SPECTRAL STRUCTURE AND SUBDECOMPOSABILITY OF $p$-HYPERSONAL OPERATORS

RUAN YINGBIN AND YAN ZIKUN

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ABSTRACT. We prove that for every $p$-hyponormal operator $A$, $0 < p \leq 1$, there corresponds a hyponormal operator $\tilde{A}$ such that $A$ and $\tilde{A}$ have “equal spectral structure”. We also prove that every $p$-hyponormal operator $A$, $0 < p \leq 1$, is subdecomposable. Then some relevant quasisimilarity results are obtained, including that two quasisimilar $p$-hyponormal operators have equal essential spectra.

1. Introduction and notation

Let $H$ be a complex separable Hilbert space and let $L(H)$ denote the algebra of all bounded linear operators on $H$. An operator $A \in L(H)$ is said to be $p$-hyponormal, $0 < p \leq 1$, denoted as $A \in p-H$, if $(AA^*)^p \leq (A^*A)^p$. An $1$-hyponormal operator is hyponormal, and a $\frac{1}{2}$-hyponormal operator is said to be semi-hyponormal. In the sequel, for every $A \in L(H)$, we define $\tilde{A}$ by $\tilde{A} = |A|^\frac{1}{p}U|A|^\frac{1}{p}$ where $U, |A|$ are as in the polar decomposition $A = U|A|$. Let $\tilde{A}$ have the polar decomposition $\tilde{A} = V|\tilde{A}|$. The operator $\tilde{A}$ is then defined by $\tilde{A} = |\tilde{A}|^\frac{1}{2}V|\tilde{A}|^\frac{1}{2}$. Aluthge \cite{1} showed that for $A \in p-H$, $0 < p \leq 1$, $A$ is semi-hyponormal and $\tilde{A}$ is hyponormal. Some authors paid attention to the relations between the spectral structure of $A$ and $\tilde{A}$ (e.g. \cite{2}, \cite{3}, \cite{4}). In this note, we prove that for general $A \in p-H$, $0 < p \leq 1$, $A, \tilde{A}$ and $\tilde{A}$ have “equal spectral structure”, i.e. $\sigma_s(A) = \sigma_s(\tilde{A}) = \sigma_s(\tilde{A})$, where $\sigma_s = \sigma, \sigma_\alpha, \sigma_\infty, \sigma_\sup, \sigma_w, \sigma_e, \sigma_k, \sigma_p, \psi, \psi_{mn}$ or $\sigma_0^p$.

A subdecomposable operator is, up to similarity, the restriction of a decomposable operator to its invariant space. J. Eschmeier \cite{5} proved that $A \in L(H)$ is subdecomposable if and only if $A \in (\beta)$, i.e. $A$ has Bishop’s property $(\beta)$. M.Putinar and J.Eschmeier \cite{6}, \cite{7} proved that hyponormal operators are subscalar and therefore subdecomposable. B.Duggal \cite{8} asked whether a general $p$-hyponormal operator satisfies condition $(\beta)$. We give an affirmative answer to this question. Yang Liming \cite{9} proved that two quasisimilar hyponormal operators have equal essential spectra.

By means of the subdecomposability of $A \in p-H$, we generalize and strengthen this result to general $p$-hyponormal operators ($0 < p \leq 1$).

For $T \in L(H), \sigma(T), \sigma_p(T), \sigma_e(T)$ and $\sigma_c(T)$ denote the spectrum, point spectrum, approximate point spectrum and essential spectrum of $T$, respectively. Write

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\[ \nu(T) = \dim \ker T, \quad \mu(T) = \dim \ker T^*; \] the index of \( T \) is defined by \( \text{ind} T = \nu(T) - \mu(T) \) if at least one of \( \nu(T) \) and \( \mu(T) \) is finite. Let \( \psi \) denote the set of all semi-Fredholm operators on \( H \). Write \( \rho - F(T) = \{ \lambda \in \mathbb{C} : T - \lambda \in \psi \} \), \( \psi_n(T) = \{ \lambda \in \mathbb{C} : R(T - \lambda) \text{ is closed}, \text{ind} (T - \lambda) = n \} \) \( (n = 0, \pm 1, \pm 2, \ldots, \pm \infty) \), \( \psi_{mn}(T) = \{ \lambda \in \mathbb{C} : R(T - \lambda) \text{ is closed}, \nu(T - \lambda) = m, \mu(T - \lambda) = n \} \) \( (m, n = 0, 1, 2, \ldots, \infty) \). Also write \( \psi_+(T) = \bigcup_{1 \leq n \leq \infty} \psi_n(T), \quad \psi_-(T) = \bigcup_{1 \leq n \leq \infty} \psi_{-n}(T), \quad \sigma_+(T) = \{ \lambda \in \mathbb{C} : \ker(T - \lambda) = \{0\}, \check{R}(T - \lambda) \text{ is closed}, \check{R}(T - \lambda) \notin \mathbb{H} \}, \quad \sigma_-(T) = \{ \lambda \in \mathbb{C} : \ker(T - \lambda) \text{ is not closed} \} \). \( \sigma_k(T) = \mathbb{C} \setminus \rho_{\text{reg}}(T) \). \( \sigma_0^b(T) \) denotes the set of all isolated eigenvalues of \( T \) with finite algebraic multiplicity. \( \mathbb{K}(H) \) denotes the set of all compact operators on \( H \). \( \sigma_p(T) = \bigcap_{K \in \mathbb{K}(H)} \sigma(T + K) = \sigma(T) \setminus \sigma_0^b(T), \quad \sigma_{w}(T) = \bigcap_{K \in \mathbb{K}(H)} \sigma(T + K) = \sigma(T) \setminus \sigma_0^b(T) \).

Suppose \( \lambda \in \mathbb{C}, T \in L(H) \). \( \lambda \) is called a regular point of the operator \( T \) if \( \| P_{\ker(T - \mu)} - P_{\ker(T - \lambda)} \| \to 0 (\mu \to \lambda) \), where \( P_{\ker(T - \lambda)} \) denotes the orthogonal projection onto \( \ker(T - \lambda) \). \( \tau^+(T) \) denotes the set of all regular points of \( T \), \( \tau^+(T) = \mathbb{C} \setminus \tau^-(T) \). For every set-valued function \( B(\cdot) : L(H) \to 2^\mathbb{C} \), write \( B^+(T) = B(T) \cap \tau^+(T), B^0(T) = B(T) \setminus \tau^0(T) \) (see [2]).

Let \( T \in L(H) \). Suppose that the closed subspace \( M \) of \( H \) reduces \( T \); then \( M \) is said to be a normal subspace of \( T \) if \( T|_M \) is a normal operator. The operator \( T \) is said to be pure if \( T \) has no non-trivial normal subspace.

\( O(\Omega, H) \) denotes the Fréchet space of all \( H \)-valued analytic functions on the open set \( \Omega \subset \mathbb{C} \) with the topology defined by uniform convergence on every compact subset of \( \Omega \). Suppose \( T \in L(H) \); \( T \) is said to have Bishop’s property (\( \beta \)) (denoted by \( T \in (\beta) \)) if the mapping \( \alpha_{T, \Omega} : O(\Omega, H) \to O(\Omega, H), f \mapsto (T - z)f \) is injective and has closed range for every open subset \( \Omega \) of \( \mathbb{C} \). Write \( E_2(T) = \{ \lambda \in \mathbb{C} : \exists \delta > 0 \text{ such that for } \Omega = O(\lambda, \delta'), 0 < \delta' < \delta, \alpha_{T, \Omega} \text{ has closed range}\} \), \( A(T) = \{ \lambda \in \mathbb{C} : \exists \delta > 0 \text{ such that for } \Omega = O(\lambda, \delta'), 0 < \delta' < \delta, \alpha_{T, \Omega} \text{ is injective}\} \). Write \( T \in (E_2)(A) \) if for every \( \lambda \in \mathbb{C}, \lambda \in E_2(T)(A(T)) \). \( T \in (A) \) is equivalent to \( T \) has the single-valued extension property. It is clear by definitions (see [10] Proposition 4)) that \( T \in (\beta) \) if and only if \( T \in (A) \) and \( T \in (E_2) \).

Let \( T_1, T_2 \in L(H) \); we say \( T_1 \) is a dense (quasiaffine) transform of \( T_2 \), denoted as \( T_1 \overset{dr}{\sim} T_2 \) \( (T_1 \overset{\xi}{\sim} T_2) \), if there exists operator \( X \) with dense range (injective and dense range) such that \( XT_1 = T_2X \). We said \( T_1 \) and \( T_2 \) are densely (quasi-) similar, denoted as \( T_1 \overset{dr}{\sim} T_2 \) \( (T_1 \overset{\xi}{\sim} T_2) \), if \( T_1 \overset{dr}{\rightarrow} T_2 \overset{dr}{\rightarrow} T_1 \overset{\xi}{\rightarrow} T_2 \overset{\xi}{\rightarrow} T_1 \).

2. Spectral structure of \( A \) and \( \hat{A} \)

**Lemma 1.** If \( T \) is a pure \( p \)-hyponormal operator, then \( \sigma_p(T) = \emptyset \).

**Proof.** Let \( T \) have the polar decomposition \( T = U|T| \). If \( \lambda \in \sigma_p(T) \), then \( \ker(T - \lambda) \neq \{0\} \). By [11] Proof of Theorem 4, \( \ker(T - \lambda) \subset \ker(T - \lambda)^* \). This implies that \( \ker(T - \lambda) \) is a reducing subspace of \( T \) and \( T|_{\ker(T - \lambda)} \) is normal, a contradiction to the purity of \( T \). Hence \( \sigma_p(T) = \emptyset \).

**Lemma 2.** Let \( S, T \in L(H) \). If \( A = TS, B = ST, \) then
\[ \dim \ker (A - \lambda) = \dim \ker (B - \lambda), \quad \lambda \neq 0; \]
moreover, if \( \ker S = \ker T \), then \( \sigma_p(A) = \sigma_p(B) \).

**Proof.** Suppose \( \lambda \neq 0 \), and \( x_i \in \ker (A - \lambda), i = 1, 2, \ldots, n \); \( x_1, x_2, \ldots, x_n \) are linearly independent. Then \( BSx_i = STSx_i = SAx_i = \lambda Sx_i, \quad Sx_i \in \overline{\mathbb{C}x_i} \).
Ker(B − λ), i = 1, 2, · · · , n. We claim that Sx1, Sx2, · · · , Sxn are linearly independent too. For otherwise, there would exist constants α1, α2, · · · , αn, not all zero, such that n \sum_{i=1}^{n} \alpha_i Sx_i = 0 and hence \sum_{i=1}^{n} \alpha_i Ax_i = \sum_{i=1}^{n} \alpha_i TSx_i = 0. Since \sum_{i=1}^{n} \alpha_i Ax_i = \lambda \sum_{i=1}^{n} \alpha_i x_i, and λ ≠ 0, therefore \sum_{i=1}^{n} \alpha_i x_i = 0, a contradiction with the supposition that x1, x2, · · · , xn are linearly independent. Therefore dimKer(A − λ) ≤ dimKer(B − λ). A similar argument shows that dimKer(B − λ) ≤ dimKer(A − λ). It follows that dimKer(A − λ) = dimKer(B − λ), λ ≠ 0.

This equality implies \sigma_p(A) \setminus \{0\} = \sigma_p(B) \setminus \{0\}.

If 0 ∈ \sigma_p(A), then there exists x ∈ H, x ≠ 0 and Ax = TSx = 0. This implies BSx = 0. If Sx ≠ 0, then 0 ∈ \sigma_p(B). If Sx = 0, then it follows from Ker S = Ker T that Tx = 0 and Bx = 0; this implies 0 ∈ \sigma_p(B), too. Similarly 0 ∈ \sigma_p(B) implies 0 ∈ \sigma_p(A). The conclusion \sigma_p(A) = \sigma_p(B) now follows.

Lemma 3. Suppose that A ∈ L(H); then dimKer(A − λ) = dimKer(A − λ)*, \lambda ≠ 0, and \sigma_p(A) = \sigma_p(A)*.


Lemma 4 ([12]). Let T ∈ L(H) be a semi-Fredholm operator. Then there exists \delta > 0 such that S ∈ L(H), \|T − S\| < \delta implies that S is semi-Fredholm and \nu(S) ≤ \nu(T), \mu(S) ≤ \mu(T), \text{ind} S = \text{ind} T.

Theorem 5. Let A ∈ L(H). If A0, the pure part of A, has no eigenvalue, then

(i) \sigma(A) = \sigma_p^0(A) \cup (\bigcup_{1 \leq n \leq \infty} \psi_{\infty}(A)) \cup \psi_{\infty}(A) \cup \sigma_{p}^{0}(A),

(ii) A and A* have “equal spectral structure”, i.e. \sigma_s(A) = \sigma_s(A*), where \sigma_s = \sigma, \sigma_r, \sigma_p, \sigma_w, \sigma_e, \sigma_k, \sigma_d, \psi_n (−\infty < n < \infty), \psi_{mn} (0 < m, n < \infty) or \sigma^0_p.

Proof. Decompose A into normal and pure parts: A = N + A0. If N = W|N| and A0 = V|A0| are the polar decompositions of N and A0 respectively, then it is easy to derive that W|N| = |N|W, N = |N|^*W|N|^* = N and that A = N + A0.

Since \sigma_p(A0) = \sigma_p(A0) = \emptyset , we have

(1) \sigma(A0) = (\bigcup_{1 \leq n \leq \infty} \psi_{\infty}(A0)) \cup \sigma_{p}^{0}(A0),

(2) \sigma(A0*) = (\bigcup_{1 \leq n \leq \infty} \psi_{\infty}(A0)) \cup \sigma_{p}^{0}(A0*).

By the property of normal operators, we have

(3) \sigma(N) = \sigma_p^0(N) \cup \psi_{\infty}(N) \cup \sigma_{p}^{0}(N).

(1) and (3) implies that (see [9])

\sigma(A) = \sigma_p^0(A) \cup (\bigcup_{1 \leq n \leq \infty} \psi_{\infty}(A)) \cup \psi_{\infty}(A) \cup \sigma_{p}^{0}(A),

i.e. (i) holds.

Now let us turn to the spectral parts of A0 and A0*. Let A0^0, (A0*) = A0^0 be the Berberian extension (see [13], Chapter I) of A0 and A0 respectively; then
\[ \sigma_a(A_0) = \sigma_p(A_0^0) \text{ and } \sigma_a(\hat{A}_0) = \sigma_p(\hat{A}_0^0). \]

By Lemma 3, \( \sigma_p(A_0^0) = \sigma_p(\hat{A}_0^0) \); by (1), (2), \( \sigma_a(A_0) = \sigma_a(\hat{A}_0) \) and \( \sigma_a(\hat{A}_0) = \sigma_a(\hat{A}_0) \). It follows then that
\[ \sigma_p(A_0) = \sigma_p(\hat{A}_0). \]

It follows from Lemma 3 and (4) that
\[ \psi_{0n}(A_0) \setminus \{0\} = \psi_{0n}(\hat{A}_0) \setminus \{0\}, \ 0 \leq n \leq \infty. \]

Since \( \psi_{0n}(A_0) \) and \( \psi_{0n}(\hat{A}_0) \) are open sets by Lemma 4, and \( \sigma_p(\hat{A}_0) = \sigma_p(\hat{A}_0) \), it is easy to derive that
\[ \psi_{0n}(A_0) = \psi_{0n}(\hat{A}_0), \ 0 \leq n \leq \infty. \]

(1)–(5) imply that
\[ \sigma_p(A) = \sigma_p(\hat{A}), \]
\[ \psi_{mn}(A) = \psi_{mn}(\hat{A}) = \emptyset, \ m > n, \]
\[ \psi_{mn}(A) = \psi_{mn}(N) \cap \psi_{0,n-m}(A_0) = \psi_{mn}(N) \cap \psi_{0,n-m}(\hat{A}_0) = \psi_{mn}(\hat{A}), \]
\[ 0 \leq m \leq n, m < \infty. \]

\[ \psi_{\infty\infty}(A) = \psi_{\infty\infty}(N) \setminus \sigma_p(A_0) = \psi_{\infty\infty}(N) \setminus \sigma_p(\hat{A}_0) = \psi_{\infty\infty}(\hat{A}), \]

or briefly,
\[ \psi_{mn}(A) = \psi_{mn}(\hat{A}), \ 0 \leq m, n \leq \infty. \]

The equalities \( \sigma_s(A) = \sigma_s(\hat{A}) \) in (ii) come now immediately from (6), (7) and the definitions and fundamental properties of various \( \sigma_s \). \( \square \)

**Theorem 6.** If \( A \in p-H \), \( 0 < p \leq 1 \), then \( A \) has “equal spectral structure” (see Theorem 5) with the semi-hyponormal operator \( \hat{A} \) and the hyponormal operator \( \hat{A} \).

**Proof.** Obvious from Lemma 1 and Theorem 5. \( \square \)

3. SUBDECOMPOSABILITY AND QUASISIMILARITY

**Theorem 7.** Let \( T \in L(H), \lambda \in C \). If \( \text{Ker}T \subset \text{Ker}T^* \), then
(1) \( \lambda \in A(T) \) if and only if \( \lambda \in A(\hat{T}) \),
(2) \( \lambda \in E_2(T) \) if and only if \( \lambda \in E_2(\hat{T}) \),
(3) \( T \in (\beta) \) if and only if \( \hat{T} \in (\beta) \).

**Proof.** Since \( \text{Ker}T \subset \text{Ker}T^* \), we can write \( T = \theta \oplus T_1 \), where \( \theta = T|_{\text{Ker}T}, T_1 = T|_{(\text{Ker}T)^-} \). Write \( \mathbf{H}_1 = (\text{Ker}T)^+ \) and let \( T_1 = U|T_1| \) be the polar decomposition. It is clear that \( \text{Ker}T_1 = \{0\} \) and \( \hat{T} = \theta \oplus \tilde{T}_1 \). Thus, to prove the required result it suffices to show that \( \lambda \in A(T_1) \) \( (E_2(T_1)) \) if and only if \( \lambda \in A(\hat{T}_1) \) \( (E_2(\hat{T}_1)) \).

(1) Suppose that \( \lambda \in A(T_1) \); then there exists \( \delta > 0 \) such that for \( \Omega = O(\lambda, \delta^*) \), \( 0 < \delta^* \leq \delta \), \( \alpha_{T_1, \Omega} \) is injective. Let \( f \in O(\Omega, \mathbf{H}_1), (\hat{T}_1 - z)f(z) = 0(z \in \Omega) \). Then \( (\hat{T}_1 - z)|U|T_1|f(z) = U|T_1|\hat{T}_1(z) - z)f(z) = 0 \). Since \( \alpha_{T_1, \Omega} \) is injective, \( U|T_1|\hat{T}_1(z) = 0 \) \( (z \in \Omega) \). It follows from \( \text{Ker}U|T_1|\hat{T}_1 = \{0\} \) that \( f(z) = 0 \) \( (z \in \Omega) \). Thus \( \lambda \in A(\hat{T}_1) \).

The argument for the converse statement is similar.
(2) Suppose that \( \lambda \in E_2(\hat{T}_1) \); then there exists \( \delta > 0 \) such that for \( \Omega = O(\lambda, \delta'), 0 < \delta' < \delta, \alpha_{T, \Omega} \) has closed range. Suppose that \( f_n \in O(\Omega, \mathbf{H}_1), n = 1, 2, \ldots, (T_1 - z)f_n \to g \in O(\Omega, \mathbf{H}_1) \); then \( |T_1|^z (U/T_1 - z)f_n \to |T_1|^z g \), i.e., \( (T_1 - z)|T_1|^z f_n \to |T_1|^z g \). By the hypothesis \( \lambda \in E_2(\hat{T}_1) \), there exists \( f \in O(\Omega, \mathbf{H}_1) \) such that \( |T_1|^z g = (T_1 - z)f \). A simple calculation shows that

\[
f(z) = \frac{1}{z}(\hat{T}_1f(z) - |T_1|^z g(z)) = |T_1|^z \left( \frac{U}{z} |T_1|^z f(z) - g(z) \right) \quad (z \in \Omega \setminus \{0\}).
\]

If \( \lambda \neq 0 \), we may assume that \( 0 \notin \Omega \). Let \( \phi(z) = \frac{U}{z} |T_1|^z f(z) - g(z) \) (\( z \in \Omega \)). It is obvious then that \( \phi \in O(\Omega, \mathbf{H}_1) \) and \( |T_1|^z g = (T_1 - z)f \), \( \phi = |T_1|^z (T_1 - z)\phi \). But since \( \text{Ker}|T_1|^z = \text{Ker}T_1 = \{0\} \), hence \( g = (T_1 - z)\phi \), and so \( \alpha_{T, \Omega} \) has closed range. Thus \( \lambda \in E_2(T_1) \).

If \( \lambda = 0 \), let \( h(z) = U|T_1|^z f(z) - g(z) \) (\( z \in \Omega \)); then \( h \in O(\Omega, \mathbf{H}_1) \). Since \( |T_1|^z g(0) = |T_1|^z U|T_1|^z f(0) \) and \( \text{Ker}|T_1|^z = \text{Ker}T_1 = \{0\} \), we have

\[
h(0) = U|T_1|^z f(0) - g(0) = 0.
\]

Define

\[
\phi(z) = \begin{cases} 
  h(z)/z, & 0 \neq z \in \Omega, \\
  h'(0), & z = 0.
\end{cases}
\]

It can be verified by calculation that \( \phi \in O(\Omega, \mathbf{H}_1) \). It follows from the preceding section that \( g(z) = (T_1 - z)\phi(z) \) (\( z \in \Omega \setminus \{0\} \)) and this, by virtue of the analyticity of \( g(z), \phi(z) \), implies that \( g(z) = (T_1 - z)\phi(z) \) (\( z \in \Omega \)), and so \( \alpha_{T, \Omega} \) has closed range. Thus \( 0 \in E_2(T_1) \).

The converse argument is similar.

(3) The statement is obvious from (1),(2) and \( T \in \langle \beta \rangle \) if and only if \( T \in \langle A \rangle \) and \( T \in (E_2) \).

\[ \square \]

**Theorem 8.** Suppose that \( A \in p \cdot \mathbf{H}, 0 < p \leq 1 \); then \( A \in \langle \beta \rangle \), i.e., \( A \) is subdecomposable.

**Proof.** Suppose \( A \in p \cdot \mathbf{H} \); then \( \text{Ker}A \subset \text{Ker}A^* \) and \( \text{Ker}\hat{A} \subset \text{Ker}(\hat{A})^* \) by \[ \text{Lemma} 1 \]. Being a hyponormal operator, \( \hat{A} \in \langle \beta \rangle \); consequently \( \hat{A} \in \langle \beta \rangle \) and hence \( A \in \langle \beta \rangle \) by Theorem 7,(3).

\[ \square \]

**Lemma 9 ([11], Corollary 3).** Let \( S \in L(\mathbf{H}) \) be subdecomposable, \( T \in L(\mathbf{H}) \). If \( T \overset{dr}{\longrightarrow} S \), then \( \sigma(S) \subset \sigma(T) \); if \( T \overset{dr}{\sim} S \), then \( \sigma_e(S) \subset \sigma_e(T) \).

**Theorem 10.** Let \( A \in p \cdot \mathbf{H}, 0 < p \leq 1, B \in L(\mathbf{H}) \). If \( B \overset{dr}{\rightarrow} A \), then \( \sigma(A) \subset \sigma(B) \); if \( B \overset{\sim}{\sim} A \), then \( \sigma_e(A) \subset \sigma_e(B) \).

**Proof.** Obvious from Theorem 8 and Lemma 9.

\[ \square \]

**Corollary 11.** If \( A \) or \( A^* \in p \cdot \mathbf{H}, 0 < p \leq 1, B \in L(\mathbf{H}), A \not\sim B \), then \( \sigma_e(A) \subset \sigma_e(B), \sigma(A) \subset \sigma(B) \).

**Corollary 12.** If \( A, B \in p \cdot \mathbf{H}, 0 < p \leq 1, A \not\sim B \), then \( \sigma_e(A) = \sigma_e(B), \sigma(A) = \sigma(B) \).
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Department of Mathematics, Fujian Normal University, Fuzhou, 350007, The People’s Republic of China
E-mail address: xhyan@fjtu.edu.cn