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# MULTIDIMENSIONAL ANALOGUES OF BOHR'S THEOREM ON POWER SERIES

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ABSTRACT. Generalizing the classical result of Bohr, we show that if an $n$-variable power series converges in $n$-circular bounded complete domain $D$ and its sum has modulus less than 1, then the sum of the maximum of the modulii of the terms is less than 1 in the homothetic domain $r \cdot D$, where $r = 1 - \sqrt[n]{2/3}$. This constant is near to the best one for the domain $D = \{z : |z_1| + \ldots + |z_n| < 1\}$.

## 1. PRELIMINARIES

The following formulation of the Bohr's result [5] (due to the work of M. Riesz, I. Schur and F. Wiener) is known.

**Theorem 1.** *If a power series*

$$\sum_{k=0}^{\infty} c_k z^k \tag{1}$$

*converges in the unit disk and its sum has modulus less than* 1*, then*

$$\sum_{k=0}^{\infty} | c_k z^k | < 1 \tag{2}$$

*in the disk* $\{z : \ |z| < 1/3\}$*. Moreover, the constant* $1/3$ *cannot be improved.*

Recently H.P. Boas and D. Khavinson obtained some multidimensional generalizations of this result ([4], see also for more references there).

Denote by $K_n$ the largest number such that if the series

$$\sum_{\alpha} c_\alpha z^\alpha \tag{3}$$

converges in the unit polydisk $U_1 = \{z : \ |z_j| < 1, \ j = 1, ..., n\}$ and the estimate

$$| \sum_{\alpha} c_\alpha z^\alpha | < 1 \tag{4}$$

is valid there, then

$$\sum_{\alpha} | c_\alpha z^\alpha | < 1 \tag{5}$$

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holds in $K_n \cdot U_1$; here $\alpha = (\alpha_1, ..., \alpha_n)$, all $\alpha_j$ are non-negative integers, $z = (z_1, ..., z_n)$, $z^\alpha = z_1^{\alpha_1}, ..., z_n^{\alpha_n}$.

**Theorem 2** (Boas, Khavinson). *It is true for $n > 1$ that*

$$\frac{1}{3\sqrt{n}} < K_n < \frac{2\sqrt{\log n}}{\sqrt{n}}. \tag{6}$$

**Theorem 3** (Boas, Khavinson). *Let the series (3) converge in a complete $n$-circular domain (Reinhardt domain) $D$ and (4) holds in $D$. Then (5) is true in the homothetic domain $K_n \cdot D$.*

Notice that Remark 1 from [4], in fact, contains a result stronger than the left part of the inequality (6), namely

$$K_n > \begin{cases} \frac{2}{5\sqrt{n}} & \text{for} \quad n > 1, \\ \frac{1}{2\sqrt{n}} & \text{for} \quad \text{large enough } n. \end{cases}$$

## 2. Main results

We consider some other multidimensional variations of Bohr's problem. Denote by $B_n(D)$ the largest number $r$ such that if the series (3) converges in a complete $n$-circular bounded domain $D$ and (4) holds in it, then

$$\sum_\alpha \sup_{D_r} | c_\alpha z^\alpha | < 1, \tag{7}$$

where $D_r = r \cdot D$ is the homothetic transformation of $D$. If $D = U_1$, then $B_n(D) = K_n$. We point out that our consideration is also a natural generalization of Bohr's theorem (Theorem 1), because it was shown in [2] that any power series (3), converging in $D$, converges also in the sense of the left part of (7) for all $r$, $0 < r < 1$.

**Theorem 4.** *The inequality*

$$1 - \sqrt[n]{\frac{2}{3}} < B_n(D) \tag{8}$$

*is true for any complete, bounded $n$-circular domain $D$.*

This estimate can be improved for concrete domains.

**Theorem 5.** *For the unit ball $D^1 = \{z : |z_1|^2 + ... + |z_n|^2 < 1\}$ the following estimate is true:*

$$B_n(D^1) > \begin{cases} \frac{2}{5n} & \text{for} \quad n > 1, \\ \frac{1}{2n} & \text{for large enough } n. \end{cases}$$

**Theorem 6.** *For the unit hypercone $D^\circ = \{z : |z_1| + ... + |z_n| < 1\}$ the following inequality holds:*

$$B_n(D^\circ) < \frac{0,446663}{n}. \tag{9}$$

**Corollary.**

$$1 - \sqrt[n]{\frac{2}{3}} < B_n(D^\circ) < \frac{0.446663}{n}. \tag{10}$$

*Remark* 1. The asymptotic equality

$$1-\sqrt[n]{\frac{2}{3}}=\frac{\log 3/2}{n}+O(\frac{1}{n^2})$$

is true, where $\log 3/2 \approx 0.405465$. Denoting by $B_-$ the left part of (10) and by $B_+$ the right one, we get

$$1<\lim_{n\to\infty}\sup\frac{B_+}{B_-}<1,1016.$$

We now turn our attention to a related problem. Denote by $L_n(D^\circ)$ the biggest number $r$ such that if the series (3) converges in the hypercone $D^\circ$ and (4) holds in it, then

$$\sum_\alpha \|c_\alpha z^\alpha\|_{L^1(\partial D_r^\circ)} \ < \ 1, \tag{11}$$

where the $L^1$-norm is considered with respect to the measure $\mu_r$,. The measure $\mu_r$ is the image of the measure

$$d\mu=\frac{(n-1)!}{(2\pi i)^n}d\mid z_1\mid\wedge\cdots\wedge d\mid z_{n-1}\mid\wedge\frac{dz_1}{z_1}\wedge\cdots\wedge\frac{dz_n}{z_n}$$

by the homothetic transformation $z\to rz$. Usually the measure $d\mu$ is used for calculating the Szëgo kernel for $D^\circ$ (see [3]); notice that $\mu(\partial D^\circ)=1$. For $n=1$ this problem coincides with Bohr's problem. The analogous problem for the polydisk $U_1$ (with $L^1$-norm on its Shilov boundary with respect to usual Lebesgue measure on it) is equivalent to the problem considered in Theorem 2.

**Theorem 7.** *For the hypercone $D^\circ$ the following estimates are true:*

$$\frac{1}{3e^{1/3}}<L_n(D^\circ)\le 1/3\ . \tag{12}$$

Surprisingly, the estimates in (12) do not depend on $n$. This is different from the results of Theorems 2–6.

We consider now a variation of the multidimensional Bohr problem, dealing with the expansions into a series of homogeneous polynomials, which is also a natural generalization of the power series expansion.

Let $Q$ be a complete circular domain (Cartan's domain) centered at $0\in Q$. Then any function $f(z)$, holomorphic in $Q$, can be expanded into the series

$$f(z)=\sum_{k=0}^{\infty}P_k(z), \tag{13}$$

where $P_k(z)$ is a homogeneous polynomial of degree $k$ for every $k\in\mathbf{N}$.

**Theorem 8.** *If the series (13) converges in the domain $Q$ and the estimate $|f(z)|<1$ holds in it, then*

$$\sum_{k=0}^{\infty}\mid P_k(z)\mid<1 \tag{14}$$

*in the homothetic domain $\frac{1}{3}Q$. Moreover, if $Q$ is convex, then $1/3$ is the best possible constant.*

Next we consider for the hypercone $D^\circ$ the same problem, which was considered in Theorems 2–3 for $U_1$. Denote by $K_n(D^\circ)$ the largest number such that if series (3) converges in $D^\circ$ and estimate (4) is valid there, then (5) holds in $K_n(D^\circ)\cdot D^\circ$.

**Theorem 9.** *For the hypercone $D^\circ$ the following estimates are true:*

$$\frac{1}{3e^{1/3}} < K_n(D^\circ) \le 1/3 \,.$$

*Moreover, if $z \notin 1/3 \cdot D^\circ$, then there exists a series of the form (3) such that it converges in $D^\circ$ and the estimate (4) is valid there, but (5) fails at the point $z$.*

## 3. Proofs

*Proof of Theorem 4.* The following generalization of Cauchy inequalities was considered in [2]: if (4) holds in $D$, then

$$| \, c_\alpha \, | \le \frac{1}{d_\alpha(D)}, \tag{15}$$

where $d_\alpha(D) = \max_{\bar{D}} |z^\alpha|$. Using Wiener's method, it is easy to strengthen the estimates (15):

$$| \, c_\alpha \, | \le (1- | \, c_0 \, |^2) \frac{1}{d_\alpha(D)} \tag{16}$$

for $|\alpha| = \alpha_1 + ... + \alpha_n > 1$. We do not present the proof of (16), because it repeats the proof for polydisk $U_1$ (see [4]), but deals with inequalities (15) instead of Cauchy inequalities. If (4) holds in $D$, then, applying (16), we get

$$\begin{aligned} \sum_\alpha \sup_{D_r} |c_\alpha z^\alpha| &= \sum_\alpha |c_\alpha| d_\alpha(D_r) \\ &\le |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^{\infty} \frac{d_\alpha(D_r)}{d_\alpha(D)} \\ &= |c_0| + (1 - |c_0|^2) \sum_{k=1}^{\infty} \binom{n+k-1}{k} r^k \\ &= |c_0| + (1 - |c_0|^2) \left[ \frac{1}{(1-r)^n} - 1 \right]. \end{aligned}$$

Now if

$$\frac{1}{(1-r)^n} - 1 \le \frac{1}{2}, \tag{17}$$

then

$$\sum_\alpha \sup_{D_r} | \, c_\alpha z^\alpha \, | \le | \, c_0 \, | + (1- | \, c_0 \, |^2) \frac{1}{2} = 1 - \frac{1}{2} (1- | \, c_0 \, |)^2 < 1.$$

The condition (17) means that (7) is true if $r \le 1 - \sqrt[n]{\frac{2}{3}}$.

*Proof of Theorem 5.* Consider Borel probability measure on $\partial D^1$, which is invariant under all unitary transformations of $\mathbf{C}^n$ :

$$d\mu = \frac{(n-1)!}{(2\pi i)^n} d \, | \, z_1 \, |^2 \wedge \cdots \wedge d \, | \, z_{n-1} \, |^2 \wedge \frac{dz_1}{z_1} \wedge \cdots \wedge \frac{dz_n}{z_n}.$$

The monomials $z^\alpha$ are orthogonal with respect to integration by $\mu$ and

$$\int_{\partial D^1} | \, z^{2\alpha} \, | \, d\mu = \frac{\alpha_1! \cdots \alpha_n! (n-1)!}{(| \, \alpha \, | + n - 1)!} \,. \tag{18}$$

Next, we repeat the proof of Theorem 2 from [4] using Wiener's method, but integrating on the sphere $\partial D^1$ with respect to the measure $\mu$ instead of integrating on the unit torus as in [4]. Thus, we obtain

$$\sum_{|\alpha|=k} | c_\alpha |^2 \frac{\alpha_1! \cdots \alpha_n!(n-1)!}{(| \alpha | + n-1)!} \leq (1- | c_0 |^2)^2.$$

Furthermore, notice that from the Schwarz Lemma for the ball $D^1$ it follows (again by using Wiener's method from [4]) that

$$\sum_{|\alpha|=1} | c_\alpha |^2 \leq (1- | c_0 |^2)^2.$$

Then, recalling that

$$d_\alpha(D^1) = \sqrt{\frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{| \alpha |^{|\alpha|}}},$$

where $0^0 = 1$, we get

$$\begin{aligned}\sum_\alpha \sup_{D_r} |c_\alpha z^\alpha| &= |c_0| + (\sum_{|\alpha|=1} |c_\alpha|)r + \sum_{k=2}^\infty \sum_{|\alpha|=k} |c_\alpha| \sqrt{\frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{|\alpha|^{|\alpha|}}} r^k \\ &\leq |c_0| + (1-|c_0|^2) r\sqrt{n} + (1-|c_0|^2) \sum_{k=2}^\infty \left( \sum_{|\alpha|=k} \frac{(k+n-1)! \alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{k^k \alpha_1! \cdots \alpha_n! (n-1)!} \right)^{1/2} r^k \\ &< |c_0| + (1-|c_0|^2) r\sqrt{n} + (1-|c_0|^2) \sum_{k=2}^\infty \left( \frac{(k+n-1)!}{k!(n-1)!} \sum_{|\alpha|=k} \frac{k!}{\alpha_1! \cdots \alpha_n!} \right)^{1/2} r^k \\ &= |c_0| + (1-|c_0|^2) r\sqrt{n} + (1-|c_0|^2) \sum_{k=2}^\infty \sqrt{\begin{pmatrix} k+n-1 \\ k \end{pmatrix}} (r\sqrt{n})^k.\end{aligned}$$

It remains now to apply the estimate, obtained in Remark 1 from [4].

*Proof of Theorem 6.* Consider the function

$$f_a(z) = \frac{1+a}{2} \frac{1-(z_1 + \cdots + z_n)}{1-a(z_1+\cdots+z_n)} = \sum_\alpha c_\alpha z^\alpha,$$

where $0 < a < 1$. Then $|f_a(z)| < 1$ in hypercone $D^\circ$ and

$$\sum_\alpha | c_\alpha z^\alpha | = \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^\infty a^{k-1} \sum_{|\alpha|=k} \frac{k!}{\alpha_1! \cdots \alpha_n!} | z^\alpha | .$$

Since

$$d_\alpha(D^\circ) = \frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{| \alpha |^{|\alpha|}},$$

it follows that

$$\sum_{\alpha} \mid c_\alpha \mid d_\alpha(D_r^\circ) = \frac{1+a}{2} + \frac{1-a^2}{2}\sum_{k=1}^{\infty}\sum_{|\alpha|=k} a^{k-1}\frac{k!\alpha_1^{\alpha_1}\cdots\alpha_n^{\alpha_n}}{\alpha_1!\cdots\alpha_n!k^k}r^k$$

$$> \frac{1+a}{2} + \frac{1-a^2}{2}\sum_{k=1}^{\infty} a^{k-1}\frac{k!}{k^k}\left(\sum_{|\alpha|=k}\frac{1}{\alpha_1!\cdots\alpha_n!}\right)r^k$$

$$= \frac{1+a}{2} + \frac{1-a^2}{2a}\sum_{k=1}^{\infty}\frac{(anr)^k}{k^k} \geq 1,$$

if

$$\sum_{k=1}^{\infty}\frac{(anr)^k}{k^k} \geq \frac{a}{1+a}.$$

Let $x_0(a)$ be the root of the equation

$$\sum_{k=1}^{\infty}\frac{x^k}{k^k} = \frac{a}{1+a};$$

then, if $anr \geq x_0(a)$, (7) fails for that $r$ and $D = D^\circ$. Now, considering $a \to 1$, we obtain that (7) is not true for $D = D_\circ$ if $r \geq \frac{x_0}{n}$, where $x_0 = x_0(1)$. Notice that $x_0$ is a root of the equation

$$\sum_{k=1}^{\infty}\frac{x^k}{k^k} = 1/2; \tag{19}$$

hence $B_n(D^\circ) \leq x_0/n$. Using the program "Mathematica 3.0" [7], we estimated $x_0$ from above. We obtained that the equation

$$\sum_{k=1}^{p}\frac{x^k}{k^k} = 1/2 \tag{20}$$

has a root 0.446662 (where the last decimal digit is precise) if $p$ runs from 5 till 25. So, the equation (20) has a root less than 0.446663 if $5 \leq p \leq 25$; hence this estimate is true for $x_0$.

*Proof of Theorem 7.* Notice that by analogy with (18) it is true that

$$\int_{\partial D_r^\circ} \mid z^\alpha \mid d\mu_r = \frac{\alpha_1!\cdots\alpha_n!(n-1)!}{(\mid \alpha \mid + n - 1)!}r^{|\alpha|}.$$

Using (16), we get

$$\sum_{\alpha} \|\, c_\alpha z^\alpha \,\|_{L^1(\partial D_r^\circ)}$$

$$\leq \mid c_0 \mid + (1 - \mid c_0 \mid^2)\sum_{k=1}^{\infty}\sum_{|\alpha|=k}\frac{k^k(n-1)!}{(n+k-1)!}\frac{\alpha_1!\cdots\alpha_n!}{\alpha_1^{\alpha_1}\cdots\alpha_n^{\alpha_n}}r^{|\alpha|}$$

$$< \mid c_0 \mid + (1 - \mid c_0 \mid^2)\sum_{k=1}^{\infty}\frac{k^k(n-1)!}{(n+k-1)!}\sum_{|\alpha|=k} r^{|\alpha|}$$

$$= \mid c_0 \mid + (1 - \mid c_0 \mid^2)\sum_{k=1}^{\infty}\frac{k^k}{k!}r^k.$$

Denote by $r_0$ the root of the equation

$$\sum_{k=1}^{\infty} \frac{k^k}{k!} r^k = \frac{1}{2}. \tag{21}$$

Then (11) holds for $r = r_0$; therefore $L_n(D^\circ) \geq r_0$. The identity

$$\sum_{k=1}^{\infty} \frac{k^k}{k!} x^k e^{-kx} = -1 + \frac{1}{1-x} \tag{22}$$

holds for $x$ in a neighborhood of 0. This can be verified by computing the Maclaurin series coefficients of the left-hand side and observing that they reduce to the value 1 by a standard theorem on sums of binomial coefficients. Now put $x = 1/3$ in equation (22) to recognize the solution of equation (21).

Finally, consider, as in the proof of Theorem 6, the function $f_a(z)$ for which

$$\sum_{\alpha} || \, c_\alpha z^\alpha \, ||_{L^1(\partial D_r^\circ)} = \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^{\infty} \frac{k!(n-1)!a^{k-1}}{(n+k-1)!} \sum_{|\alpha|=k} r^{|\alpha|}$$

$$= \frac{1+a}{2} + \frac{1-a^2}{2a} \sum_{k=1}^{\infty} (ar)^k \geq 1,$$

if

$$\sum_{k=1}^{\infty} (ar)^k \geq \frac{a}{1+a}.$$

When $a \to 1$, we get that the inequality (11) fails if $r > 1/3$.

*Proof of Theorem 8.* In each section of the domain $Q$ by a complex line

$$\alpha = \{z : z_j = a_j t, \quad j = 1, \cdots, n; \quad t \in \mathbf{C}\}$$

the series turns into the power series by $t$

$$f(at) = \sum_{k=0}^{\infty} P_k(a) t^k$$

and, in addition, $|f(at)| < 1$. By Theorem 1

$$\sum_{k=0}^{\infty} | \, P_k(a) t^k \, | < 1$$

in the section $\alpha \cap (\frac{1}{3} \cdot Q)$. But it is just (14), since $\alpha$ is an arbitrary complex line passing through the origin. Conversely, let the domain $Q$ be convex; then $Q$ is an intersection of half-spaces

$$Q = \bigcap_{a \in J} \{z : Re(a_1 z_1 + \cdots + a_n z_n) < 1\}$$

for some $J$. Since $Q$ is circular, we obtain

$$Q = \bigcap_{a \in J} \{z : | \, a_1 z_1 + \cdots + a_n z_n \, | < 1\}.$$

It is sufficient now to show that the constant $1/3$ cannot be improved for each domain $P_a = \{z : |a_1 z_1 + ... + a_n z_n| < 1\}$. From Theorem 1 it follows that for any $r > 1/3$ there exists a function $f(z)$, represented by (1) and such that $|f(z)| < 1$ in the unit disk, but (2) fails in the disk $\{z : |z| < r\}$. To complete the proof we use the functions $f(a_1 z_1 + ... + a_n z_n)$.

*Proof of Theorem 9.* If $z \in r \cdot D^\circ$, then from (16) it follows

$$\sum_\alpha |c_\alpha z^\alpha| \le |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^\infty \frac{|\alpha|^{|\alpha|}}{{\alpha_1}^{\alpha_1} \dots {\alpha_n}^{\alpha_n}} |z^\alpha|$$

$$= |c_0| + (1 - |c_0|^2) \sum_{|\alpha|=1}^\infty \frac{|\alpha|^{|\alpha|}}{|\alpha|!} \frac{\alpha_1! \dots \alpha_n!}{{\alpha_1}^{\alpha_1} \dots {\alpha_n}^{\alpha_n}} \frac{|\alpha|!}{\alpha_1! \dots \alpha_n!} |z^\alpha|$$

$$< |c_0| + (1 - |c_0|^2) \sum_{k=1}^\infty \frac{k^k}{k!} r^k.$$

Thus (5) holds for $r > r_0$, where $r_0$ is the root of equation (21).

Suppose $z \notin 1/3 \cdot D^\circ$; then we have for the function $f_a(z)$ (which was used in the proof of Theorem 6 )

$$\sum_\alpha |c_\alpha z^\alpha| = \frac{1+a}{2} + \frac{1-a^2}{2} \sum_{k=1}^\infty a^{k-1} (|z_1| + \dots + |z_n|)^k \ge 1$$

if

$$\sum_{k=1}^\infty a^k (|z_1| + \dots + |z_n|)^k \;\ge\; \frac{a}{1+a}.$$

Hence inequality (5) fails as $a \to 1$.

## 4. Final remarks

*Remark* 2. In the proof of Theorem 5 we used the facts that in equality (18) the domain in consideration is the ball $D^1$ and that the estimates $d_\alpha(D^1) \le 1$ are valid. Therefore, an analogous theorem holds for any complete $n$-circular domain $D$ if $D^1 \subset D \subset U_1$. But for the hypercone $D^\circ$ this is not true (see Theorem 6).

*Remark* 3. In the proof of Theorem 6 was used the following quite rough inequality:

$$\sum_{|\alpha|=k} \frac{\alpha_1^{\alpha_1} \cdots \alpha_n^{\alpha_n}}{\alpha_1! \cdots \alpha_n!} \ge \sum_{|\alpha|=k} \frac{1}{\alpha_1! \cdots \alpha_u!}.$$

Therefore, in fact, the estimate from above in the theorem might be decreased. For example, if $n = 2$ (10) implies $B_2(D^\circ) < 0.223332$, but using a PC one can show that $B_2(D^\circ) < 0.191373$. Hence, $0.183502 < B_2(D^\circ) < 0.191373$.

*Remark* 4. Comparing Theorem 2 and the corollary we can see that $B_n(D)$ depends essentially on the domain $D$ since $B_n(U_1) = K_n$. It seems that it is more natural to consider not a single number in the problem considered in Theorem 2 and Theorem 3, but the largest subdomain $D_B$ of $D$ such that (5) holds. From [4] it follows, for example, that $U_{1B}$ contains the ball $\frac{1}{3} \cdot D^1$, and from Theorem 9 it follows that ${D^\circ}_B \subset 1/3 \cdot D^\circ$.

*Remark* 5. Notice that there exist such unbounded $n$-circular domains $D$, for which the problems defined by the conditions (5) and (7) are equivalent. Consider, for example, the domains $D = \{z : |z|^\beta < c\}$, where $\beta$ is a multiindex with coprime components. Bounded holomorphic functions in such domains depend only on one variable $z^\beta$ and the exact value of Bohr's radius equals $(\frac{1}{3})^{1/|\beta|}$. Thus there exist $n$-circular domains with Bohr's radius arbitrarily close to 1. Therefore it is impossible to remove the assumption about convexity in Theorem 8.

*Remark* 6. Unlike Theorems 1–7, in Theorem 8 the series (13) is not a basis expansion. In [1] it was shown that there exists a basis in the space of all holomorphic functions in $Q$, consisting of homogeneous polynomials $P_{k,m}(z)$, where $k$ is the degree of the polynomial and $m = 1, ..., \begin{pmatrix} k+n-1 \\ k \end{pmatrix}$. It is reasonable to consider Bohr's problem for such basis expansions, but there are no results yet in this direction. This question is a particular case of a more general problem in Bohr spirit for expansions by an arbitrary basis in a domain $D \subset \mathbf{C}^n$ (under some restrictions on the basis, because, for example, the basis $\{1, (z-1)/2, z^2, z^3, \ldots\}$ in the unit disk has no Bohr's constant).

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