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# WEAKLY COMPACT COMPOSITION OPERATORS BETWEEN ALGEBRAS OF BOUNDED ANALYTIC FUNCTIONS

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Abstract. We prove a characterization (up to the approximation property) of weakly compact composition operators $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ in terms of their inducing analytic maps $\phi : B_E \to B_F$.

Let $E$ denote a complex Banach space with open unit ball $B_E$ and let $\phi : B_E \to B_F$ be an analytic map, where $F$ is also a complex Banach space. We will consider the composition operator $C_\phi$ defined by $C_\phi(f) = f \circ \phi$, acting from the uniform algebra $H^\infty(B_F)$ of all bounded analytic functions on $B_F$ into $H^\infty(B_E)$.

In [U] A. Ülger proved that every weakly compact homomorphism from a logmodular uniform algebra into any uniform algebra is compact. This was also later proved in [GL] by a slightly different proof. In his paper A. Ülger also asks whether every weakly compact homomorphism between uniform algebras is compact without the logmodularity condition. An example showing that this is not generally true was given in [AGL]. In fact it was shown that the uniform algebra $H^\infty(B_E)$ when $E$ is the Tsirelson space and $\phi(x) = \frac{x}{2}$ give rise to a weakly compact homomorphism which is not compact. In this note we characterize (modulo the approximation property) the weakly compact composition operators $C_\phi$ which makes it possible, in a general way, to produce noncompact weakly compact composition operators $C_\phi$. As a byproduct of our technique we characterize the completely continuous composition operators. In [GG] M. González and J. Gutiérrez have studied weakly compact composition operators between the Fréchet algebras $H_b(B_E)$ of analytic functions of bounded type endowed with the topology of uniform convergence on $B_E$-bounded sets.

**Preliminaries.** The reader is referred to [D] and [M2] for background information on analytic functions on an infinite dimensional Banach space. The algebra $H^\infty(B_E)$ is a Banach algebra with the natural norm $\|f\| = \sup_{x\in B_E} |f(x)|$. This algebra, which is a natural generalization of the classical algebra $H^\infty(\Delta)$ of analytic functions on the complex open disk $\Delta$, has been studied in [ACG]. A homomorphism between Banach algebras is a continuous linear multiplicative map. By an operator we mean a continuous linear map from a Banach space into another Banach space. The space of all operators from $E$ into $F$ is denoted by $L(E, F)$. We denote

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the adjoint operator of $T \in L(E,F)$ by $T^t : F^* \to E^*$. We say that $T \in L(E,F)$ is *(weakly) compact* if $T$ maps bounded sets in $E$ into relatively (weakly) compact sets in $F$. If $T$ maps the closed unit ball of $E$ onto a conditionally weakly compact set, $T$ is called a *Rosenthal* operator. Clearly every weakly compact operator is Rosenthal. The operator $T$ is called *completely continuous*, if it maps every weakly convergent sequence of $E$ into a norm convergent one of $F$. Every compact operator is completely continuous. A subset $A \subset E$ is called a *Dunford-Pettis set*, if for every weakly null sequence $(l_n) \subset E^*$ and every sequence $(x_n) \subset A$ we have that $l_n(x_n) \to 0$, when $n \to \infty$. If $E^*$ has the Schur property, then every bounded set in $E$ is a Dunford-Pettis set.

Let $P(E)$ denote the algebra of all continuous polynomials on $E$. We denote by $(E, \sigma(E, P(E)))$ (respectively $(B_E, \sigma(E, H^\infty(B_E)))$ the set $E$ (respectively $B_E$) endowed with the weakest topology making all $p \in P(E)$ (respectively $f \in H^\infty(B_E)$) continuous. The topology $\sigma(E, P(E))$ is a regular Hausdorff topology such that $(E, ||\cdot||) \geq (E, \sigma(E, P(E))) \geq (E, \sigma(E, E^*))$. Thus it follows that $\sigma(E, P(E))$ is angelic, and consequently the concepts (relatively) countably compact, (relatively) sequentially compact and (relatively) compact all agree with respect to this topology. A Banach space $E$ is called a $\Lambda$-*space*, if all null sequences in $(E, \sigma(E, P(E)))$ are norm convergent, and hence convergent sequences in $(E, \sigma(E, P(E)))$ are also norm convergent. All superreflexive spaces and $\ell_1$ are $\Lambda$-spaces [JaP].

The space $H^\infty(B_E)$ is a dual space, i.e., there is a Banach space $G^\infty(B_E)$ such that $H^\infty(B_E) = G^\infty(B_E)^*$. This fact follows from a theorem of K. Ng [N] and has been pointed out by S. Dineen in his book [D] and developed by J. Mujica in [M1]. By $\tau_0$ we denote the compact-open topology on $H^\infty(B_E)$. $G^\infty(B_E)$ is defined as the subspace of $H^\infty(B_E)^*$ of those functionals which are $\tau_0$ continuous when restricted to the unit ball of $H^\infty(B_E)$ or equivalently to the bounded subsets. The correspondence $f \in H^\infty(B_E) \to T_f \in G^\infty(B_E)^*$ given by $T_f(u) = u(f), u \in G^\infty(B_E)$, is an isometric isomorphism. The map $\delta : B_E \to G^\infty(B_E)$ is defined by $\delta(x) = \delta_x$, where $\delta_x(f) = f(x)$ for all $f \in H^\infty(B_E)$. The space $G^\infty(B_E)$ is a closed subspace of $H^\infty(B_E)^*$ and the proof of Theorem 2.1 in [M1] shows that the closed unit ball of $G^\infty(B_E)$ coincides with the closed, convex, balanced hull $\overline{\Gamma}\{\delta_x : x \in B_E\}$ of $\{\delta_x : x \in B_E\}$. In particular, we have that $G^\infty(B_E)$ is the closed span of $\{\delta_x : x \in B_E\}$ in $H^\infty(B_E)^*$.

In [AGL] the following characterization of compactness of the composition operator $C_\phi$ was obtained in the case $E = F$. The same proof works also in this more general situation.

**Theorem.** *Consider the composition operator $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$. The following statements are equivalent:*

(i) *$C_\phi$ is compact;*
(ii) *$C_\phi$ is weakly compact and $\phi(B_E)$ is relatively compact in $F$;*
(iii) *$\phi(B_E)$ lies strictly inside $B_F$ and $\phi(B_E)$ is relatively compact in $F$.*

**Weak compactness of composition operators.** We start with an elementary observation.

**Lemma 1.** *Let $(x_\alpha)$ be a net in $E$ such that $||x_\alpha|| \leq r$ for some $0 < r < 1$ and let $x \in B_E$. If $p(x_\alpha) \to p(x)$ for all $p \in P(E)$, then $f(x_\alpha) \to f(x)$ for all $f \in H^\infty(B_E)$. Hence for given $0 < r < 1$, we have that $\sigma(E, H^\infty(B_E))$ and $\sigma(E, P(E))$ coincide on $r\overline{B}_E$.*

*Proof.* Take $f \in H^\infty(B_E)$. It is a uniform limit on every $sB_E$, $0 < s < 1$, of the partial sums of its Taylor series. Thus there is a sequence $(p_n) \subset P(E)$ such that $p_n \to f$ uniformly on $sB_E$ for every $0 < s < 1$. Let $\varepsilon > 0$. Hence there is $n_0$ such that $\sup_{||x_\alpha|| \le r} |p_{n_0}(x_\alpha) - f(x_\alpha)| < \frac{\varepsilon}{2}$ and $|p_{n_0}(x) - f(x)| < \frac{\varepsilon}{2}$, so for all $\alpha$ we have

$$|f(x_\alpha) - f(x)| \le \varepsilon + |p_{n_0}(x_\alpha) - p_{n_0}(x)|.$$

Thus $f(x_\alpha) \to f(x)$, when $\alpha \to \infty$. □

**Proposition 2.** *If $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is Rosenthal or completely continuous, then $\phi(B_E)$ lies strictly inside $B_F$.*

*Proof.* If not, then there is a sequence $(x_j) \subset B_E$ such that $||\phi(x_j)|| > 1 - \frac{1}{j}$ for all $j$. Hence $\lim ||\phi(x_j)|| = 1$. By the proof of Theorem 10.5 in [ACG] there is a $g \in H^\infty(B_F^{**})$ which satisfies $|g| < 1$ and such that $(g(\phi(x_k)))_k$ is interpolating for $H^\infty(\Delta)$, where $\Delta$ is the open unit disk in $\mathbb{C}$ and $(x_k)$ is a subsequence of $(x_j)$. Hence, by Theorem 2.1 in [G, p. 294], there is a sequence $(f_n) \subset H^\infty(\Delta)$ and a constant $M > 0$ such that

$$f_n(g(\phi(x_k))) = 0, \text{ if } n \ne k, \; f_n(g(\phi(x_n))) = 1,$$

and

$$\sum_{n=1}^{\infty} |f_n(z)| \le M \text{ for all } z \in \Delta.$$

Now we define a map $T : H^\infty(B_E) \to l^\infty$ by $T(f) = (f(x_k))_k$ and another map $S : l^\infty \to H^\infty(B_F)$ by $S((\xi_n)) = \sum_{n=1}^\infty \xi_n f_n \circ g$. These two maps are both well-defined, continuous and linear. Further it can be seen that $T \circ C_\phi \circ S = id_{l^\infty}$. Since $id_{l^\infty}$ is neither Rosenthal nor completely continuous, we get a contradiction. □

A portion of the next proposition relies on the proof of Proposition 3 in [GGM].

**Proposition 3.** *Assume $F$ has the approximation property. If $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is a weakly compact composition operator, then $\phi(B_E)$ is relatively compact in $(B_F, \sigma(F, H^\infty(B_F)))$.*

*Proof.* Let us denote $\sigma_F = \sigma(F, H^\infty(B_F))$ and $\sigma_{G^\infty} = \sigma(G^\infty(B_F), H^\infty(B_F))$. By weak compactness of $C_\phi^t$ and the fact that $C_\phi^t(\delta_{B_E}) = \delta_{\phi(B_E)}$, we get that its weak closure in $H^\infty(B_F)^*$, $\overline{\delta_{\phi(B_E)}} \subset G^\infty(B_F)$, endowed with the weak topology is compact. Hence $\overline{\delta_{\phi(B_E)}}$ is compact in $G^\infty(B_F)$ with the induced topology, i.e., in $\sigma_{G^\infty}$. Let $(\phi(x_\alpha)) \subset \phi(B_E)$ be an arbitrary net. We will show that it has a $\sigma_F$ converging subnet to a point in $B_F$ and that will prove that $\phi(B_E)$ is a $\sigma_F$ relatively compact set in $B_F$. Since $\overline{\delta_{\phi(B_E)}}^{\sigma_{G^\infty}}$ is a compact set, there is a subnet (which we denote the same) $\sigma_{G^\infty}$ converging to some $u \in \overline{\delta_{\phi(B_E)}}^{\sigma_{G^\infty}}$. Let $y = u_{|_{F^*}} \in F^{**}$. Observe that by Proposition 2, $\phi(B_E) \subset rB_F$ for some $0 < r < 1$, hence $||y|| \le r$.
We claim that $y \in F$ and $u = \delta_y$. Recall that $u$ is $(H^\infty(B_F), \tau_0)$ continuous on bounded subsets of $H^\infty(B_F)$. An application of the Banach-Dieudonné theorem [M3, Theorem 2.1], for $P(^mF)$, $m \in \mathbb{N}$, leads to the $(P(^mF), \tau_0)$ continuity of $u$. In particular $y \in F$ since the compact open topology on $F^*$ belongs to the dual pair $(F, F^*)$. Then $u(P) = P(y)$ for all finite type polynomials and hence for all polynomials because of the approximation property. Moreover, for each $f \in H^\infty(B_F)$ the sequence of the Cesáro sums $(\sigma_m f)$ of its Taylor series $\tau_0$ converges to $f$ and constitutes a bounded set in $H^\infty(B_F)$ ([M1, 5.2.c])),

thus $u(f) = \lim_m u(\sigma_m f) = \lim_m \sigma_m f(y) = f(y)$. Therefore $\delta_{\phi(x_\alpha)} \to \delta_y$ in $(G^\infty(B_F), \sigma_{G^\infty})$ or in other words, $\phi(x_\alpha) \to y$ in $(B_F, \sigma_F)$. □

**Proposition 4.** *If $\phi(B_E) \subset rB_F$ for some $0 < r < 1$ and $\phi(B_E)$ is relatively compact in $(B_F, \sigma(F, H^\infty(B_F)))$, then $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is weakly compact.*

*Proof.* Consider $C_\phi^t : H^\infty(B_E)^* \to H^\infty(B_F)^*$. Since $H^\infty(B_E)^* = G^\infty(B_E)^{**}$ we consider the continuous restriction map $C_\phi^t|_{G^\infty} : G^\infty(B_E) \to G^\infty(B_F)^{**}$. We have that $G^\infty(B_E)$ is the closed span of $\{\delta_x : x \in B_E\}$ in $H^\infty(B_E)^*$ and $C_\phi^t(\delta_x) = \delta_{\phi(x)}$, so it follows that $C_\phi^t(G^\infty(B_E)) \subset \overline{span}\{\delta_{\phi(x)} : x \in B_E\} \subset G^\infty(B_F)$. Thus the restriction map $S = C_\phi^t|_{G^\infty}$ can be considered as a map from $G^\infty(B_E)$ into $G^\infty(B_F)$. We first show that $S : G^\infty(B_E) \to G^\infty(B_F)$ is weakly compact. Let $\overline{B}_{G^\infty}$ be the closed unit ball in $G^\infty(B_E)$. Then $S(\overline{B}_{G^\infty}) \subset \overline{\Gamma}\{\delta_{\phi(x)} : x \in B_E\}$. By Krein's theorem it is enough to show that $\{\delta_{\phi(x)} : x \in B_E\}$ is relatively $\sigma(G^\infty(B_F), H^\infty(B_F))$-compact. Since $\phi(B_E) \subset rB_F$, it follows that

$$\overline{\phi(B_E)}^{\sigma(F, H^\infty(B_F))} \subset B_F,$$

which is $\sigma(F, H^\infty(B_F))$-compact, so $\delta_{\phi(B_E)}$ is relatively $\sigma(G^\infty(B_F), H^\infty(B_F))$-compact.

Observe that for $f \in H^\infty(B_F)$ and $x \in B_E$, we have $\langle S^t(f), \delta_x\rangle = \langle f, S(\delta_x)\rangle = \langle f, C_\phi^t(\delta_x)\rangle = \langle C_\phi(f), \delta_x\rangle$. Therefore, $S^t(f) = C_\phi(f)$, so $S^t = C_\phi$. Since $S^t : G^\infty(B_F)^* \to G^\infty(B_E)^*$ is weakly compact, $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is weakly compact. □

Collecting the above propositions and recalling Lemma 1, we get

**Theorem 5.** *The composition operator $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is weakly compact if* (i) *$\phi(B_E) \subset rB_F$ for some $0 < r < 1$ and* (ii) *$\phi(B_E)$ is relatively compact in $(F, \sigma(F, P(F)))$. The converse holds if moreover $F$ has the approximation property.*

**Corollary 6.** *If $F$ is a $\Lambda$-space with the approximation property, then every weakly compact composition operator $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is compact.*

**Corollary 7.** *If $\phi(B_E)$ is a relatively weakly compact set strictly inside $B_F$, and $P(F) = P_{wsc}(F)$, where $P_{wsc}(F)$ is the subspace of $P(F)$ of all weakly sequentially continuous polynomials, then $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is weakly compact.*

*Proof.* From any sequence in $\phi(B_E)$ we get a subsequence which is weakly convergent, hence $(F, \sigma(F, P(F)))$-convergent. Thus $\phi(B_E)$ is relatively compact in the angelic space $(F, \sigma(F, P(F)))$. □

**Example 8.** Should we have an infinite dimensional reflexive Banach space $E$, a Banach space $F$ so that $P(F) = P_{wsc}(F)$ and an embedding $\phi : E \to F$ with $||\phi|| < 1$, we will have found a weakly compact noncompact composition operator $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$.

The Banach spaces (and their closed subspaces ) $c_0$, $T^*$, the original (reflexive) Tsirelson space, the Tsirelson*-James space, $T_J^*$, and $c_0 \times T^*$ have property $P_\alpha$ of Pelczynski for all $\alpha < 1$, so they fulfill the assumption regarding $F$ in the above corollary ([AF, Corollary 3]). Also any Banach space $F$ with the Dunford-Pettis property satisfies that $P(F) = P_{wsc}(F)$ [R1]. Therefore we have a bunch of triads $(E, F, \phi)$ fulfilling the quoted conditions. For instance, taking $E$ any separable reflexive Banach space, $F = C([0,1])$ -where $E$ may be isometrically embedded since

$C([0,1])$ is universal among the separable Banach spaces- and $\phi$ any contraction of the embedding. Another example is provided by taking $\phi : x \in T^* \rightsquigarrow rx \in c_0$ $(0 < r < 1)$, which is linear, continuous and weakly compact but noncompact (the unit ball of $T^*$ contains the sequence of units of $c_0$) by the original construction of Tsirelson as can be seen in [HHZ]. The same will happen if we replace $T^*$ by $\ell_2$.

Also, if $E \subset T^*$ is an infinite dimensional Banach space and $\phi : x \in E \rightsquigarrow rx \in T^*$ $(0 < r < 1)$, then $C_\phi : H^\infty(B_{T^*}) \to H^\infty(B_E)$ is weakly compact, but noncompact.

**Corollary 9.** *If $P(^nF)$ is reflexive for all $n$, $F$ has the approximation property and $\phi(B_E)$ is strictly inside $B_F$, then $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is weakly compact.*

*Proof.* Since $P(^nF)$ is reflexive for all $n$, $F$ is reflexive and $P(F) = P_{wsc}(F)$ holds when $F$ has the approximation property (see [AAD], [R]). □

We next describe the completely continuous composition operators $C_\phi$.

**Proposition 10.** *The composition operator $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is completely continuous if and only if* (i) *$\phi(B_E) \subset rB_F$ for some $0 < r < 1$ and* (ii) *$\phi(B_E)$ is a Dunford-Pettis set in $F$.*

*Proof.* Assume first that $C_\phi$ is completely continuous. By Proposition 2, $\phi(B_E)$ is strictly inside $B_F$. Let $(l_n) \subset F^*$ be a weak-null sequence. The map $J : F^* \to H^\infty(B_F)$, $l \mapsto l|_{B_F}$, is weak-weak continuous, so $(J(l_n))$ is a weak-null sequence in $H^\infty(B_F)$. Hence $(l_n)$ converges to zero uniformly on $\phi(B_E)$, so $\phi(B_E)$ is a Dunford-Pettis set in $F$.

Conversely, suppose that the assumptions are satisfied. Take a weak-null sequence $(f_n) \subset H^\infty(B_F)$. Then for every $m$ the sequence of their Taylor polynomials at 0, $(P_m f_n)_n \subset P(^mF)$, is also weakly null. Moreover it follows from [GG, Theorem 2.2] that $\phi(B_E) \otimes \cdots \otimes \phi(B_E)$ is a Dunford-Pettis set in the projective tensor product $F \otimes_\pi \cdots \otimes_\pi F$, thus the sequence $(P_m f_n)_n$ converges uniformly to 0 on $\phi(B_E)$.

Since $(||f_n||)$ is bounded, it follows from Cauchy inequalities that on every ball of radius less than 1 the approximation of each $f_n$ by its Taylor series $\sum P_m f_n$ can be chosen independently of $n$. That is, for a given $\epsilon > 0$, there is $k \in \mathbb{N}$ such that $|\sum_{m=0}^k P_m f_n(x) - f_n(x)| < \epsilon$ for all $n \in \mathbb{N}$ and $x \in rB_F$. Once we have found $k$ we also find, by the above, $n_0 \in \mathbb{N}$ so that $|P_m f_n(z)| < \frac{\epsilon}{k+1}$ for $n > n_0$ and for all $z \in \phi(B_E)$.

So, if $n > n_0$, then $|C_\phi(f_n)(x)| = |f_n(\phi(x))| \leq |\sum_{m=0}^k P_m f_n(\phi(x)) - f_n(\phi(x))| + |\sum_{m=0}^k P_m f_n(\phi(x))| \leq ||\sum_{m=0}^k P_m f_n - f_n||_{rB_F} + ||\sum_{m=0}^k P_m f_n||_{\phi(B_E)} < 2\epsilon$ for all $x \in B_E$. Hence $(||C_\phi(f_n)||)$ converges to 0. □

**Corollary 11.** *Assume that $F^*$ has the Schur property. The composition operator $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is completely continuous if and only if $\phi(B_E)$ lies strictly inside $B_F$.*

**Corollary 12.** *If $F$ has the Dunford-Pettis property as well as the approximation property, then every weakly compact composition operator $C_\phi : H^\infty(B_F) \to H^\infty(B_E)$ is completely continuous.*

*Proof.* By the Dunford-Pettis property of $F$, every weakly null sequence in $F^*$ converges uniformly to 0 in $\phi(B_E)$ since it is a relatively weakly compact subset of

$F$ by Proposition 3. Thus $\phi(B_E)$ is a Dunford-Pettis set lying strictly inside $B_F$ (Proposition 2), so Proposition 10 leads to the result. □

*Remark 13.* There is no characterization of Rosenthal composition operators similar to Proposition 10 . Take $E = F = \ell_2$ and $\phi(x) = \frac{x}{2}$; then $\phi(B_E)$ is conditionally weakly compact. But $C_\phi : P(^2\ell_2) \to P(^2\ell_2)$ is not a Rosenthal operator, since if it were, the identity on $P(^2\ell_2)$, which coincides with $4C_\phi$, would also be a Rosenthal operator which is prevented by the fact that $P(^2\ell_2)$ contains a copy of $\ell^\infty$ [D1, Corollary 4].

**Example 14.** *A Rosenthal nonweakly compact composition operator.* Let $E = F = T_J^*$. Since $E^{**}$ has the Radon-Nikodym property, $E^*$ is an Asplund space, hence $P(^mE) = P_{w*}(^mE^{**})$ is also Asplund ([V, Corollary 1.1]), so it does not contain any copy of $\ell_1$ and therefore its bounded subsets are conditionally weakly compact. If we take $\phi(x) = \frac{x}{2}$, then $C_\phi$ cannot be weakly compact since $\phi(B_E) = \frac{B_E}{2}$ is not a weakly compact set with $E$ a nonreflexive space.

On the other hand $C_\phi$ is a Rosenthal operator. Indeed: Put $U$ for the unit ball in $H^\infty(B_E)$ and as in the proof of the above Proposition, for a given $\epsilon > 0$, there is $k \in \mathbb{N}$ such that $|\sum_{m=0}^k P_m f(x) - f(x)| < \epsilon$ for all $f \in U$ and $x \in \frac{1}{2}B_E$. So, $|| \sum_{m=0}^k \frac{P_m f}{2^m} - C_\phi(f)|| < \epsilon$ for all $f \in U$. Since each of the sets $K_m = \{\frac{P_m f}{2^m} : f \in U\}$ is conditionally weakly compact and $C_\phi(U) \subset \epsilon U + \sum_{m=0}^k K_m$, it follows that $C_\phi(U)$ is a conditionally weakly compact set (see [Di, Example 2, p. 237]).

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