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THE FUCIK SPECTRUM AND CRITICAL GROUPS

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Abstract. We compute critical groups of zero for variational functionals arising from semilinear elliptic boundary value problems with jumping nonlinearities when the asymptotic limits of the nonlinearity fall in certain parts of Type (II) regions between curves of the Fucik spectrum.

1. Introduction

In this paper we consider the problem of determining the critical groups of zero for the functional

$$I(u) = I(u,a,b) = \int_\Omega |\nabla u|^2 - a\,(u^-)^2 - b\,(u^+)^2, \quad u \in H = H_0^1(\Omega), \tag{1.1}$$

associated with the problem

$$\begin{cases} -\Delta u &= b\,u^+ - a\,u^- & \text{in } \Omega, \\ u &= 0 & \text{on } \partial\Omega \end{cases} \tag{1.2}$$

where Ω is a smooth bounded domain in $\mathbb{R}^n$ and $u^\pm(x) = \max\{\pm u(x),\, 0\}$.

The set Σ of those points $(a,b) \in \mathbb{R}^2$ for which (1.2) has nontrivial solutions is called the Fucik spectrum of $-\Delta$. It was shown in Schechter [7] that, if $0 < \lambda_1 < \lambda_2 < \cdots$ are the distinct Dirichlet eigenvalues of $-\Delta$, there are decreasing curves C_{l1}, C_{l2} (which may coincide) passing through the point (λ_l, λ_l) such that all points on the curves are in Σ, while points in the square $Q_l := (\lambda_{l-1},\, \lambda_{l+1})^2$ that are either in the region I_{l1} below the lower curve C_{l1} or in the region I_{l2} above the upper curve C_{l2} are not in Σ. When the curves do not coincide, points in the region II_l between them may or may not belong to Σ. We set $\mathrm{I}_l = \mathrm{I}_{l1} \cup \mathrm{I}_{l-1,\,2}$.

When $(a,b) \notin \Sigma$, the origin is an isolated critical point of I and hence the critical groups $C_*(I,0)$ are defined. The following partial result on the critical groups was obtained by Dancer [2, 1].

Theorem 1.1. *Let $(a,b) \in Q_l \backslash \Sigma$ and let d_l denote the dimension of the subspace N_l spanned by the eigenfunctions corresponding to $\lambda_1, \cdots, \lambda_l$.*

(i) *If $(a,b) \in \mathrm{I}_l$, then* $C_q(I,0) = \begin{cases} \mathbb{Z}, & q = d_{l-1}, \\ 0, & q \neq d_{l-1}. \end{cases}$

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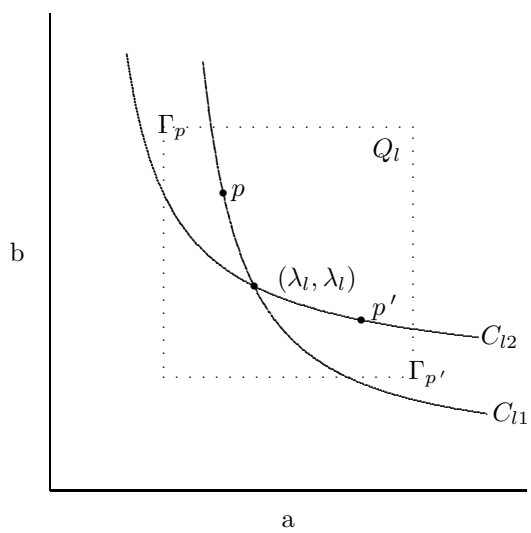
(ii) *If* $(a,b) \in \mathrm{II}_l$, *then* $C_q(I,0) = 0$ *for* $q \le d_{l-1}$ *and for* $q \ge d_l$. *In particular,* $C_q(I,0) = 0$ *for all* q *when* λ_l *is a simple eigenvalue.*

A different proof of Theorem 1.1 based on some ideas developed in Schechter [7, 8] was given in Perera and Schechter [5]. Note that this theorem does not determine $C_q(I,0)$ for $d_{l-1} < q < d_l$ when $(a,b) \in \mathrm{II}_l$ (and λ_l is a multiple eigenvalue). Here we compute these critical groups for some of the points in II_l.

Definition 1.2. For $p \in Q_l \cap C_{l2}$, denote by Γ_p the set of those points $(a,b) \in \mathbb{R}^2 \backslash \Sigma$ for which there is a curve $\gamma = (\gamma_1, \gamma_2) \in C([0,2]) \cap C^1([0,1])$, $\gamma(0) = p$, $\gamma(2) = (a,b)$ such that

(i) $\gamma((0,2]) \cap \Sigma = \emptyset$,
(ii) $\gamma_1', \gamma_2' \le 0$ on $[0,1]$,
(iii) $\gamma(0) + \gamma'(0) = \gamma(1)$.

We can take γ to be the line segment joining $p = (p_1, p_2)$ and (a,b) if it intersects Σ only at p and $a \le p_1$, $b \le p_2$. For example, suppose C_{l1} and C_{l2} intersect only at (λ_l, λ_l) and $\mathrm{II}_l \cap \Sigma = \emptyset$, as in the figure below. Then the region II_l consists of two connected components. The component to the left (resp. right) of (λ_l, λ_l) is contained in Γ_p for each $p \in Q_l \cap C_{l2}$ to the left (resp. right) of (λ_l, λ_l). Also note that $\Gamma_{(\lambda_l,\lambda_l)}$ always contains I_l.



Observe that, by (i) and the homotopy invariance of the critical groups, the $C_*(I(\cdot,\gamma(t)),0)$ are defined for all $t \in (0,2]$ and are independent of t. Let K_p denote the set of critical points of $I_p = I(\cdot,p)$ and let $\widehat{K}_p = \{u \in K_p : \|u\| = 1\}$.

Theorem 1.3. *If* $p \in Q_l \cap C_{l2}$ *and* $(a,b) \in \Gamma_p$, *then*

$$C_q(I,0) \cong \begin{cases} H^{d_l-q-1}(\widehat{K}_p), & q \ne d_l - 1, \\ H^0(\widehat{K}_p)/\mathbb{Z}, & q = d_l - 1. \end{cases} \tag{1.3}$$

Theorem 1.3 is proved in Section 2 and should be compared with Theorem 2 of Dancer [2]. Note that Theorem 1.3 completely determines the critical groups for all $(a,b) \in \mathrm{II}_l$ when the region II_l is free of Σ.

As an application we consider the problem

$$\begin{cases} -\Delta u &= f(x,u) & \text{in } \Omega, \\ u &= 0 & \text{on } \partial\Omega \end{cases} \tag{1.4}$$

where $f \in C(\overline{\Omega} \times \mathbb{R})$ and

$$f(x,t) = \begin{cases} b_0\, t^+ - a_0\, t^- + o(t) & \text{as } t \to 0, \\ b\, t^+ - a\, t^- + o(t) & \text{as } t \to \infty \end{cases} \tag{1.5}$$

with (a_0, b_0), $(a,b) \notin \Sigma$. It was shown in Perera and Schechter [4] that this problem has a nontrivial solution if, for some q,

$$C_q(I_0, 0) \ncong C_q(I, 0) \tag{1.6}$$

where $I_0 = I(\cdot, a_0, b_0)$. By Theorem 1.1, this is the case when $(a_0, b_0) \in \mathrm{I}_j$ and $(a,b) \in \mathrm{I}_l$ with $j \neq l$ and when $(a_0, b_0) \in \mathrm{I}_j$ and $(a,b) \in \mathrm{II}_l$ (or $(a,b) \in \mathrm{I}_l$ and $(a_0, b_0) \in \mathrm{II}_j$). We use Theorem 1.3 to consider the case where $(a_0, b_0) \in \mathrm{II}_j$, $(a,b) \in \mathrm{II}_l$, and λ_l is a multiple eigenvalue. Let $i_q(p)$ denote the right hand side of (1.3) and let $Q = \bigcup_l Q_l$.

Theorem 1.4. *If (a_0, b_0), $(a,b) \notin \Sigma$, then (1.4) has a nontrivial solution in each of the following cases:*

(i) *$(a_0, b_0) \in \Gamma_p$ for some $p \in (Q_l \cap C_{l2}) \setminus C_{l1}$ with $i_q(p) \neq 0$ for some q, and $(a,b) \in Q \setminus \mathrm{II}_l$,*

(ii) *$(a,b) \in \Gamma_p$ for some $p \in (Q_l \cap C_{l2}) \setminus C_{l1}$ with $i_q(p) \neq 0$ for some q, and $(a_0, b_0) \in Q \setminus \mathrm{II}_l$.*

Note that, since $p \notin C_{l1}$ and γ'_1, $\gamma'_2 \leq 0$ on $[0,1]$, $\gamma(t) \in \mathrm{II}_l$ for $t > 0$ sufficiently small, and hence $i_q(p) \neq 0$ implies $d_{l-1} < q < d_l$ by the remark following Definition 1.2 and part (ii) of Theorem 1.1. For example, $i_{d_l - 1}(p) \neq 0$ if $\widehat{K}_p$ is disconnected (this may be caused by a symmetry of Ω, and hence of I_p).

Theorem 1.3 also implies that $H^*(\widehat{K}_{p_1}) \cong H^*(\widehat{K}_{p_2})$ whenever p_1, $p_2 \in Q_l \cap C_{l2}$ and $\Gamma_{p_1} \cap \Gamma_{p_2} \neq \emptyset$. Moreover, it can be combined with Theorem 1.1 to obtain extra information on K_p. Let $M_l = N_l^{\perp}$. It was shown in Schechter [8] that there are continuous and positive homogeneous functions $\tau : N_l \to M_l$, $\theta : M_{l-1} \to N_{l-1}$ such that $w_0 = \tau(v)$, $v_0 = \theta(w)$ are the unique solutions of

$$I_p(v + w_0) = \inf_{w \in M_l} I_p(v + w), \qquad v \in N_l, \tag{1.7}$$

$$I_p(v_0 + w) = \sup_{v \in N_{l-1}} I_p(v + w), \qquad w \in M_{l-1}, \tag{1.8}$$

respectively. Let

$$S_{l1} = \{v + \tau(v) : v \in N_l\}, \tag{1.9}$$

$$S_{l2} = \{\theta(w) + w : w \in M_{l-1}\}, \tag{1.10}$$

$$S_l = S_{l1} \cap S_{l2}. \tag{1.11}$$

We will see in Section 3 that S_l is a radial manifold modeled on the eigenspace $E(\lambda_l)$. Let $\widehat{S}_l = \widehat{S}_l(p) = \{u \in S_l : \|u\| = 1\}$. It follows from Lemmas 3.10 and 3.16 of Schechter [7] that $\widehat{K}_p \subset \widehat{S}_l$. Moreover, by Lemmas 3.6 and 3.7 of Schechter [6], $I_p \leq 0$ on S_{l1} and $K_p = S_{l1} \cap I_p^{-1}(0)$ if $p \in C_{l2}$, while $I_p \geq 0$ on S_{l2} and $K_p = S_{l2} \cap I_p^{-1}(0)$ if $p \in C_{l1}$, so $\widehat{K}_p = \widehat{S}_l$ if $p \in Q_l \cap C_{l1} \cap C_{l2}$. We shall prove

Theorem 1.5. *Let $p \in (Q_l \cap C_{l2}) \backslash C_{l1}$. If $\Gamma_p \neq \emptyset$, then $\widehat{K}_p$ is a proper subset of $\widehat{S}_l$. In particular, $\widehat{K}_p$ is a single point if λ_l is simple.*

As we will see in Section 2, the proof of Theorem 1.3 involves considering $I|_{S_{l1}}$. We suspect that there is a counterpart of this theorem for the lower curve C_{l1}, but the proof seems to be more complicated because the manifold S_{l2} is infinite dimensional. However, we believe that such a result can be obtained by looking at $I|_{S_l}$. In preparation for a future work carrying out this idea, we prove the following theorem in Section 4.

Theorem 1.6. *If $(a,b) \in \mathrm{II}_l \backslash \Sigma$, then*

$$C_q(I,0) \cong \begin{cases} H^{d_l-q-1}(\widehat{S}_l^+), & q \neq d_l - 1, \\ H^0(\widehat{S}_l^+)/\mathbb{Z}, & q = d_l - 1, \end{cases} \tag{1.12}$$

where $\widehat{S}_l^+ = \{u \in \widehat{S}_l(a,b) : I(u) > 0\}$.

Notation. We will use the customary notation

$$I^\alpha := \{u \in H : I(u) \leq \alpha\}, \qquad \alpha \in \mathbb{R}, \tag{1.13}$$

for the sublevel sets.

2. Proof of Theorem 1.3

Let

$$J(u,t) = I(u, \gamma(t)), \qquad u \in H,\ t \in [0,2], \tag{2.1}$$

where γ is as in Definition 1.2. Then $J(\cdot,0) = I_p$, $J(\cdot,2) = I(\cdot,a,b) = I$, and the $C_*(J(\cdot,t),0)$ are independent of $t \in (0,2]$, so

$$C_q(I,0) \cong C_q(\tilde{I},0) \tag{2.2}$$

where $\tilde{I} = J(\cdot,1)$. By (ii), $\gamma(t) \in \mathrm{I}_{l1} \cup \mathrm{II}_l$ for $t > 0$ sufficiently small, so both sides of (1.3) are zero for $q \geq d_l$ by Theorem 1.1, and hence we may assume that $q \leq d_l - 1$.

Fix $\alpha < 0$. Then

$$C_q(\tilde{I},0) \cong H_q(H, \tilde{I}^\alpha) \tag{2.3}$$

since 0 is the only critical point of $\tilde{I}$. By (ii), $J(u,t)$ is nondecreasing in t on $[0,1]$, so $\tilde{I}^\alpha \subset I_p^\alpha \subset I_p^0 \backslash K_p$. We claim that $\tilde{I}^\alpha$ is a strong deformation retract of $I_p^0 \backslash K_p$. To see this let $\eta = \eta(u,t)$ be the flow generated by

$$\begin{cases} \dfrac{d\eta}{dt} = \dfrac{\alpha - I_p(u) - \partial_t J(\eta,t)}{\|\partial_u J(\eta,t)\|^2}\ \partial_u J(\eta,t) \quad \text{for } t \in [0,1], \\[2ex] \eta(u,0) = u \quad \text{on } I_p^0 \backslash (K_p \cup \tilde{I}^\alpha). \end{cases} \tag{2.4}$$

Then

$$\frac{d}{dt} J(\eta,t) = (\partial_u J(\eta,t), \frac{d\eta}{dt}) + \partial_t J(\eta,t) = \alpha - I_p(u), \tag{2.5}$$

so

$$J(\eta(u,t),t) = (1-t)\, I_p(u) + \alpha\, t. \tag{2.6}$$

In particular, $I_p(\eta) = J(\eta, 0) \le J(\eta, t) < 0$ for $t \in (0, 1]$ since $I_p(u) \le 0$, so the flow stays in $I_p^0 \setminus K_p$. Also, $\tilde{I}(\eta(u, 1)) = J(\eta(u, 1), 1) = \alpha$. Moreover, at $t = 0$,

$$\begin{aligned} \alpha - I_p(u) - \partial_t J(\eta, t) &= \alpha - I_p(u) - \partial_t J(u, 0) = \alpha - I(u, p + \gamma'(0)) \\ &= \alpha - \tilde{I}(u) \end{aligned} \tag{2.7}$$

by (iii), so $\eta(u, t) = u$ for all $t \in [0, 1]$ if $\tilde{I}(u) = \alpha$. It follows that η can be continuously extended to $(I_p^0 \setminus K_p) \times [0, 1]$ by setting $\eta(u, t) = u$ on $\tilde{I}^\alpha \times [0, 1]$, defining a strong deformation retraction of $I_p^0 \setminus K_p$ onto $\tilde{I}^\alpha$. Hence

$$H_q(H, \tilde{I}^\alpha) \cong H_q(H, I_p^0 \setminus K_p). \tag{2.8}$$

Since $p \in C_{l2}$,

$$I_p(u) \le 0, \qquad u \in S_{l1}, \tag{2.9}$$

and

$$K_p = \{u \in S_{l1} : I_p(u) = 0\}. \tag{2.10}$$

Moreover, $I_p(v + w)$ is convex in $w \in M_l$ for fixed $v \in N_l$ and $I_p(u + w) > 0$ for $u \in K_p$, $w \in M_l \setminus \{0\}$. Hence the mapping $(I_p^0 \setminus K_p) \times [0, 1] \to I_p^0 \setminus K_p$,

$$(v + w,\, t) \mapsto v + (1 - t)\, w + t\, \tau(v), \tag{2.11}$$

is a strong deformation retraction of the pair $(H, I_p^0 \setminus K_p)$ onto the pair $(S_{l1}, S_{l1} \setminus K_p)$. Since S_{l1} is a radial manifold and I is positive homogeneous, $(S_{l1}, S_{l1} \setminus K_p)$ is homotopic to $(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p)$ where $B_{l2} = \{u \in S_{l1} : \|u\| \le 1\}$ and $\widehat{S}_{l1} = \partial B_{l2}$. Hence

$$H_q(H, I_p^0 \setminus K_p) \cong H_q(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p). \tag{2.12}$$

Combining (2.2), (2.3), (2.8), and (2.12) we have

$$C_q(I, 0) \cong H_q(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p). \tag{2.13}$$

Now consider the exact sequence of the triple $(B_{l2}, \widehat{S}_{l1}, \widehat{S}_{l1} \setminus \widehat{K}_p)$:

$$\begin{aligned} \cdots \longrightarrow H_{q+1}(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p) &\longrightarrow H_{q+1}(B_{l2}, \widehat{S}_{l1}) \longrightarrow H_q(\widehat{S}_{l1}, \widehat{S}_{l1} \setminus \widehat{K}_p) \\ \longrightarrow H_q(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p) &\longrightarrow H_q(B_{l2}, \widehat{S}_{l1}) \longrightarrow \cdots. \end{aligned} \tag{2.14}$$

Since B_{l2} is homeomorphic to the unit ball in N_l,

$$H_q(B_{l2}, \widehat{S}_{l1}) = \begin{cases} \mathbb{Z}, & q = d_l, \\ 0, & q \ne d_l, \end{cases} \tag{2.15}$$

so it follows that

$$H_q(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p) \cong H_q(\widehat{S}_{l1}, \widehat{S}_{l1} \setminus \widehat{K}_p), \qquad q < d_l - 1. \tag{2.16}$$

Since $H_{d_l}(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p) \cong C_{d_l}(I, 0) = 0$, for $q = d_l - 1$ we get the short exact sequence

$$0 \longrightarrow \mathbb{Z} \longrightarrow H_{d_l - 1}(\widehat{S}_{l1}, \widehat{S}_{l1} \setminus \widehat{K}_p) \longrightarrow H_{d_l - 1}(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p) \longrightarrow 0, \tag{2.17}$$

so

$$H_{d_l - 1}(B_{l2}, \widehat{S}_{l1} \setminus \widehat{K}_p) \cong H_{d_l - 1}(\widehat{S}_{l1}, \widehat{S}_{l1} \setminus \widehat{K}_p)/\mathbb{Z}. \tag{2.18}$$

Since

$$H_q(\widehat{S}_{l1}, \widehat{S}_{l1} \setminus \widehat{K}_p) \cong H^{d_l - q - 1}(\widehat{K}_p) \tag{2.19}$$

by the Alexander duality theorem, (1.3) follows from (2.13), (2.16), and (2.18).

3. Proof of Theorem 1.5

Lemma 3.1. *There are continuous and positive homogeneous functions* $\xi : E(\lambda_l) \to N_{l-1},\ \eta : E(\lambda_l) \to M_l$ *such that*

$$S_l = \{\xi(y) + y + \eta(y) : y \in E(\lambda_l)\}. \tag{3.1}$$

Proof. We have to show that for each $y \in E(\lambda_l)$, there is exactly one $u = v + y + w \in N_{l-1} \oplus E(\lambda_l) \oplus M_l$ that is in S_l. Fix $y \in E(\lambda_l)$ and let

$$L(v, w) = I_p(u). \tag{3.2}$$

Then $u \in S_{l1}$ if and only if

$$L(v, w) = \inf_{h \in M_l} L(v, h), \tag{3.3}$$

while $u \in S_{l2}$ if and only if

$$L(v, w) = \sup_{g \in N_{l-1}} L(g, w). \tag{3.4}$$

Thus $u \in S_l$ if and only if

$$L(v, h) \geq \alpha, \quad h \in M_l, \qquad L(g, w) \leq \alpha, \quad g \in N_{l-1}, \tag{3.5}$$

where

$$\alpha = \sup_{g \in N_{l-1}} \inf_{h \in M_l} L(g, h) = \inf_{h \in M_l} \sup_{g \in N_{l-1}} L(g, h). \tag{3.6}$$

Let $u_i = v_i + y + w_i \in S_l,\ i = 1, 2$, so

$$L(v_i, h) \geq \alpha, \quad h \in M_l, \qquad L(g, w_i) \leq \alpha, \quad g \in N_{l-1}. \tag{3.7}$$

In particular

$$L(v_i, w_1) \geq \alpha, \qquad i = 1, 2. \tag{3.8}$$

Since $L(v, w)$ is strictly concave in v, if $v_1 \neq v_2$ this implies

$$L\left((1-t)\, v_1 + t\, v_2, w_1\right) > \alpha, \qquad t \in (0, 1), \tag{3.9}$$

contradicting (3.7). Similarly $w_1 = w_2$. □

We are now ready to prove Theorem 1.5. Since $\Gamma_p \neq \emptyset$ and $p \notin C_{l1}$, using (ii) of Definition 1.2 we can take a point $(a, b) \in \Gamma_p \cap \Pi_l$. Then

$$C_{d_{l-1}}(I, 0) = 0 \tag{3.10}$$

by Theorem 1.1, so

$$i_{d_{l-1}}(p) = 0 \tag{3.11}$$

by Theorem 1.3. Since $\widehat{K}_p \subset \widehat{S}_l$ and $\widehat{S}_l$ is homeomorphic to the $(d_l - d_{l-1} - 1)$-dimensional sphere by Lemma 3.1, this implies that $\widehat{K}_p$ is a proper subset of $\widehat{S}_l$. If λ_l is simple, $\mathrm{II}_l \cap \Sigma = \emptyset$ by Gallouët and Kavian [3], so $\Gamma_p \neq \emptyset$ for any $p \in Q_l \cap C_{l2}$.

4. Proof of Theorem 1.6

Since 0 is the only critical point of I,

$$C_q(I, 0) \cong H_q(H, I^0 \setminus \{0\}). \tag{4.1}$$

Arguments similar to those in the proof of Theorem 1.3 show that (2.11) defines a strong deformation retraction of $(H, I^0 \setminus \{0\})$ onto $(S_{l1}, (I^0 \cap S_{l1}) \setminus \{0\})$ and

$$H_q(S_{l1}, (I^0 \cap S_{l1}) \setminus \{0\}) \cong \begin{cases} H^{d_l - q - 1}(\widehat{S}_{l1}^+), & q \neq d_l - 1, \\ H^0(\widehat{S}_{l1}^+)/\mathbb{Z}, & q = d_l - 1, \end{cases} \tag{4.2}$$

where $\widehat{S}_{l1}^+ = \{u \in \widehat{S}_{l1} : I(u) > 0\}$. For $u \in N_l$, let $\zeta(u) = u + \tau(u)$. Then $\widehat{S}_{l1} = \{\zeta(v + y) : v + y \in N_{l-1} \oplus E(\lambda_l),\ \|\zeta(v + y)\| = 1\}$. We claim that

$$(\zeta(v + y),\, t) \mapsto \frac{\zeta((1 - t)\, v + t\, \xi(y) + y)}{\|\zeta((1 - t)\, v + t\, \xi(y) + y)\|}, \quad \zeta(v + y) \in \widehat{S}_{l1}^+,\ t \in [0, 1], \tag{4.3}$$

where ξ is as in Lemma 3.1, defines a strong deformation retraction of $\widehat{S}_{l1}^+$ onto $\widehat{S}_l^+$. We have to show that

$$I(\zeta((1 - t)\, v + t\, \xi(y) + y)) > 0. \tag{4.4}$$

First note that

$$\begin{aligned} I(\zeta(\xi(y) + y)) &\geq I(v + y + \tau(\xi(y) + y)) \\ &\geq I(\zeta(v + y)). \end{aligned} \tag{4.5}$$

So, since $I(v + y + w)$ is concave in v for any $w \in M_l$,

$$\begin{aligned} I((1 - t)\, v + t\, \xi(y) + y + w) &\geq (1 - t)\, I(v + y + w) + t\, I(\xi(y) + y + w) \\ &\geq (1 - t)\, I(\zeta(v + y)) + t\, I(\zeta(\xi(y) + y)) \\ &\geq I(\zeta(v + y)) > 0, \end{aligned} \tag{4.6}$$

and (4.4) follows. This completes the proof of Theorem 1.6.

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