset Z$. Then the intersection of Z and $R^{C(M|X)}$ is a closed subspace F of Z such that $C_p C_p(M|X) \subset F$. It follows that F is K -analytic and $C_p C_p(M|X)$ is K -analytic-framed in $R^{C(M|X)}$. Since M has a countable network, it follows that $C_p(M|X)$ also has a countable network (see [3]). Thus, by Theorem 2.4, $C_p(M|X)$ is σ -compact. This implies by Theorem 3.2 that M is a P -space. Since M is countable, it follows that M is discrete.

Corollary 3.4. *If $C_p C_p(X)$ is K -analytic-framed in $R^{C(X)}$, and Y is a subspace of X such that Y is countably compact in X , then Y is finite.*

Corollary 3.5. *If $C_p C_p(X)$ is K -analytic-framed in $R^{C(X)}$, and Y is a dense subspace of X such that Y is countably compact in X , then X is finite.*

Corollary 3.6. *If $C_p C_p(X)$ is K -analytic-framed in $R^{C(X)}$, and X is countably compact, then X is finite.*

Remark. The last result cannot be generalized to the class of pseudocompact spaces. Indeed, in [2], Proposition 9.31, a space X constructed by D. Shakhmatov is described. The space X is infinite, pseudocompact, and the space $C_p(X)$ is σ -bounded (even σ -pseudocompact) and, therefore, projectively σ -compact. By virtue of Okunev's result in [18], the space $C_p C_p(X)$ is K -analytic-framed in $R^{C(X)}$.

Problem 1. Let X be a Tychonoff space such that $C_p(X)$ is regularly subanalytic. Is X then σ -compact? Is the space $C_p(\omega^\omega)$ regularly subanalytic? Is $C_p C_p(\omega^\omega)$ regularly subanalytic?

This problem is a version of Christensen's problem (see [11]), in which no restrictions on separation axioms were explicitly stated. Note that A. Bešlagić [5] and J. Calbrix [6] proved that every regular cosmic space can be embedded in a (Hausdorff) analytic space, while there exists a Hausdorff cosmic space which cannot be embedded in any (Hausdorff) analytic space. It seems that the descriptive structure of $C_p(X)$ tends to become more and more complicated when we take iterated function spaces, so they may be useful in the search for a counterexample to the first part of Problem 1.

Problem 2. Let $C_p(X)$ be K -analytic-framed ($K_{\sigma\delta}$ -framed) in R^X . Is X then σ -bounded?

The positive answer to this question would mean that the conditions 2), 3), and 4) (2) and 3)) in Theorem 2.3 are equivalent.

Problem 3. Let X be a Lindelöf space such that $C_p(X)$ is K -analytic. Is X then σ -compact?

Problem 4. Let X be a Lindelöf space such that $C_p(X)$ is K -analytic-framed. Is X then σ -compact?

Problem 5. Characterize X as being analytic by a natural topological property of $C_p(X)$.

That this might be possible is suggested by the following result of Okunev [19]: If X is analytic, and $C_p(X)$ and $C_p(Y)$ are homeomorphic, then Y is also analytic.

REFERENCES

- [1] A.V. Arhangel'skii, *An addition theorem for the weight of sets lying in bicomacta*, DAN CCCP, 126, (1959), 239-241. (In Russian.) MR **21**:5176
- [2] A.V. Arhangel'skii, *Function spaces in the topology of pointwise convergence and compact sets*, Russian Math. Surveys 39:5 (1984), 9-56.
- [3] A.V. Arhangel'skii, *Topological function spaces*, Mathematics and its Applications (Soviet Series), v.78 Kluwer Academic Publishers Group, Dordrecht, 1992. MR **92i**:54022
- [4] A.V. Arhangel'skii, *C_p -theory*, in: Recent Progress in General Topology, Hušek, van Mill Ed-rs, Elsevier Sciences Publishers B.V., (1992), 3-48.
- [5] A. Bešlagić, *Embedding cosmic spaces in Lusin spaces*, Proc. Amer. Math. Soc. 89:3 (1983), 515-518. MR **84h**:54037
- [6] J. Calbrix, *Une propriété des espaces topologiques réguliers, images continues d'espaces métrisables séparables*, C. R. Acad. Sc. Paris, t. 295, (1982), 81-82. MR **83j**:54029
- [7] J. Calbrix, *Espaces K_σ et espaces des applications continues*, Bull. Soc. Math. France, 113, (1985), 183-203. MR **87d**:54064
- [8] J. Calbrix, *Filtres sur les entiers et ensembles analytiques*, C. R. Acad. Sc. Paris, t. 305, (1987), 109-111. MR **88j**:54051
- [9] J. Calbrix, *k -spaces and Borel filters on the set of integers*, Trans. Amer. Math. Soc., 348, 1996, 2085-2090. (In French.) MR **96h**:54025
- [10] G. Choquet, *Ensembles K -analytiques et K -Sousliniens*, Ann. Inst. Fourier, 9, 1959, 77-81. MR **22**:3692a
- [11] J.P.R. Christensen, *Topology and Borel Structure*, North Holland, Amsterdam, (1974). MR **50**:1221
- [12] R. Engelking, *General Topology*, Heldermann, (1989). MR **91c**:54001
- [13] Z. Frolik, *On analytic spaces*, Bull. Acad. Polon. Sci., 8, (1961), 747-750.
- [14] J.E. Jayne, *Structure of analytic Hausdorff spaces*, Mathematika 23, (1976), 208-211. MR **57**:1451
- [15] J.E. Jayne and C.A. Rogers, *Borel isomorphisms at the first level*, I, II, Mathematika 26, (1979), 125-179, and 27 (1980), 236-260. MR **81h**:54048a; MR **81h**:54048b; MR **82e**:54043
- [16] C.A. Rogers, J.E. Jayne, C. Dellacherie, F. Topsoe, J. Hoffman-Jorgensen, D.A. Martin, A.S. Kehris, and A.H. Stone, *Analytic sets*, Academic Press, London, 1980.
- [17] E. Michael, \aleph_0 -Spaces, J. Math. Mech., 15, (1966), 983-1002. MR **34**:6723
- [18] O.G. Okunev, *On Lindelöf Σ -spaces of continuous functions in the pointwise topology*, Topology and Appl., 49, (1993), 149-166. MR **94b**:54055
- [19] O.G. Okunev, *Weak topology of an associated space, and t -equivalence*, Math. Notes 41:1-3 (1990), 534-539. MR **91h**:46008

- [20] O.G. Okunev, *On analyticity in cosmic spaces*, Comment. Math. Univ. Carolinae 34:1 (1993), 185–190. MR **94h**:54032
- [21] J. Saint-Raymond, *Caractérisation d'espaces polonais*, Seminaire d'initiation à l'Analyse (Choquet), 5, (1971-1973), 10 pages. MR **57**:12811

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