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ON $\mathbf{R}^+$ AND C COMPLETE HOLOMORPHIC VECTOR FIELDS

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Abstract. We show that, on holomorphic manifolds that have a plurisubharmonic exhaustion function and that do not carry nonconstant bounded plurisubharmonic functions (e.g. $\mathbf{C}^n$), holomorphic vector fields that are complete in positive time are complete in complex time.

Introduction

A holomorphic vector field $\mathbf{X}$ on a complex manifold M is said to be $\mathbf{R}^+$ complete, respectively $\mathbf{R}$ complete or $\mathbf{C}$ complete if for each $z \in M$, the initial value problem

$$\phi(0) = z,\ \phi'(t) = \mathbf{X}(\phi(t))$$

can be solved in forward time ($t > 0$), respectively in real time ($-\infty < t < +\infty$) or in complex time ($t \in \mathbf{C}$). Of course, complete in complex time implies complete in real time implies complete in positive time, and in general these notions are distinct. However, on any Stein manifold that supports no bounded, nonconstant, plurisubharmonic function, complete in real time implies complete in complex time [4], Corollary 2.2. On some manifolds, fields complete in positive time are much more abundant than those complete in real time. For example, in the unit disc, among nonconstant fields vanishing at the origin, only the rotation fields are complete in real time but any small perturbation of the field $\mathbf{X}(\zeta) = -\zeta$ is complete in positive time. In this paper we shall strengthen the theorem mentioned above as follows:

Theorem. *Suppose that M is a complex manifold that has a plurisubharmonic exhaustion function and that any bounded plurisubharmonic function on M is constant. Then any $\mathbf{R}^+$ complete field on M is $\mathbf{C}$ complete.*

This Theorem has immediate consequences on Fatou Bieberbach domains and maps: No Fatou Bieberbach domain can be star shaped and no Fatou Bieberbach map can be a time t map because by the above Theorem, in $\mathbf{C}^n$, time t maps are automorphisms. The time t maps topologically generate the full automorphism group of $\mathbf{C}^n$, [2] and [5]; however it is exceptional that an automorphism be a time t map, [3].

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We wish to say a few words about the hypothesis on M. Note that we are <u>*not*</u> requiring that the exhaustion function be <u>*strictly*</u> plurisubharmonic. Hence any compact manifold will satisfy the hypothesis of our theorem (the conclusion is obvious in this case of course!). Also the class of manifolds that satisfy the hypotheses is closed under products. Moreover, if a manifold M satisfies our hypotheses and if we blow up a point of M, then the new manifold satisfies the hypotheses as well.

As seen above (the example of the disc) the hypothesis about the nonexistence of nonconstant bounded plurisubharmonic functions is needed. Considering $M = \mathbf{C}^n \setminus \{z : |z| < 1\}$ and $\mathbf{X}(z) = z$ we see that the hypothesis of the existence of a plurisubharmonic exhaustion function is necessary as well.

The paper is organized as follows: in the first section we introduce our notation and then give a very short direct proof of our main result in the special case $M = \mathbf{C}^n$. Then in the second section we develop the ideas necessary to prove the general case. This is not logically necessary since the special case is not used in the proof of the general case, but the proof of the special case is so short and direct that we provide it for those who may be interested only in the case $M = \mathbf{C}^n$.

1. The case of $\mathbf{C}^n$

We shall follow standard notation and think of the mapping ϕ discussed above as a function of the variables (ζ, z). Hence for each $z \in M$ we have a neighborhood D of the origin in $\mathbf{C}$ and a mapping $\phi(.,z) : D \to M$ such that $\phi(0, z) = z$ and $\frac{\partial}{\partial \zeta}\phi(\zeta, z) = \mathbf{X}(\phi(\zeta, z))$ for $\zeta \in D$. Now we assume that $M = \mathbf{C}^n$ and that the field $\mathbf{X}$ is $\mathbf{R}^+$ complete. We will show that for each fixed z_0 in $\mathbf{C}^n$ the solution $\phi(\zeta, z_0)$ originally defined in a neighborhood of the origin in $\mathbf{C}$ extends to all of $\mathbf{C}$. To do this, it is enough to show that there is a neighborhood Δ of the origin in $\mathbf{C}$ such that for every $z \in \mathbf{C}^n$ the solution ϕ extends to Δ. By general properties of solutions of ordinary differential equations there is a $\delta > 0$ such that if $|z| \leq 1$, then $\phi(\zeta, z)$ exists for $|\zeta| < 2\delta$. We will show that for all $z \in \mathbf{C}^n$ the solution exists for $\zeta \in \Delta$ where $\Delta = \{\zeta : |\zeta| < \delta\}$. Now fix $t_0 \in \Delta$ and $z_0 \in \mathbf{C}^n$. Consider a curve γ in the ζ plane. It consists of the circle of radius 2δ with a 'notch' near the point 2δ. More precisely, γ consists of the circle of radius 2δ with the arc from 2δ to $2\delta e^{-i\epsilon}$ replaced by two straight line segments, one, called I, from 2δ to δ and the other from δ to $2\delta e^{-i\epsilon}$. Let O be the interior of γ. We can find a holomorphic mapping $z : \overline{O} \to \mathbf{C}^n$ such that $|z(\zeta)| < 1$ on $\gamma \setminus I$ and such that $z(t_0) = z_0$. This is possible since we have placed no restriction on the values of z in the interval I. Using the fact that the field $\mathbf{X}$ is $\mathbf{R}^+$ complete $\phi(s\zeta, z(\zeta))$ is defined for all $\zeta \in \gamma$ and all $0 \leq s \leq 1$. Now define

$$\Phi(s) = \frac{1}{2\pi i} \int_\gamma \phi(s\zeta, z(\zeta)) \frac{d\zeta}{\zeta - t_0}.$$

Since $\{z(\zeta) : \zeta \in \gamma\} = K$ is compact, there is some fixed neighborhood of the origin such that $\phi(\zeta, z)$ is defined for $z \in K$ and ζ in this neighborhood. Hence for s small, $\Phi(s) = \phi(st_0, z_0)$ and hence $\Phi'(s) = t_0\mathbf{X}(\Phi(s))$. Now since Φ is real analytic it satisfies the above differential equation for all $0 \leq s \leq 1$. This means that the solution ϕ, originally defined in some neighborhood of the origin, has an analytic continuation along the straight line segment from 0 to t_0. This completes the proof in the case $M = \mathbf{C}^n$.

2. Proof of the Theorem

We will begin this section assuming only that $\mathbf{X}$ is an $\mathbf{R}^+$ complete holomorphic vector field on a complex manifold M. We will add hypotheses on M as we proceed. We recall Lemma 8 of [1] which says that for each $z \in M$ there is a simply connected domain $\mathbf{R}_z \subset \mathbf{C}$ containing the origin such that $\phi(\zeta, z)$ exists for all $\zeta \in \mathbf{R}_z$. $\mathbf{R}_z$ also has the property that if $\zeta \in \mathbf{R}_z$, then $\zeta + t \in \mathbf{R}_z$ for all $t > 0$ (this just reflects the $\mathbf{R}^+$ completeness of $\mathbf{X}$). Finally if $\zeta_n \in \mathbf{R}_z$ and $\zeta_n \to \zeta_0 \in \partial\mathbf{R}_z$, then $\phi(\zeta_n, z)$ leaves every compact subset of M. We start with a basic geometrical fact about the domains $\mathbf{R}_z$ that will play a crucial role in what follows. We define $d(x, z) = sup\{t : x + is \in \mathbf{R}_z, 0 \le s \le t\}$. Then d is lower semicontinuous in (x, z) by properties of flows. Moreover $d(x, z)$ increases with x by the $\mathbf{R}^+$ property so $d(z) = lim_{x\to\infty} d(x, z)$ exists and is lower semicontinuous.

We now give a series of lemmas which will lead to a proof of the Theorem. The first two lemmas are completely straightforward and standard, but we state them separately for convenience.

Lemma 1. *If $K \subset M$ is compact and a continuous function $f(z) < d(z)$ is given on K, then there is a positive number C such that if $x \ge C$, then $d(z) \ge d(x, z) > f(z)$ for all $z \in K$.*

Proof. Fix $z_0 \in K$; then there is a $C = C(z_0)$ such that $d(x, z_0) > f(z_0)$ for $x \ge C$. By semicontinuity this holds in a neighborhood of z_0 and the Heine Borel theorem finishes the proof of the lemma.

Lemma 2. *If M has a plurisubharmonic exhaustion function ρ, then it has the maximum property; that is, if $K \subset M$ is compact, then there is a compact $L \subset M$ such that if Δ is the unit disc in $\mathbf{C}$ and $g : \overline{\Delta} \to M$ is holomorphic and $g(\partial\Delta) \subset K$, then $g(\Delta) \subset L$.*

Proof. The exhaustion function ρ is bounded above on K, say $\rho(z) \le c$ for $z \in K$. That is, $K \subset \{z : \rho(z) \le c\} = M_c$. By the maximum principle for plurisubharmonic functions $\rho(g(\lambda)) \le c$ for $|\lambda| \le 1$ and hence $g(\Delta) \subset M_c$. M_c is compact, by definition.

Lemma 3. *If M has a plurisubharmonic exhaustion function, then d is plurisuperharmonic on M.*

Proof. We have already noted that d is lower semicontinuous so we must show that if Δ is the unit disc in the complex plane and if $F : \overline{\Delta} \to M$ is holomorphic and for all θ we have $d(F(e^{i\theta})) > \Im m\ p(e^{i\theta})$ for some polynomial p, then $d(F(0)) \ge \Im m\ p(0)$. Since d is lower semicontinuous and positive it is bounded below on compact sets. Hence, we may assume that $\Im m\ p(e^{i\theta}) > 0$ for all θ and we may also assume that $\Re e\ p(0) = 0$. By Lemma 1 there is a $C > 0$ such that if $x > C$, then $d(x, F(e^{i\theta})) > \Im m\ p(e^{i\theta})$ for all θ, $0 \le \theta \le 2\pi$. Now choose a $C_1 > 0$ so that $s\Re e\ p(e^{i\theta}) + C_1 > C$ for all θ and all $0 \le s \le 1$. By choice of C_1

$$\phi(sp(e^{i\theta}) + C_1, F(e^{i\theta}))$$

is defined and describes a compact subset $K \subset M$ for $0 \le \theta \le 2\pi$ and $0 \le s \le 1$. For z in the compact set $\{F(\lambda) : |\lambda| \le 1\}$ there is a uniform disc about 0 for which the solution $\phi(\zeta, z)$ is defined, and hence by the $\mathbf{R}^+$ property, a uniform half-strip to the right. Hence if s is sufficiently small, then

$$\phi(sp(\lambda)+C_1, F(\lambda))$$

is defined for all $|\lambda| \leq 1$. We claim that $sp(\lambda)+C_1 \in \mathbf{R}_{F(\lambda)}$ for all $0 \leq s \leq 1$ and all $|\lambda| \leq 1$. If not, there would be an $s_0 \leq 1$ and a $|\lambda_0| < 1$ so that $sp(\lambda)+C_1 \in \mathbf{R}_{F(\lambda)}$ for all $s < s_0$ and all $|\lambda| \leq 1$ but $s_0 p(\lambda_0)+C_1 \notin \mathbf{R}_{F(\lambda_0)}$. We have the analytic discs

$$G_s(\lambda) = \phi(sp(\lambda)+C_1, F(\lambda))$$

for $0 \leq s < s_0$. The boundaries of these discs all lie in the fixed compact set K, described above. Hence the discs themselves lie in the fixed compact L by the maximum property. In particular, $\phi(sp(\lambda_0)+C_1, F(\lambda_0)) \in L$ for all $s < s_0$. This contradicts the fact that, for a fixed $z \in M$, $\phi(\zeta, z)$ must leave every compact subset of M as $\zeta \to \partial\mathbf{R}_z$. It follows that $sp(0)+C_1 \in \mathbf{R}_{F(0)}$ for all $0 \leq s \leq 1$. Of course this means that $d(F(0)) > \Im m\ p(0)$, as required.

We remark that if we define $l(x,z) = sup\{t : z - is \in \mathbf{R}_z, 0 \leq s \leq t\}$, then l is plurisuperharmonic on M as well.

Corollary. *If M has a plurisubharmonic exhaustion function and supports no nonconstant bounded plurisubharmonic function, then $d(z) \equiv \infty$ (and similarly for l).*

Proof. d is plurisuperharmonic and positive and hence constant, say $d(z) \equiv T$. Suppose T were finite. It follows from iteration that if $\zeta \in \mathbf{R}_z$ and $\tau \in \mathbf{R}_{\phi(\zeta,z)}$, then $\zeta + \tau \in \mathbf{R}_z$. Now fix $z \in M$ and $t > \frac{T}{2}$; then $x + is \in \mathbf{R}_z$ for some $x > 0$ and all $0 \leq s \leq t$. In the same way $u + iv \in \mathbf{R}_{\phi(x+is,z)}$ for some $u > 0$ and all $0 \leq s \leq t$ and $0 \leq v \leq t$. Hence $x + u + ir \in \mathbf{R}_z$, for all $0 \leq r \leq 2t$, contradicting the definition of T. So T must be ∞.

Our analysis will be based on a study of the domain of ϕ; that is, $\Omega = \{(\zeta, z) \in \mathbf{C} \times M : \zeta \in \mathbf{R}_z\}$.

Lemma 4. *Assume that M has a plurisubharmonic exhaustion function ρ and supports no nonconstant bounded plurisubharmonic function. Suppose $(\zeta_k, z_k) \in \Omega$ and $(\zeta_k, z_k) \to (\zeta_0, z_0) \in \partial\Omega$; it then follows that $\phi(\zeta_k, z_k)$ leaves every compact set.*

Proof. If the conclusion of the lemma were false we could find a subsequence of (ζ_k, z_k), again called (ζ_k, z_k), such that $\phi(\zeta_k, z_k) \to w_0 \in M$. By the corollary to Lemma 3 there is a $C > 0$ such that $(\zeta_0 + t, z_0) \in \Omega$ for all $t > C$. Now $(\zeta_k + t, z_k) \to (\zeta_0 + t, z_0)$ in Ω so $\phi(\zeta_k + t, z_k) \to \phi(\zeta_0 + t, z_0)$. But $\phi(\zeta_k + t, z_k) = \phi(t, \phi(\zeta_k, z_k)) \to \phi(t, w_0)$. So we have

$$\phi(\zeta_0 + t, z_0) = \phi(t, w_0)$$

for all $t > C$. Now by the $\mathbf{R}^+$ property the right hand side is defined for $t \geq 0$, so we have an analytic continuation of the left hand side for all $t \geq 0$. But this means that $\zeta_0 + t \in \mathbf{R}_z$ for all $t \geq 0$. In particular, $(\zeta_0, z_0) \in \Omega$ which contradicts the hypotheses. Hence the lemma is proved.

Corollary. *Under the hypotheses of the lemma Ω has a plurisubharmonic exhaustion function.*

Proof. Define $\Lambda(\zeta, z) = |\zeta|^2 + \rho(z) + \rho(\phi(\zeta, z))$; then we see immediately that Λ is an exhaustion function for Ω.

In the next 3 lemmas we temporarily forget about the vector field $\mathbf{X}$. We have a complex manifold M and an open subset $\Omega \subset \mathbf{C} \times M$ and we will show that under appropriate conditions on Ω and M, Ω must be a product $O \times M$.

Lemma 5. *Let $\Omega \subset \mathbf{C} \times M$ have a plurisubharmonic exhaustion function. Define $d(\zeta, z) = sup\{r : \Delta(\zeta, r) \times z \subset \Omega\}$. Then $-log\ d(\zeta, z)$ is plurisubharmonic on Ω.*

Proof. The fact that Ω is open implies that $-log\ d$ is upper semicontinuous. Suppose that $F : \overline{\Delta(0,1)} \to \Omega$ is holomorphic and

$$-log\ d(F(e^{i\theta})) = -log\ d(\zeta(e^{i\theta}), z(e^{i\theta})) < \Re e\ p(e^{i\theta})$$

for some polynomial p. This means that we have $d(\zeta(e^{i\theta}), z(e^{i\theta})) > |e^{-t-p(e^{i\theta})}|$ for all $t \geq 0$. This means that $(\zeta(e^{i\theta}) + e^{-t-p(e^{i\theta})}, z(e^{i\theta})) \in \Omega$ for all θ and all $t \geq 0$. We have the continuous family of analytic discs $g_t : \overline{\Delta} \to \mathbf{C} \times M$ defined by $g_t(\lambda) = (\zeta(\lambda) + e^{-t-p(\lambda)}, z(\lambda))$. We also have $(\zeta(\lambda) + e^{-t-p(\lambda)}, z(\lambda)) \in \Omega$ for all $|\lambda| \leq 1$ for all large t. The maximum property now implies that $(\zeta(0) + e^{-t-p(0)}, z(0)) \in \Omega$ for all $t \geq 0$. Since we can change the imaginary part of p without changing the result we see that $-log\ d(\zeta(0), z(0)) < \Re e\ p(0)$, as required.

Lemma 6. *Assume $\Omega \subset \mathbf{C} \times M$ has a plurisubharmonic exhaustion function and M supports no nonconstant plurisubharmonic function. Suppose there is a nonempty open set ω such that $\omega \times M \subset \Omega$ and that for each $z \in M$ the fibre $\mathbf{R}_z = \{\zeta \in \mathbf{C} : (\zeta, z) \in \Omega\}$ is connected. Then $\Omega = O \times M$ for some open set $O \subset \mathbf{C}$.*

Proof. We fix $z \in M$ and we will show $\mathbf{R}_z \subset \mathbf{R}_w$ for all $w \in M$. By hypothesis there is a disc $\Delta(\zeta_0, \epsilon) \subset \omega \subset \mathbf{R}_w$ for all $w \in M$. Now take $\zeta \in \mathbf{R}_z$; we will show $\zeta \in \mathbf{R}_w$ for all w. There is a chain of discs $\Delta_k = \Delta(\zeta_k, r_k) \subset \mathbf{R}_z, k = 1, \ldots, N$ such that $\Delta_0 = \Delta(\zeta_0, \epsilon)$, ζ_{k+1} lies in Δ_k, and $\zeta \in \Delta_N$. We will show by induction that $\Delta_k \subset \mathbf{R}_w$ for all w. By hypothesis the statement is true for $k = 0$. Suppose it is true for some k. Since $\zeta_{k+1} \in \Delta_k$ there is an $\eta > 0$ so that $\Delta(\zeta_{k+1}, \eta) \subset \Delta_k \subset \mathbf{R}_w$ for all $w \in M$. Now $-log\ d(\zeta_{k+1}, w)$ is plurisubharmonic in the variable w and bounded above and hence $d(\zeta_{k+1}, w)$ is a constant. Clearly this constant is at least as big as r_{k+1}, from which it follows that $\Delta_{k+1} \subset \mathbf{R}_w$ for all $w \in M$, and this completes the proof.

Lemma 7. *Assume that Ω is an open subset of $\mathbf{C} \times M$. We also assume that Ω and M have plurisubharmonic exhaustion functions and that M supports no nonconstant bounded plurisubharmonic function. Suppose that there is a nonempty line segment I such that $I \times M \subset \Omega$ and for each $z \in M$ the fibre $\mathbf{R}_z = \{\zeta : (\zeta, z) \in \Omega\}$ is connected. Then $\Omega = O \times M$ for some open set $O \subset \mathbf{C}$.*

Proof. Let ψ be a conformal map of $\mathbf{C} \setminus I$ onto the complement of the closed unit disc $\overline{\Delta}$. Define $\mathbf{Q}_z = \psi(\mathbf{R}_z \setminus I) \cup \overline{\Delta}$. Now let $\Omega_1 = \{(\zeta, z) : \zeta \in \mathbf{Q}_z\}$. Since Ω is open it follows that Ω_1 is open. Now we wish to show that Ω_1 has a plurisubharmonic exhaustion function. Let Λ be the plurisubharmonic exhaustion function for Ω and ρ be the plurisubharmonic exhaustion function for M. Choose a convex increasing function χ so that $\chi(\rho(z)) > sup\{\Lambda(\zeta, z) : \zeta \in I\}$. Now define, on Ω_1,

$$U(\zeta, z) = max\big(\Lambda(\psi^{-1}(\zeta, z)), \chi(\rho(z))\big).$$

Of course $\psi^{-1}(\zeta)$ is only defined of $|\zeta| > 1$, but by the the definition of χ, $U(\zeta, z) = \chi(\rho(z))$ if $|\zeta|$ is close to 1. Moreover U gives a plurisubharmonic exhaustion function for Ω_1. Now by construction $\Delta \times M \subset \Omega_1$ and so by Lemma 6 $\Omega_1 = O \times M$ and hence $\Omega = V \times M$, where $V = \psi^{-1}(O \setminus \overline{\Delta}) \cup I$.

Proof of the Theorem. By the corollary to Lemma 4, Ω has a plurisubharmonic exhaustion function and by the $\mathbf{R}^+$ property it contains $[0,1] \times M$(in fact it contains $\mathbf{R}^+ \times M$) and hence by Lemma 7 it is a product $O \times M$. So, O contains a fixed neighborhood of the origin and now by iteration of the flow we may conclude that $O = \mathbf{C}$.

Now we give some consequences of our result.

Corollary. *Suppose* $\mathbf{X}$ *is a* $\mathbf{C}$ *complete field on* $\mathbf{C}^n$ *with flow* $\phi(\zeta, z)$. *Suppose there is a point* $p \in \mathbf{C}^n$ *such that* $\phi(t, z) \to p$ *as* $t \to \infty$, $t \in \mathbf{R}^+$. *Suppose that* $O \subset \mathbf{C}^n$ *is a pseudoconvex domain, containing* p, *that supports no nonconstant bounded plurisubharmonic function and that for all* $z \in O$, $\phi(t, z) \in O$ *for all* $t > 0$, *and then* $O = \mathbf{C}^n$.

Proof. The hypotheses imply that the field $\mathbf{X}$, restricted to O, is $\mathbf{R}^+$ complete, and hence $\mathbf{C}$ complete, by the theorem. The corollary follows.

Notice that one consequence of this corollary is that if O is a pseudoconvex subset of $\mathbf{C}^n$ which admits no nonconstant bounded plurisubharmonic function and if O is star-like with respect to some point $p \in O$, then $O = \mathbf{C}^n$. To see this we just consider the field $\mathbf{X}(z) = p - z$ and apply the corollary.

It is known that if $\mathbf{X}$ is a $\mathbf{C}$ complete field on $\mathbf{C}^n$ and has an attracting fixed point p and if the basin of attraction of this fixed point is invariant under the flow for both positive and negative time, then this basin of attraction is a Fatou-Bieberbach domain; that is, it is biholomorphic to $\mathbf{C}^n$. We can now give a sort of converse to this result.

Corollary. *If* $\mathbf{X}$ *is a* $\mathbf{C}$ *complete field on* $\mathbf{C}^n$ *with an attracting fixed point* p *and its basin of attraction* B *is biholomorphic to* $\mathbf{C}^n$, *then* B *is invariant under the flow for all complex time.*

Proof. $\mathbf{X}$ restricted to B is $\mathbf{R}^+$ complete and hence $\mathbf{C}$ complete by the theorem.

We finish with a question. There are many differences between continuous and discrete dynamics. As pointed out earlier Fatou Bieberbach maps can never be time t maps. We have also seen that a Fatou Bieberbach domain cannot be star shaped. That is, a Fatou Bieberbach domain can never be invariant under the maps $z \to tz$ for all $0 \le t \le 1$. Question: can a Fatou Bieberbach domain be invariant under the map $z \to \frac{z}{2}$?

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