PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 129, Number 7, Pages 2085–2092
S 0002-9939(00)05843-3
Article electronically published on December 28, 2000

SIMILARITY TO A CONTRACTION AND HYPERCONTRACTIVITY OF COMPOSITION OPERATORS

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(Communicated by David R. Larson)

ABSTRACT. On the Hardy spaces H^p with $1 \leq p < \infty$, we consider the composition operators induced by analytic self-maps of the open unit disc D. First, we characterize those which are similar to contractions. Then, we give some necessary and sufficient conditions for them to be hypercontractive. Finally, we prove that, among those ones, only the zero-symbol composition operator sends H^p into H^∞ with a norm less than or equal to 1.

1. INTRODUCTION

Throughout this paper, we denote by D the open unit disc in the complex plane, by $H(D)$ the space of holomorphic functions on D and by $H(D, D)$ the subset of $H(D)$ consisting of all self-maps of D. We also denote by $\mathbb{N}^*$ the set of integers larger than one : $\mathbb{N}^* = \{1, 2, \cdots\}$.

Let φ be in $H(D, D)$. On appropriate subspaces of $H(D)$, the composition operator induced by the symbol φ is defined by $C_\varphi f := f \circ \varphi$. We recall that the *Hardy space* H^p $(0 < p < \infty)$ is the subspace of $H(D)$ consisting of all functions satisfying

$$\|f\|_p := \Big(\sup_{0 \leq r < 1} \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta \Big)^{\frac{1}{p}} < \infty.$$

For $p \geq 1$, this gives a norm for which H^p is a Banach space. We also recall that the space H^∞, endowed with the norm $\|f\|_\infty := \sup_{z \in D} |f(z)|$, is a Banach space too. From these definitions, we get the following proper inclusions:

$$H^\infty \subset H^{\beta p} \subset H^p \quad (0 < p < \infty \ , \ 1 < \beta < \infty).$$

It is well known that C_φ is continuous on H^p $(0 < p < \infty)$ (see [10] and [14]) and that, for $p \geq 1$, C_φ is a *contraction* (i.e. $\|C_\varphi\| \leq 1$) if and only if $\varphi(0) = 0$ (see Theorem 2.1).

Here, we will characterize a subclass of composition operators that contains each C_φ which is *similar to a contraction.* In Section 3, we show that the existence of a fixed point in D, for the symbol φ, is a necessary and sufficient condition for C_φ to

Received by the editors March 10, 1999 and, in revised form, November 17, 1999.
1991 *Mathematics Subject Classification.* Primary 47B38, 47B65.
Key words and phrases. Composition operators, Hardy spaces, contraction, hypercontractive (or β-contractive) operator.

be in that class. In particular, we observe that if C_φ is polynomially bounded, then it is similar to a contraction. G. Pisier has shown that this is not true for every operator on a Hilbert space (see [13] for details). In the case where the contraction is isometric and the symbol is analytic on a neighborhood of $\overline{D}$, we give a more precise characterization.

In Section 4, we study the class of all C_φ's sending H^p into $H^{\beta p}$ ($\beta > 1$) with a norm less than or equal to 1. The qualitative aspect of this problem (i.e. sending H^p into $H^{\beta p}$) has been solved by H. Hunziker and H. Jarchow ([7]). For the quantitative aspect (i.e. sending H^p into $H^{\beta p}$ with norm ≤ 1), we give some necessary and sufficient conditions. C_φ's satisfying the second aspect are said to be *hypercontractive* or also *β-contractive*. At the end of Section 4, we show that, among those ones, only the zero-symbol composition operator is a contraction from H^p into H^∞.

Section 2 is devoted to some results focused on the symbols φ and on the operators C_φ sending H^p into $H^{\beta p}$ ($1 \leq \beta, p < \infty$).

2. Preliminaries

It is well known (cf. [5]) that, for each $f \in H^p$, $f^*(e^{i\theta}) := \lim_{r\to 1} f(re^{i\theta})$ exists almost everywhere on the unit circle ∂D. A function φ in $H(D, D)$ is said to be inner if $|\varphi^*| = 1$ almost everywhere.

Theorem 2.1 (cf. [2], p. 123)**.** *For all $\varphi \in H(D, D)$ and $p \in [1, +\infty[$, C_φ is bounded on H^p and we have*

$$\sup_{|z|<1} \left(\frac{1-|z|^2}{1-|\varphi(z)|^2}\right)^{\frac{1}{p}} \leq \|C_\varphi\| \leq \left(\frac{1+|\varphi(0)|}{1-|\varphi(0)|}\right)^{\frac{1}{p}}.$$

Moreover, C_φ is an isometry if and only if φ is inner and vanishes at 0.

Let X be a Banach space. We recall that $T : X \to X$ is an isometry if $\|Tx\| = \|x\|$ for all $x \in X$. The following well-known proposition (see [1], p. 213) describes the spectrum $\sigma(T)$ of such an operator. For sake of completeness, we give an elementary proof.

Proposition 2.2. *If $T : X \to X$ is an isometry , then either $\sigma(T) \subseteq \partial D$ or $\sigma(T) = \overline{D}$.*

Proof. If T is onto, $\sigma(T) \subseteq \partial D$; otherwise $E := \sigma(T) \cap D$ is a closed subset of D containing 0. Let $F = D - E = \sigma(T)^c \cap D$ be the complement of E in D. If $\lambda \in D$, $\lambda_n \in F$ and $\lambda_n \to \lambda$, then for all $x \in X$, we have

$$\|(T - \lambda_n I)x\| \geq \|Tx\| - |\lambda_n|\|x\| = (1 - |\lambda_n|)\|x\| \geq \frac{1-|\lambda|}{2}\|x\| = \delta\|x\|$$

for large n, which implies that $\|T - \lambda_n I)^{-1}\| \leq \delta^{-1}$. And this implies (cf. [4]) that $\lambda \in \sigma(T)^c$. Therefore, F is equally closed in D. The connectedness of D implies that $E = D$, and $\sigma(T) = \overline{D}$. ☐

Remark. Proposition 2.2 can be seen as a consequence of the fact that $\partial\sigma(T)$ is contained in $\sigma_{ap}(T)$ (Proposition 6.7 in [1]). This fact implies that, for any $\lambda \in \partial\sigma(T)$, there is a sequence $(x_n)_n$ with $\|x_n\| = 1$ such that $\|(T - \lambda I)x_n\| \to 0$. Since T is an isometry, it is easy to see that $|\lambda| = 1$, and the result follows by connectedness of D.

The following proposition can be proved by using the factorization theorem of F. Riesz ([5], p. 20). It justifies the reason for which one can study hypercontractivity only on the Hilbert space H^2. If it sends H^p into $H^{\beta p}$, for some $1 < \beta < \infty$, C_φ is said to be β-*bounded*, and we denote its norm by $\|C_\varphi\|_{p,\beta p}$.

Proposition 2.3. *If C_φ is hypercontractive on H^p for some $1 \le p < \infty$, then the same is true for all $1 \le p < \infty$.*

In the next theorem, $(\varphi_n)_{n\in\mathbb{N}}$ denotes the iterate sequence of the map φ. The uniform convergence on every compact set of D is denoted by $\overset{u.c}{\to}$.

Theorem 2.4 (cf. [3] or [16])**.** *If $\varphi \in H(D, D)$ has no fixed point in D, then there exists a unique point $\zeta \in \partial D$ (called the Denjoy-Wolff point of φ) such that $\varphi_n \overset{u.c}{\to} \zeta$.*

3. Similarity to a contraction

Let X be a Banach space. $T : X \to X$ is said to be polynomially bounded if there exists $M > 0$ such that $\|P(T)\| \le M\|P\|_\infty$ for every polynomial P, where $\|P\|_\infty = \sup\{|P(z)|; |z| \le 1\}$.

Theorem 3.1. *In the following assertions,* (1), (3) *and* (4) *are equivalent for all $1 \le p < \infty$. In particular, for $p = 2$, they are equivalent to* (2).

(1) *C_φ is similar to a contraction.*
(2) *C_φ is polynomially bounded.*
(3) *C_φ is power bounded.*
(4) *φ has a fixed point in D.*

Proof. (1) $\Longrightarrow$ (2) By hypothesis, there exists an invertible operator S on H^2 such that

$$\begin{aligned} C_\varphi &= S^{-1}CS \quad \text{where} \quad C \text{ is a contraction on } H^2. \\ P(C_\varphi) &= S^{-1}P(C)S. \\ \|P(C_\varphi)\| &\le \|S^{-1}\|\|P(C)\|\|S\| \le \|S^{-1}\|\|S\|\|P\|_\infty. \end{aligned}$$

The last estimate follows from the famous von Neumann's inequality.

(2)$\Longrightarrow$(3) This is immediate.

(3)$\Longrightarrow$(4) Assume that φ has no fixed point in D. By the weak form of the Denjoy-Wolff theorem, we have $|\varphi_n| \overset{u.c}{\to} 1$. Since

$$\|C_\varphi^n\| = \|C_{\varphi_n}\| \ge (1 - |\varphi_n(0)|^2)^{-\frac{1}{p}},$$

we conclude that

$$\varliminf_{n\to\infty} \|C_\varphi^n\| = \infty.$$

This is in contradiction with (3).

(4)$\Longrightarrow$(1) Let $a \in D$ be a fixed point of φ. Set

$$\psi = \varphi_a \circ \varphi \circ \varphi_a \quad \text{where} \quad \varphi_a(z) = \frac{a - z}{1 - \bar{a}z}.$$

φ_a is a holomorphic automorphism of D and $\varphi_a^{-1} = \varphi_a$. Since $\psi \in H(D, D)$ and $\psi(0) = 0$, by Theorem 2.1, C_ψ is a contraction on H^p. Now, the identity $\varphi = \varphi_a \circ \psi \circ \varphi_a^{-1}$ implies that $C_\varphi = C_{\varphi_a}^{-1} C_\psi C_{\varphi_a}$, and this completes the proof. $\square$

E. Nordgren ([10]) has shown that if C_φ^n is compact for some $n \in \mathbb{N}^*$, then φ has a fixed point in D. As a consequence of this result and Theorem 3.1, we have the following.

Corollary 3.2. *If C_φ^n is compact for some $n \in \mathbb{N}^*$, then C_φ is similar to a contraction.*

In order to study the similarity with an isometry on H^p, we need the following theorem (cf. [9]) which provides the spectrum of C_φ in a special case.

Theorem 3.3. *Suppose φ is analytic on a neighborhood of $\overline{D}$, not inner and with a fixed point a in D. If C_φ^n is compact on H^p $(1 \le p < \infty)$ for no $n \in \mathbb{N}^*$, then there exists $0 < \rho < 1$ such that*

$$\sigma(C_\varphi) = \{\lambda \in \mathbb{C}; |\lambda| \le \rho\} \cup \{(\varphi'(a))^n; n \in \mathbb{N}^*\} \cup \{1\}.$$

Theorem 3.4. *Suppose φ is analytic on a neighbourhood of $\overline{D}$. Then the following are equivalent:*

(1) *C_φ is similar to an isometry.*
(2) *φ is inner and has a fixed point in D.*

Proof of Theorem 3.4. (1)$\Longrightarrow$(2) By Theorem 3.1, φ has necessarily a fixed point in D. Now, assume that φ is not inner. Since C_φ^n is compact for no $n \in \mathbb{N}^*$ (an operator which is similar to an isometry cannot be compact in infinite dimension), by Theorem 3.3, $\sigma(C_\varphi) \neq \overline{D}$ and $\sigma(C_\varphi) \not\subseteq \partial D$. This deduction together with Theorem 2.2 leads to a contradiction with (1). Consequently, φ is necessarily inner.

(2)$\Longrightarrow$(1) This can be proved in the same way as (4)$\Longrightarrow$(1) of Theorem 3.1. Note that, here, C_ψ is an isometry since ψ is inner and vanishes at 0 (see Theorem 2.1). $\square$

Question. Theorem 3.3 provides the spectrum of C_φ with φ analytic on a neighborhood of $\overline{D}$. Is this condition on φ necessary in Theorem 3.4 ?

The next corollary follows immediately from Theorem 3.4 and a known result saying that C_φ is invertible on H^p if and only if φ is an automorphism of D (see [11] or [2] or more recently [6]).

Corollary 3.5. *A composition operator is similar to an isometric isomorphism on H^p if and only if its symbol is an elliptic automorphism of D.*

4. Hypercontractivity

F. B. Weissler ([15]) has characterized the hypercontractivity of convolution operators by Poisson kernels $P_r(\theta) := \frac{1-r^2}{1+r^2-2r\cos\theta}$ $(0 < r < 1)$. Those operators are defined on $L^p(\partial D, m)$ by : $([P_r]f)(e^{i\theta}) = \sum_{-\infty}^{+\infty} \hat{f}(n) r^{|n|} e^{in\theta}$. He has shown that $\|[P_r]\|_{L^p \to L^q} = 1$ (resp., $\|[P_r]\|_{H^p \to H^q} = 1$) if and only if $r^2 \le \frac{p-1}{q-1}$ (resp., $r^2 \le \frac{p}{q}$) for all p, q such that $1 < p < q < \infty$.

This result and the following lemma will allow us to characterize the hypercontractive $C_{\alpha z}$ with $|\alpha| \le 1$. Note that the symbols $\varphi(z) = \alpha z$ are exactly those for which $C_\varphi : H^2 \to H^2$ is normal (see [11]).

Lemma 4.1. *Let $f \in H^\infty$ with $f(0) = 0$ and $q \in]0, +\infty[$. For all $\varepsilon > 0$ small enough, we have*

$$\|1 + \varepsilon f\|_q = 1 + \frac{q}{4}\|f\|_2^2 \varepsilon^2 + o(\varepsilon^2).$$

Proof. For all $z \in D$, one has

$$(1+\varepsilon f(z))^{\frac{q}{2}} = 1 + \varepsilon \frac{q}{2} f(z) + \varepsilon^2 \frac{q}{4}(\frac{q}{2}-1) f^2(z) + o(\varepsilon^2).$$

Here, $o(\varepsilon^2)$ does not depend on z since f is bounded. So, we obtain

$$\begin{aligned} |1+\varepsilon f(z)|^q &= |(1+\varepsilon f(z))^{\frac{q}{2}}|^2 = (1+\varepsilon f(z))^{\frac{q}{2}} \overline{(1+\varepsilon f(z))^{\frac{q}{2}}} \\ &= 1 + \varepsilon q Re(f(z)) + \varepsilon^2 \tfrac{q}{2}(\tfrac{q}{2}-1) Re(f^2(z)) + \varepsilon^2 \tfrac{q^2}{4} |f(z)|^2 + o(\varepsilon^2). \end{aligned}$$

As $Re(f)$ and $Re(f^2)$ are harmonic in D and since $f(0) = 0$, by integration on $[0, 2\pi]$ with respect to the measure $\frac{d\theta}{2\pi}$, one gets

$$\frac{1}{2\pi} \int_0^{2\pi} |1+\varepsilon f(e^{i\theta})|^q \, d\theta = 1 + \varepsilon^2 \frac{q^2}{4} \|f\|_2^2 + o(\varepsilon^2).$$

This leads to the desired assertion. $\square$

Theorem 4.2. *Let $\alpha \in \overline{D}$ and $1 < \beta < \infty$. We have equivalence between* (1) *and* (2) *and between* (3) *and* (4).

(1) *$C_{\alpha z}$ is β-bounded.*
(2) $|\alpha| < 1$.
(3) *$C_{\alpha z}$ is β-contractive.*
(4) $|\alpha| \le \beta^{-\frac{1}{2}}$.

Proof. (1) $\Longrightarrow$ (2) Suppose that $|\alpha| = 1$. That means φ is a rotation of D, and then C_φ sends H^2 onto itself: this is a contradiction with (1).

(2)$\Longrightarrow$(1) Since $\|\varphi\|_\infty = |\alpha| < 1$, one has

$$C_\varphi(H^2) \subset H^\infty \subset H^{2\beta}.$$

Thus, C_φ is β-bounded.

(3)$\Longrightarrow$(4) Let g be the function defined on D by $g(z) = 1 + \varepsilon z$ with $\varepsilon > 0$ small enough. Since $C_{\alpha z} g = 1 + \varepsilon f$ where $f(z) = \alpha z$ for all $z \in D$, by Lemma 4.1 applied to this f and to $q = 2\beta$, we get

$$\|C_\varphi g\|_{2\beta} = 1 + \frac{\beta|\alpha|^2}{2} \varepsilon^2 + o(\varepsilon^2).$$

On the other hand, this same lemma, applied to $f \equiv z$ and $q = 2$, gives

$$\|g\|_2 = 1 + \frac{1}{2} \varepsilon^2 + o(\varepsilon^2).$$

Now, by hypothesis, we have $\|C_\varphi g\|_{2\beta} \le \|g\|_2$. Let us simplify, then take the limit as $\varepsilon \to 0$ to obtain $|\alpha|^2 \le \frac{1}{\beta}$. This implies (4).

(4)$\Longrightarrow$(3) This is nothing else but another formulation of Weissler's theorem mentioned at the beginning of this section. Indeed, just observe that $\|C_{\alpha z}\|_{2,2\beta} = \|[P_r]\|_{H^2 \to H^{2\beta}}$ with $r = |\alpha|$. $\square$

Corollary 4.3. *If $\varphi(0) = 0$ and $\|\varphi\|_\infty \le \beta^{-\frac{1}{2}}$, then C_φ is β-contractive.*

Proof. Set $r = \|\varphi\|_\infty$ and $\psi = \frac{1}{r}\varphi$. By Schwarz' lemma, one has $|\varphi(z)| \le r|z|$. Thus, $\psi \in H(D, D)$. On the other hand, the identities $C_\varphi = C_{r\psi} = C_\psi C_{rz}$ imply that

$$\|C_\varphi\|_{1,\beta} \le \|C_\psi\|_{\beta,\beta} \|C_{rz}\|_{1,\beta} \le 1.$$

The last estimate comes from the fact that $\psi(0) = 0$ and from Theorem 4.2 which one can apply because $r \le \beta^{-\frac{1}{2}}$. $\square$

In the following theorem, we denote by H_0^2 the subspace of H^2 consisting of all functions vanishing at 0.

Theorem 4.4. *If C_φ is β-contractive, then we have the following:*

(1) $\|C_\varphi\|_{B(H_0^2)} \leq \beta^{-\frac{1}{2}}$.
(2) $\frac{1}{2\pi}\int_0^{2\pi} |g(\varphi(e^{i\theta}))|^2 |\varphi(e^{i\theta})|^2 \, d\theta \leq \frac{1}{\beta}\|g\|_2^2$ *for all* $g \in H^2$. *In particular,* $\|\varphi\|_2 \leq \beta^{-\frac{1}{2}}$.

Proof. (1) First, let us suppose that $f \in H^\infty$ with $f(0) = 0$. Since the contractivity of C_φ implies that $\varphi(0) = 0$, we get $f \circ \varphi \in H^\infty$ and $(f \circ \varphi)(0) = 0$. Let $\varepsilon > 0$ be small enough. By Lemma 4.1, we obtain

$$\begin{cases} \|1 + \varepsilon f \circ \varphi\|_{2\beta} & = 1 + \frac{\beta}{2}\|f \circ \varphi\|_2^2 \varepsilon^2 + o(\varepsilon^2), \\ \|1 + \varepsilon f\|_2 & = 1 + \frac{1}{2}\|f\|_2^2 \varepsilon^2 + o(\varepsilon^2). \end{cases}$$

Now, by hypothesis, one has

$$\|1 + \varepsilon f \circ \varphi\|_{2\beta} \leq \|1 + \varepsilon f\|_2.$$

Thus, after simplifying and taking the limit as $\varepsilon \to 0$, one concludes that $\beta\|f \circ \varphi\|_2^2 \leq \|f\|_2^2$. Hence, $\|f \circ \varphi\|_2 \leq \beta^{-\frac{1}{2}}\|f\|_2$.

Let us suppose now that $f \in H_0^2$. If we denote by $(f_n)_n$ the partial sums of f, that is, $f_n(z) = \sum_{k=1}^{n} \hat{f}(k) z^k$, we have $\|f_n - f\|_2 \to 0$. The inequality $\|f_n \circ \varphi\|_2^2 \leq \beta^{-1}\|f_n\|_2^2$ follows from the previous part of the proof (since $f_n \in H_0^\infty$), and the desired result is obtained by passing to the limit in this inequality (since C_φ is bounded on H^2).

(2) This follows from (1) by using the fact that $zg \in H_0^2$ for all $g \in H^2$. □

Remark. So far, we have not been able to discover whether the condition $\|\varphi\|_\infty < 1$ is necessary for C_φ to β-contractive. However, one can show that this is true in the case where $\varphi(0) = 0$ and $|\varphi(u) - \varphi(v)| \leq k|u - v|^{\frac{1}{\beta}}$ on $(\partial D)^2$ with $0 < k < 2^{\frac{1}{\beta} - 1}$ (see [8]).

H. Hunziker and H. Jarchow ([7]) have characterized β-boundedness of C_φ's for all $1 \leq \beta < \infty$. The following theorem gives a version for the limit case $\beta = \infty$.

Theorem 4.5. *The following are equivalent:*

(1) $\|\varphi\|_\infty < 1$.
(2) $C_\varphi(H^p) \subset H^\infty$ *for all* $1 \leq p < \infty$.
(3) $C_\varphi(H^p) \subset H^\infty$ *for some* $1 \leq p < \infty$.

Proof. (1) $\Longrightarrow$ (2) It is well known (cf. [17]) that the point evaluation operator δ_z, defined by $\delta_z(f) = f(z)$, is bounded on H^p and $\|\delta_z\| = (1 - |z|^2)^{-\frac{1}{p}}$. For all $f \in H^p$ and $z \in D$, we have

$$|f(\varphi(z))| \leq \|\delta_{\varphi(z)}\|_{(H^p)^*}\|f\|_p = (1 - |\varphi(z)|^2)^{-\frac{1}{p}}\|f\|_p \leq (1 - \|\varphi\|_\infty^2)^{-\frac{1}{p}}\|f\|_p.$$

Thus, $f \circ \varphi \in H^\infty$ and $\|f \circ \varphi\|_\infty \leq (1 - \|\varphi\|_\infty^2)^{-\frac{1}{p}}\|f\|_p$.

(2)$\Longrightarrow$(3) This is immediate.

(3)$\Longrightarrow$(1) For every $\zeta \in \partial D$, the function $z \longmapsto (\zeta - z)^{-\frac{1}{2p}}$ belongs to H^p. So by hypothesis, there exists $c > 1$ such that $|\zeta - \varphi(z)|^{-\frac{1}{2p}} \leq c$. In other words, there exists $\delta \in]0,1[$ such that

$$(*) \qquad |\zeta - \varphi(z)| \geq \delta \quad \text{for all } z \in D.$$

Let z be in D such that $\varphi(z) \neq 0$. By $(*)$, applied to $\zeta = \frac{\varphi(z)}{|\varphi(z)|}$, we obtain

$$1 - |\varphi(z)| = |\varphi(z)||\frac{1}{|\varphi(z)|} - 1| = |\varphi(z)(\frac{1}{|\varphi(z)|} - 1)| \geq \delta.$$

Thus, $|\varphi(z)| \leq 1 - \delta$; z being arbitrary, we conclude that $\|\varphi\|_\infty \leq 1 - \delta < 1$. □

We end this paper with the following theorem which says that C_0 is the only contractive composition operator from H^p $(1 \leq p < \infty)$ into H^∞.

Theorem 4.6. *The following are equivalent:*

(1) $C_\varphi : H^p \to H^\infty$ *is a contraction.*
(2) $\varphi \equiv 0$.

Proof. (1)$\Longrightarrow$(2) For all $z \in D$ and $f \in H^p$ with $\|f\|_p = 1$, one has $|f(\varphi(z))| \leq \|f \circ \varphi\|_\infty \leq 1$. Thus, $\|\delta_{\varphi(z)}\|_{(H^p)^*} = (1 - |\varphi(z)|^2)^{-\frac{1}{p}} \leq 1$. So, we deduce that $1 - |\varphi(z)|^2 = 1$ for all $z \in D$, and this gives (2).

(2)$\Longrightarrow$(1) For all $f \in H^p$, $C_\varphi f = f \circ \varphi \equiv f(0) \in H^\infty$. Consequently, $\|C_\varphi f\|_\infty = |f(0)| \leq \|f\|_p$. □

Acknowledgements

This work is part of the author's thesis written under the supervision of Professor H. Queffelec. We would like to thank the referee for his very careful reading and his clever remarks.

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