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PERIODIC SOLUTIONS IN PERIODIC STATE-DEPENDENT DELAY EQUATIONS AND POPULATION MODELS

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ABSTRACT. Sufficient and realistic conditions are obtained for the existence of positive periodic solutions in periodic equations with state-dependent delay. The method involves the application of the coincidence degree theorem and estimations of uniform upper bounds on solutions. Applications of these results to some population models are presented. These application results indicate that seasonal effects on population models often lead to synchronous solutions. In addition, we may conclude that when both seasonality and time delay are present and deserve consideration, the seasonality is often the generating force for the often observed oscillatory behavior in population densities.

1. INTRODUCTION

Frequently, we observe that populations in the real world tend to fluctuate. There are three typical approaches for modeling such behavior: (i) introduce more species into the model, and consider the higher-dimensional systems (like predator-prey interactions, May [8]); (ii) assume that the per capita growth function is time-dependent and periodic in time; (iii) take into account the time delay effect in the population dynamics (Smith and Kuang [9], Zhao et al. [12]). Generally speaking, approach (i) is rather artificial, while (ii) and (iii) emphasize only one aspect of reality. Although all of them are good mechanisms for generating periodic solutions (and therefore offer some explanations to the often observed oscillatory behavior in population densities), it does not give us any insight as to which is the real generating or dominating force behind the oscillatory behavior if only one such mechanism is considered. Naturally, more realistic and interesting models of single species growth should take into account both the seasonality of the changing environment and the effects of time delays. Therefore, it is interesting and important to study the following general nonlinear nonautonomous delayed differential equation:

$$\frac{dx}{dt} = F(t, x(t - \tau(t, x(t)))). \tag{1.1}$$

Here we assume that both F and τ are periodic with the same periodicity. Notice that we allow the delay to be state dependent.

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Existing results on the existence of periodic solutions in periodic systems (population models, in particular) often fall into one of the following three categories: (1) the results of the applications of the contraction principle or fluctuation principle, which establish both the existence and attractivity of the periodic solutions in periodic equations with time delay (Kuang [4, p. 181]); (2) the existence simply follows the observation that the periodic solution exists when there is no time delay and this periodic solution remains so when time delay is a multiple of the period of the periodic equation (Gopalsamy et al. [3], Zhang and Gopalsamy [11]); (3) the results of the application of Horn's asymptotic fixed point theorem (Freedman and Wu [1], Tang and Kuang [10]). While these methods often allow the investigator to address the stability issues of the periodic solutions, the conditions for the existence part are often unnecessarily numerous, tedious, stringent, and difficult to satisfy. Specifically, all of the above methods are ill-suited to problems with state-dependent delay equations.

By employing the powerful and effective coincidence degree method, we found that the existence of periodic solutions in periodic equations with or without state-dependent delay require only a set of natural and easily verifiable conditions. These conditions are readily satisfied in many realistic population models. Such an approach was adopted in Li [5] for a delayed Lotka-Volterra predator-prey model. This strongly suggests that seasonal effects on population models indeed often lead to synchronous solutions. In addition, we may conclude that when both seasonality and time delay are present and deserve consideration, the seasonality is often the generating force for the often observed oscillatory behavior in population densities.

2. Main result

The method to be used in this paper involves the application of the continuous theorem of coincidence degree (Gaines and Mawhin [2, p. 40]). This requires us to introduce a few notations.

Let X, Y be real Banach spaces, $L : \text{dom } L \subset X \to Y$ a Fredholm mapping of index zero (Index $L = \dim\ker L - \text{codim Im } L$) and $P : X \to X$, $Q : Y - Y$ continuous projectors such that $\text{Im } P = \text{Ker } L$, $\text{Ker } Q = \text{Im } L$ and $X = \text{Ker } L \oplus \text{Ker } P$, $Y = \text{Im } L \oplus \text{Im } Q$. Consequently, the restriction Lp of L to $\text{dom } L \cap \text{Ker } P$ is one-to-one and onto $\text{Im } L$, so that its (algebraic) inverse $K_p : \text{Im } L \to \text{dom } L \cap \text{Ker } P$ is defined. Denote by $J : \text{Im } Q \to \text{Ker } L$ an isomorphism of $\text{Im } Q$ onto $\text{Ker } L$.

For convenience we cite the continuous theorem (Gaines and Mawhin [2, p. 40]), as follows.

Theorem A. *Let $\Omega \subset X$ be an open bounded set, and let $N : X \to Y$ be a continuous operator which is L-compact on $\overline{\Omega}$ (i.e., $QN : \overline{\Omega} \to Y$ and $K_p(1-Q)N : \overline{\Omega} \to X$ are compact). Assume*

(a) *for each $\lambda \in (0,1)$, every solution x of $Lx = \lambda Nx$ is such that $x \notin \partial\Omega$,*
(b) *$QNx \neq 0$ for each $x \in \text{Ker}L \cap \partial\Omega$,*
(c) *the Brouwer degree $\deg[JQN, \Omega \cap \text{Ker}L, 0] \neq 0$.*

Then the operator equation $Lx = Nx$ has at least one solution in $\text{dom } L \cap \overline{\Omega}$.

Consider the following state-dependent delay differential equation:

$$\frac{dx}{dt} = -F[t, x(t - \tau(t, x(t)))], \tag{2.1}$$

where $F, \tau \in C(R^2, R)$ and F, τ are ω-periodic with respect to their first arguments, respectively.

Theorem 2.1. *Suppose that in* (2.1)*, there exist constants* B*,* α*,* $\beta > 0$ *such that:*

(I) *When* $|x| \le B$*,* $|F(t,x)| < \beta$ *holds uniformly for* $t \in R$*, and when* $|x| > B$*,* $xF(t,x) > 0$ *holds uniformly for* $t \in R$*.*

(II) *One of the following conditions holds:*

  (i) *when* $x < -B$*,* $F(t,x) \ge -\alpha$ *holds uniformly for* $t \in R$*,*

  (ii) *when* $x > B$*,* $F(t,x) \le \alpha$ *holds uniformly for* $t \in R$*.*

Then equation (2.1) *has at least one* ω*-periodic solution.*

Proof. In order to apply Theorem A to (2.1), we let

$$X = Y = \{x(t) \in C(R,R) \mid x(t+\omega) = x(t)\}$$

and

$$|x|_\infty = \max_{t\in[0,\omega]} |x(t)|.$$

Then X is a Banach space when it is endowed with the uniform norm $|\cdot|_\infty$. Let

$$L : \text{dom } L \cap X \to X, \quad Lx = x', \quad \text{dom } L = \{x(t) \mid x(t) \in X, x(t) \in C^1(R,R)\},$$

$$N : X \to X, \quad Nx = -F[t, x(t - \tau(t, x(t)))].$$

Define projectors P and Q as

$$Px = \frac{1}{\omega}\int_0^\omega x(\tau)d\tau, \quad Qx = \frac{1}{\omega}\int_0^\omega x(t)\, dt, \quad x \in X.$$

It is easy to see that Ker $L = R$ and Im $L = \{x \in X \mid \int_0^\omega x(t)dt = 0\}$ are closed in X and dim Ker L = codim Im $L = 1$. Therefore, L is a Fredholm mapping of index 0. Furthermore, straightforward computation shows that the inverse (to L_p) $K_p : \text{Im } L \to$ Ker $P \cap \text{dom } L$ has the form

$$K_p(x) = \int_0^t x(s)ds - \frac{1}{\omega}\int_0^\omega \int_0^u x(s)\, ds\, du, \quad t \in [0,\omega].$$

Hence,

$$QN : X \to X,$$
$$x \to -\frac{1}{\omega}\int_0^\omega F[t, x(t - \tau(t, x(t)))]dt,$$
$$K_p(I - Q)N : X \to X,$$
$$\begin{aligned} x \quad \to \quad & -\int_0^t F[s, x(s - \tau(s, x(s)))]ds \\ & -\left(\frac{1}{2} - \frac{t}{\omega}\right)\int_0^\omega F[s, x(s - \tau(s, x(s)))]ds \\ & +\frac{1}{\omega}\int_0^\omega \int_0^u F[s, x(s - \tau(s, x(s)))]ds. \end{aligned}$$

Notice that QN and $K_p(I - Q)N$ are continuous by the Lebesgue theorem, and $QN(\overline{\Omega})$ and $K_p(1 - Q)N(\overline{\Omega})$ are relatively compact for any open bounded set $\Omega \subset X$. Hence, N is L-compact on $\overline{\Omega}$ for any open bounded set $\Omega \subset X$. Consider now the equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, which is

$$x'(t) = -\lambda F[t, x(t - \tau(t, x(t)))], \quad \lambda \in (0,1). \tag{2.2}$$

Suppose that $x(t) \in X$ is a solution of Eq. (2.2) for a certain $\lambda \in (0,1)$. Integrating Eq. (2.2) over the interval $[0,\omega]$, we obtain

$$\int_0^\omega F[t, x(t-\tau(t,x(t)))]\,dt = 0. \tag{2.3}$$

In what follows, without loss of generality, we assume that (i) of condition (II) holds. Denote by $E_1 = \{t \in [0,\omega] \mid x(t-\tau(t,x(t))) > B\}$, $E_2 = \{t \in [0,\omega] \mid x(t-\tau(t,x(t))) < -B\}$ and $E_3 = \{t \in [0,\omega] \mid |x(t-\tau(t,x(t)))| \leq B\}$. Then it follows from (2.3), condition (I) and (i) of condition (II) that

$$\begin{aligned} \int_{E_1} F[t, x(t-\tau(t,x(t)))]dt &= -\int_{E_2} F[t, x(t-\tau(t,x(t)))]dt \\ &\quad -\int_{E_3} F[t, x(t-\tau(t,x(t)))]dt \\ &< \alpha\omega + \beta\omega. \end{aligned}$$

Therefore

$$\begin{aligned} \int_0^\omega |F[t, x(t-\tau(t,x(t)))]|dt &= \int_{E_1} |F[t, x(t-\tau(t,x(t)))]|dt \\ &\quad + \int_{E_2} |F[t, x(t-\tau(t,x(t)))]|dt + \int_{E_3} |F[t, x(t-\tau(t,x(t)))]|dt \\ &= \int_{E_1} F[t, x(t-\tau(t,x(t)))]dt - \int_{E_2} F[t, x(t-\tau(t))]dt \\ &\quad + \int_{E_3} |F[t, x(t-\tau(t,x(t)))]|dt \\ &< (\alpha\omega + \beta\omega) + \alpha\omega + \beta\omega \stackrel{\text{def}}{=} M. \end{aligned} \tag{2.4}$$

Moreover, in view of (2.3) and condition (I), one can see that there exists a point $t^* \in [0,\omega]$ such that

$$|x(t^* - \tau(t^*, x(t^*)))| \leq B.$$

Hence there is a $t_0 \in [0,\omega]$ such that $t^* - \tau(t^*, x(t^*)) = t_0 + n\omega$, where n is an integer. Then we have

$$|x(t_0)| \leq B. \tag{2.5}$$

By (2.2) and (2.4) it follows that

$$\begin{aligned} \int_0^\omega |x'(t)|dt &\leq \lambda \int_0^\omega |F[t, x(t-\tau(t,x(t)))]|dt \\ &< M_1. \end{aligned} \tag{2.6}$$

Therefore, from (2.4), (2.5) and (2.6), we obtain that

$$\begin{aligned} |x(t)| &\leq |x(t_0)| + \int_0^\omega |x'(t)|dt \\ &< B + M. \end{aligned}$$

Now we let $\Omega = \{x(t) \in X \mid |x|_\infty < B + M\}$. It is clear that Ω verifies requirement (a) in Theorem A.

Assume now that $x \in \partial\Omega \cap \text{Ker } L = \partial\Omega \cap R$. Then x is a constant with $|x| = B + M$. By the second part of condition (I) we see that

$$QNx = -\frac{1}{\omega}\int_0^\omega F(t,x)dt \neq 0.$$

Furthermore, let $J = I : \text{Im } Q \to \text{Ker } L$, $x \to x$, and

$$H(\gamma, x) = -(1-\gamma)x + \gamma JQNx, \quad \gamma \in [0,1].$$

Since for $x \in \partial\Omega \cap \text{Ker } L$, we have

$$xH(\gamma, x) = -(1-\gamma)x^2 - \frac{\gamma}{\omega}\int_0^\omega xF(t,x)dt < 0,$$

it follows that

$$H(\gamma, x) \neq 0, \quad x \in \partial\Omega \cap \text{Ker } L, \quad \gamma \in (0,1].$$

According to the invariance of homotopy, we obtain

$$\begin{aligned} \deg\,[JQN, \text{Ker } L \cap \Omega, 0] &= \deg\,[I, \text{Ker } L \cap \Omega, 0] \\ &\neq 0. \end{aligned}$$

By Theorem A, we have completed the proof of Theorem 2.1. □

Theorem 2.2. *Suppose that the assumptions of Theorem 2.1 hold. Then the equation*

$$\frac{dx}{dt} = F[t, x(t - \tau(t, x(t)))]$$

has at least one ω-periodic solution, where $F, \tau \in C(R^2, R)$ and F, τ are ω-periodic with respect to their first arguments, respectively.

Proof. The proof is similar to the proof of Theorem 2.1 and will be omitted. □

Consider the following state-dependent delay differential equations:

$$\frac{dx(t)}{dt} = \pm x(t)G[t, x(t - \tau(t, x(t)))], \tag{2.7}$$

where $G, \tau \in C(R^2, R)$ and G, τ are ω-periodic with respect to their first arguments, respectively. By Theorems 2.1 and 2.2, we have

Theorem 2.3. *Suppose that in* (2.7) *there exist constants B, α, $\beta > 0$ such that:*

(I) *When $|x| \leq B$, $|G(t, e^x)| \leq \beta$ holds uniformly for $t \in R$, and when $|\gamma| > B$, $xG(t, e^x) > 0$ holds uniformly for $t \in R$.*

(II) *One of the following conditions holds:*
   (i) *when $x < -B$, $G(t, e^x) > -\alpha$ holds uniformly for $t \in R$,*
   (ii) *when $x > B$, $G(t, e^x) < \alpha$ holds uniformly for $t \in R$.*

Then equation (2.7) *has at least one positive ω-periodic solution.*

Proof. Consider the equation

$$\frac{dy(t)}{dt} = G[t, e^{y(t - \tau(t, e^{y(t)}))}], \tag{2.8}$$

where G, τ are the same as those in Eq. (2.7) (with + sign). Set

$$G[t, e^{y(t-\tau(t, e^{y(t)}))}] \equiv F[t, y(t - \tau(t, y(t)))].$$

Then it is easy to see that $F(t,y)$ satisfies all the conditions of Theorem 2.2. Hence, according to Theorem 2.2, we see that Eq. (2.8) has at least one ω-periodic solution $y^*(t)$. Set

$$x^*(t) = e^{y^*(t)}.$$

Then one can see that $x^*(t)$ is a positive ω-periodic solution of Eq. (2.7). In a similar way, by using Theorem 2.1, one can prove that Eq. (2.7) has a positive ω-periodic solution when the negative sign is selected. The proof is complete. □

Next consider the following distributed delay differential equations:

$$\frac{dx}{dt} = \pm F\left[t, \int_{-\tau}^{0} x(t+\theta)d\eta(\theta)\right], \tag{2.9}$$

where $F \in C(R^2, R)$, F is ω-periodic with respect to its first argument, τ is a positive constant and η is a nondecreasing function such that $\eta(0^+) - \eta(-\tau^-) = 1$.

Theorem 2.4. *Under the assumptions of Theorem 2.1, Eq.* (2.9) *has at least one ω-periodic solution.*

Proof. The proof is entirely similar to the proofs of Theorems 2.1 and 2.2 and will be omitted. □

The following corollary of Theorem 2.4 can be easily established.

Corollary 1. *Under the assumptions of Theorem 2.3, both of the equations*

$$\frac{dx(t)}{dt} = \pm x(t)G\left[t, \int_{-\tau}^{0} x(t+\theta)d\eta(\theta)\right] \tag{2.10}$$

have at least one positive ω-periodic solution, where $G \in C(R^2, R)$, G is ω-periodic with respect to its first argument, τ is a positive constant and η is a nondecreasing function such that $\eta(0^+) - \eta(-\tau^-) = 1$.

Proof. The proof is similar to that of Theorem 2.3 and we shall omit it. □

3. Applications

As pointed out in the introduction, the main motivation of this work is to establish sufficient conditions for the existence of periodic solutions in periodic population models with state-dependent delay. The following examples will accomplish this.

Example 1. Consider the state-dependent delay logistic equation

$$\frac{dN(t)}{dt} = \gamma(t)N(t)\left[1 - \frac{N(t-\tau(t,N(t)))}{K(t)}\right], \tag{3.1}$$

and the distributed delay logistic equation

$$\frac{dN(t)}{dt} = \gamma(t)N(t)\left[1 - \frac{\int_{-\sigma}^{0} x(t+\theta)d\eta(\theta)}{K(t)}\right], \tag{3.2}$$

where $\gamma, K \in C(R,R)$, γ, K are positive ω-periodic functions, $\tau \in C(R^2, R)$, τ is ω-periodic with respect to its first argument, σ is a positive constant and η is a nondecreasing function satisfying $\eta(0^+) - \eta(-\sigma^-) = 1$.

From Theorem 2.3 and Corollary 1 it follows that

Theorem 3.1. *Both Eq.* (3.1) *and Eq.* (3.2) *have at least one positive ω-periodic solution.*

Proof. Set

$$G[t,u] = -\gamma(t)\left[1 - \frac{u}{K(t)}\right]$$

and take $B = \max\{\ln(|K|_\infty), 1\} + 1$, $\alpha = |\gamma|_\infty$, $\beta = |\gamma|_\infty(1 + (e^B/K_m))$, where $K_m = \min_{t\in[0,\omega]} K(t)$. Then one can see that $G(t, e^x)$ satisfies conditions (I) and (II)(i) of Theorem 2.3. Hence, according to Theorem 2.3, we see that Eq. (3.1) has at least one positive ω-periodic solution. According to Corollary 1 we see that Eq. (3.2) has at least one positive ω-periodic solution. The proof is complete. □

Example 2. Consider the state-dependent delay generalized food limited model

$$\frac{dN(t)}{dt} = r(t)N(t)\left[\frac{K(t) - N(t-\tau(t,N(t)))}{K(t) + c(t)\gamma(t)N(t-\tau(t,N(t)))}\right]^\theta, \tag{3.3}$$

and the distributed delay generalized food limited model

$$\frac{dN(t)}{dt} = r(t)N(t)\left[\frac{K(t) - \int_{-\sigma}^0 N(t+\theta)d\eta(\theta)}{K(t) + c(t)\gamma(t)\int_{-\sigma}^0 N(t+\theta)d\eta(\theta)}\right], \tag{3.4}$$

where $r, K, c \in C(R,R)$, γ, K, c are positive ω-periodic functions, $\tau \in C(R^2, R)$, τ is ω-periodic with respect to its first argument, θ is a positive odd integer, σ is a positive constant and η is a nondecreasing function satisfying $\eta(0^+) - \eta(-\sigma^-) = 1$.

Theorem 3.2. *Both Eq.* (3.3) *and Eq.* (3.4) *have at least one positive ω-periodic solution.*

Proof. Set

$$G(t,u) = -\gamma(t)\left[\frac{K(t) - u}{K(t) + c(t)\gamma(t)u}\right]^\theta,$$

and take $B = \max\{\ln(|K|_\infty), 1\} + 1$, $\alpha = |\gamma|_\infty$, $\beta = |\alpha/K|_\infty e^B + |\gamma|_\infty$. Then it is easy to see that $G(t, e^x)$ satisfies conditions (I) and (II)(i) of Theorem 2.3. Therefore, by Theorem 2.3 and Corollary 1, we see that both Eq. (3.3) and Eq. (3.4) have at least one positive ω-periodic solution. This completes the proof. □

Example 3. Consider the following one-species model that exhibits the so-called Allee effect:

$$\begin{aligned}\frac{dN(t)}{dt} &= N(t)[a(t) + b(t)N(t-\tau(t,N(t))) \\ &\quad -c(t)N^2(t-\tau(t,N(t)))]\end{aligned} \tag{3.5}$$

and

$$\begin{aligned}\frac{dN(t)}{dt} &= N(t)\Bigg[a(t) + b(t)\int_{-\sigma}^0 N(t+\theta)d\eta(\theta) \\ &\quad -c(t)\left(\int_{-\sigma}^0 N(t+\theta)d\eta(\theta)\right)^2\Bigg],\end{aligned} \tag{3.6}$$

where $a, b, c \in C(R,R)$, a, b, c are ω-periodic functions, $a, c > 0$, $\tau \in C(R^2, R)$, τ is ω-periodic with respect to its first argument, σ is a positive constant and η is a nondecreasing function satisfying $\eta(0^+) - \eta(-\sigma^-) = 1$.

Theorem 3.3. *Both Eq.* (3.5) *and Eq.* (3.6) *have at least one positive ω-periodic solution.*

Proof. Set

$$G(t,u) = -a(t) - b(t)u + c(t)u^2;$$

then

$$G(t,e^x) = -a(t) - b(t)e^x + c(t)e^{2x}.$$

Since $\lim_{x\to+\infty} G(t,e^x) = +\infty$ uniformly for $t \in R$ and $\lim_{x\to-\infty} G(t,e^x) = -a(t) < 0$ uniformly for $t \in R$, one can see that there exist constants $B, \alpha, \beta > 0$ such that $G(t,e^x)$ satisfies conditions (I) and (II)(i) of Theorem 2.3. Therefore, we see that the conclusion of Theorem 3.3 is true. The proof is complete. □

4. Discussion

It is easy to see that the continuation theorem of coincidence degree theory is an effective tool for establishing the existence of periodic solutions in periodic equations with certain dissipativity (such solutions are eventually uniformly bounded). It is conceivable that it can be applied to certain systems of delay differential equations as well. This was indeed carried out for some periodic Lotka-Volterra type systems with distributed or state-dependent delays in Li and Kuang [6] and for a delayed Gause-type predator-prey system in Li and Kuang [7]. More applications in population dynamics are expected as they normally possess some regulatory mechanism which mathematically ensures the necessary dissipativity.

A more abstract generalization of our work may be realized by the utilization of some Lyapunov type functional that can be used to gain suitable dissipativity. For an example, consider a delayed differential system

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(t, \mathbf{x}(t - \tau(t, \mathbf{x}(t)))). \tag{4.1}$$

Here $\mathbf{x}$ and $\mathbf{F}$ are vector functions and we assume that both $\mathbf{F}$ and τ are periodic with the same periodicity. Notice that we allow the delay to be state dependent. Let $H(\mathbf{x_t})$ be a suitable functional (for example: $H(\mathbf{x}) = \sum_{i=1}^n a_i x_i$ or $H(\mathbf{x}) = \sum_{i=1}^n (a_i x_i - b_i \ln(x_i)))$ such that

$$d(H(\mathbf{x_t}))/dt|_{(4.1)} \cdot \mathbf{F}(t, \mathbf{x}(t - \tau(t, \mathbf{x}(t)))) \le 0$$

for large $||\mathbf{x}||$. This could lead to some useful bounds on solutions of the system. Of course, one still needs certain specific structures from the system (4.1) to allow successful implementation of the continuation theorem of coincidence degree theory.

To conclude this paper, we would like to point out that we have only established sufficient conditions for the existence of periodic solutions of the same period (intrinsic period) of the model and provided no statement regarding their stabilities. For some very special cases, they can actually be globally attractive (Zhang and Gopalsamy [11]). However, we do not expect them to be globally attractive in general, since for autonomous models with large delays, locally stable periodic solutions with large period (several times of delay length) may exist (such as the slowly oscillating periodic solutions reported in Zhao et al. [12] and the references cited there). With small and appropriate periodic perturbations, it is conceivable that such types of periodic solutions may persist while a positive steady state may give rise to a periodic solution of the perturbation period (intrinsic period). This suggests that, in general, it is not true that all solutions tend to some periodic solutions of intrinsic period.

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