ON THE COMPLETENESS OF FACTOR RINGS

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(Communicated by Wolmer V. Vasconcelos)

Abstract. Let $T$ be a complete local domain containing the integers with maximal ideal $M$ such that $|T/M|$ is at least the cardinality of the real numbers. Let $p$ be a nonmaximal prime ideal of $T$ such that $T_p$ is a regular local ring. We construct an excellent local ring $A$ such that the completion of $A$ is $T$, the generic formal fiber of $A$ is local with maximal ideal $p$ and if $I$ is a nonzero ideal of $A$, then $A/I$ is complete.

1. Introduction

Let $A$ be a local ring with maximal ideal $M$ and $M$-adic completion $\hat{A}$. If $A$ is not complete, and has dimension at least two, the “usual” case is that there are many nonzero ideals $I$ of $A$ such that $A/I$ is not complete. In this paper, we construct rings that do not satisfy this “usual” condition. In other words, we are interested in noncomplete rings $A$ such that $A/I$ is complete for every nonzero ideal $I$ of $A$.

In fact, examples of such rings are known to exist. In [9], it is shown that noncomplete excellent regular local rings $R_s$ exist satisfying

$$\dim R_s = s + 1$$

and for any nonzero ideal $I$ of $R_s$, the ring $R_s/I$ is complete. In addition, $R_s$ satisfies the following property: If $q$ is a nonzero prime ideal of $R_s$, then $q \cap R_s \neq (0)$. In other words, the dimension of the generic formal fiber of $R_s$ is zero. (For definitions and background information on the formal fibers and generic formal fiber of an integral domain, see [8].) Also, in [1], S. Abhyankar, W. Heinzer, and S. Wiegand construct a two-dimensional normal local domain $S$ such that $S$ is not Henselian but $\hat{S}$ is Henselian for every height one prime ideal $P$ of $S$. In fact, one can show (using Weierstrass Preparation) that $\hat{S}$ is complete for every nonzero ideal $I$ of $S$. This ring $S$ satisfies the property that if $q$ is a nonzero prime ideal of $\hat{S}$, then $q \cap S \neq (0)$. So, once again, for this example, the dimension of the generic formal fiber is zero. As far as the authors know, these are the only known examples of noncomplete
local rings $A$ of dimension at least two satisfying the property that $A/I$ is complete for every nonzero ideal $I$ of $A$. In light of this, we ask the natural question: Let $A$ be a local integral domain such that $A/I$ is complete for all nonzero ideals $I$ of $A$. Does it follow that the dimension of the generic formal fiber of $A$ is zero?

In this paper, we construct examples showing that the answer to this question is no. Let $T$ be a complete local domain with maximal ideal $M$ containing the integers and such that $|T/M|$ is at least the cardinality of the real numbers. Let $p$ be a nonmaximal prime ideal of $T$ such that $T_p$ is a regular local ring. We construct an excellent local ring $A$ such that the completion of $A$ is $T$, the generic formal fiber of $A$ is local with maximal ideal $p$ and if $I$ is a nonzero ideal of $A$, then $A/I$ is complete. So, given $n \geq 2$ and $0 \leq t < n$, we can construct an excellent local ring $A$ with $\dim A = n$ and $A/I$ complete for all nonzero ideals $I$ of $A$. In addition, if, for example, $T$ is a regular local ring, $p$ can be any nonmaximal prime ideal of $T$. Thus, by choosing the height of $p$ to be $t$, we can ensure that the dimension of the generic formal fiber of $A$ is $t$.

The fact that $A$ is a noncomplete excellent local ring possessing a local generic formal fiber is interesting. In [4], it is shown that a certain class of complete regular local rings can be realized as the completion of a noncomplete excellent local ring possessing a local generic formal fiber. Theorem 7 in this paper (thanks to suggestions by the referee) shows that a larger class of complete local domains can be realized as the completion of a noncomplete excellent local ring possessing a local generic formal fiber.

We note here that it can be important to know which ideals $I$ of a ring $A$ satisfy that $A/I$ is complete. For example, in [3], Heinzer, Rotthaus and Sally show that if “enough” prime ideals of $A$ satisfy this condition, then there is a one-to-one correspondence between the prime ideals in $A$ that are maximal in the generic formal fiber of $A$ and certain birational extensions of $A$. (See [3] for details.)

All rings in this paper are commutative with unity. When we say a ring is local, Noetherian is implied. For a ring with exactly one maximal ideal that is not necessarily Noetherian, we will use the term quasi-local. When we write $(T, M)$ is a local ring, we mean that $T$ is a local ring with maximal ideal $M$. Following Matsumura in [3], we will use $\alpha(A)$ to denote the dimension of the generic formal fiber of a local integral domain $A$. Finally, we will use $c$ to denote the cardinality of the real numbers.

The construction of the ring $A$ in our theorem is simply a refinement of the construction used in [6]. We make use of the result found in [5] that if $(R, M \cap R)$ is a quasi-local subring of a complete local ring $(T, M)$, the map $R \to T/M^2$ is onto and $IT \cap R = I$ for every finitely generated ideal $I$ of $R$, then $R$ is Noetherian and the natural homomorphism $\bar{R} \to T$ is an isomorphism. Given a complete local domain $(T, M)$ and a prime ideal $p$ of $T$, both satisfying the conditions of our theorem, we start with the prime subring of $T$. We then build a chain of subrings of $T$ (starting with the prime subring of $T$) with each member of the chain satisfying some “nice” properties. These rings will be called $p$-subrings and defined later. Our ring $A$ will be the union of this chain of subrings. Clearly, then, we must choose our rings in the chain carefully. Specifically, we must ensure that $IT \cap A = I$ for every finitely generated ideal $I$ of $A$ and that $A \to T/M^2$ is onto. These conditions will guarantee that the completion of $A$ is $T$. In addition, we must have that $p \cap A = (0)$ and if $q$ is a prime ideal of $T$ not contained in $p$, then $q \cap A \neq (0)$. Finally, we must choose the chain so that $A/I$ is complete for every nonzero ideal $I$ of $A$. 


2. The construction

As discussed in the previous section, the ring A of our theorem will be the union of a chain of subrings of T. We use the following proposition from [3] to guarantee that the completion of A is, in fact, T.

**Proposition 1.** If \((R, M \cap R)\) is a quasi-local subring of a complete local ring \((T, M)\), the map \(R \longrightarrow T/M^2\) is onto and \(IT \cap R = I\) for every finitely generated ideal I of R, then R is Noetherian and the natural homomorphism \(R \longrightarrow T\) is an isomorphism.

The following definition is taken from [6]. In our construction, we will want to ensure that each ring in our chain satisfies the conditions needed to be a \(p\)-subring.

**Definition.** Let \((T, M)\) be a complete local ring and \((R, R \cap M)\) a quasi-local subring. Let \(p \neq M\) be a prime ideal of T. Suppose:

(i) \(|R| \leq \sup (\aleph_n, |T/M|)\) with equality implying \(T/M\) is countable,

(ii) \(R \cap p = (0)\), and

(iii) \(R \cap Q = (0)\) for every \(Q \in \text{Ass } T\).

Then we call R a \(p\)-subring of T.

Condition (i) will be used so that R is “small” compared to T. This will allow us to choose elements in T that satisfy certain transcendental properties over R. Adjoining these elements to R is the process we will use to build our chain of subrings. Condition (ii) is needed so that \(p\) will be in the generic formal fiber of A. When T is an integral domain, condition (iii) is trivial. However, all our results except for Theorem 1 hold when T is not an integral domain and so we prove them as such. In this case, we will use condition (iii) to guarantee that A will be an integral domain.

The following lemma is Lemma 2 from [3]. Note that it is really a generalization of the prime avoidance lemma.

**Lemma 2.** Let \((T, M)\) be a local ring. Let \(C \subset \text{Spec } T\), let I be an ideal such that \(I \not\subset P\) for every \(P \in C\), and let D be a subset of T. Suppose \(|C \times D| < |T/M|\). Then \(I \not\subset \bigcup \{P + r \mid P \in C, r \in D\}\).

Recall that to use Proposition 1, we need the map \(A \longrightarrow T/M^2\) to be onto. In addition, we want \(A/I\) to be complete for every nonzero ideal I of A. Note that if T is, in fact, the completion of A where I is a nonzero ideal of A and \(A \longrightarrow T/IT\) is onto, then \(A/I\) will be complete. To satisfy this condition and ensure that \(A \longrightarrow T/M^2\) is onto, we will construct A so that \(A \longrightarrow T/J\) is onto for every ideal J of T that is not contained in the ideal p. It turns out that this condition will also help us show that the ring A we construct is excellent. If J is an ideal of T not contained in p, and \(\bar{u} \in T/J\), we use the following lemma to construct an appropriate \(p\)-subring of T that contains an element of \(\bar{u}\). This lemma will be used later to guarantee that \(A \longrightarrow T/J\) is onto.

**Lemma 3.** Let \((T, M)\) be a complete local ring with \(|T/M| \geq c\), \(p\) a nonmaximal prime ideal of T and J an ideal of T with \(J \not\subset p\). Suppose \(p\) contains all associated prime ideals of T. Let R be a \(p\)-subring of T and \(\bar{u} \in T/J\). Then there exists a \(p\)-subring S of T such that \(R \subset S \subset T\) and \(\bar{u}\) is in the image of the map \(S \longrightarrow T/J\). Moreover, if \(\bar{u} = 0\), then \(S \cap J \neq (0)\).
Proof. Let \( C = \text{Ass}T \cup \{ p \} \) and suppose \( P \in C \). Define \( D_{(p)} \) to be a full set of coset representatives \( t + P \) that make \((u + t) + P\) algebraic over \( R \) as an element of \( T/P \). Now, \(|D_{(p)}| = \max(|R|, n_0)\). As \(|T/M| \geq c\), and \( R \) is a \( p \)-subring, we have \(|R| < |T/M|\). So, \(|D_{(p)}| < |T/M|\). Note that by hypothesis, \( J \not\subset P \) for every \( P \in C \). Let \( D = \bigcup_{P \in C} D_{(p)} \) and note that \(|C \times D| < |T/M|\). Now, use Lemma 2 to find an element \( x \in J \) such that \( x \not\in \bigcup\{r + P | r \in D \text{ and } P \in C\} \). We claim that \( S = R[x + u] \) localized at \( R[x + u] \cap M \) is the desired \( p \)-subring.

Since \(|R| < |T/M|\) and \(|S| = \max(|R|, n_0)\), we have \(|S| < |T/M|\). Hence, condition (i) of \( p \)-subrings is satisfied. Now, suppose that \( P \in C \). Then if \( f \in R[u + x] \cap P \), we have

\[
f = r_n(u + x)^n + \cdots + r_1(u + x) + r_0 \in P.
\]

So, \( f \equiv 0 \) modulo \( P \). By the way \( x \) was chosen, we have that \( r_i \equiv 0 \) modulo \( P \) for every \( i \). So, \( r_i \in P \cap R = (0) \). Hence, \( S \cap P = (0) \) and it follows that \( S \) is a \( p \)-subring. Note that under the map \( S \rightarrow T/J\), \( u + x \) is mapped to \( u + J \), so \( u + J \) is in the image of the map. Also, note that if \( \bar{u} = 0 \), then \( u + x \in J \). Since \((u + x) + p\) is transcendental over \( R \) as an element of \( T/p \), we have \( u + x \not\equiv 0 \). It follows that \( S \cap J \neq (0) \).

Lemma 4 is Lemma 6 from [10]. It will be needed to ensure that \( IT \cap A = I \) for every finitely generated ideal \( I \) of \( A \). This property, of course, is needed to apply Proposition 1.

Lemma 4. Let \((T, M)\) be a complete local ring, \( p \) a prime ideal of \( T \), and \( R \) a \( p \)-subring. Suppose \( I \) is a finitely generated ideal of \( R \) and \( c \in IT \cap R \). Then there exists a \( p \)-subring \( S \) of \( T \) with \( R \subset S \subset T \) and \( c \in IS \).

Definition. Let \( \Omega \) be a well-ordered set and \( \alpha \in \Omega \). We define \( \gamma(\alpha) = \sup\{\beta \in \Omega | \beta < \alpha\} \).

In Lemma 5, we construct a \( p \)-subring that simultaneously satisfies many of the properties we desire.

Lemma 5. Let \((T, M)\) be a complete local ring with \(|T/M| \geq c \) and \( p \) a nonmaximal prime ideal of \( T \). Let \( J \) be an ideal of \( T \) with \( J \not\subset p \) and suppose all associated prime ideals of \( T \) are contained in \( p \). Let \( \bar{u} \in T/J \). Suppose \( R \) is a \( p \)-subring of \( T \). Then there exists a \( p \)-subring \( S \) of \( T \) such that:

(i) \( R \subset S \subset T \),
(ii) if \( \bar{u} = 0 \), then \( S \cap J \neq (0) \),
(iii) \( u \) is in the image of the map \( S \rightarrow T/J \),
(iv) for every finitely generated ideal \( I \) of \( S \), we have \( IT \cap S = I \).

Proof. Use Lemma 4 to find a \( p \)-subring \( R_0 \) such that \( R \subset R_0 \subset T \) and \( \bar{u} \in \text{Image}(R_0 \rightarrow T/J) \) and if \( \bar{u} = 0 \), then \( R_0 \cap J \neq (0) \). The rest of the proof follows the proof of Lemma 12 in [10], but the proof is not long and so we include it here.

We will construct \( S \) to contain \( R_0 \), so conditions (i), (ii) and (iii) will follow automatically. Let

\[
\Omega = \{(I, c) | I \text{ a finitely generated ideal of } R_0 \text{ and } c \in IT \cap R_0\}.
\]

Now, since \( I \) can be \( R_0 \), we have \(|R_0| \leq |\Omega|\), and \(|\Omega| \leq |R_0|\) is clear since the number of finite subsets of \( R_0 \) is \(|R_0|\). So, \(|\Omega| = |R_0|\). Well-order \( \Omega \) so that it does not have a maximal element and let \( 0 \) denote its initial element. Now, we
will define a family of \( p \)-subrings. Begin with \( R_0 \). If \( \gamma(\alpha) \neq \alpha \) and \( \gamma(\alpha) = (I, c) \), then choose \( R_\alpha \) to be the \( p \)-subring gotten from Lemma 4 so that \( R_\gamma(\alpha) \subset R_\alpha \subset T \) and \( c \in IR_\alpha \). If \( \gamma(\alpha) = \alpha \), choose \( R_\alpha = \bigcup_{\beta < \alpha} R_\beta \). Set \( R_1 = \bigcup R_\alpha \). Now, if \( I \) is any finitely generated ideal of \( R_0 \), and \( c \in IT \cap R_0 \), then \( (I, c) = \gamma(\alpha) \) for some \( \alpha \in \Omega \). So, \( c \in IR_\alpha \subset IR_1 \). Thus, \( IT \cap R_0 \subset IR_1 \). It is easy to verify that \( R_1 \) is a \( p \)-subring.

We repeat the process to obtain a \( p \)-subring extension \( R_2 \) of \( R_1 \) such that \( IT \cap R_1 \subset IR_2 \) for every finitely generated ideal \( I \) of \( R_1 \). Continue to obtain an ascending chain \( R_0 \subset R_1 \subset R_2 \subset \cdots \) such that \( IT \cap R_n \subset IR_{n+1} \) for every finitely generated ideal \( I \) of \( R_n \). Then, \( S = \bigcup R_i \) is a \( p \)-subring. If \( I \) is a finitely generated ideal of \( S \), then some \( R_n \) contains \( I \) as a \( p \)-subring.

Proof. Define \( A = T \).

(ii) If \( P \) is a nonzero prime ideal of \( A \), then \( T \otimes_A k(P) \) is a field where \( k(P) = A_p/PA_p \).

(iii) The generic formal fiber of \( A \) is local with \( p \) the maximal ideal.

(iv) If \( I \) is a nonzero ideal of \( A \), then \( A/I \) is complete.

Proof. Define \( \Omega = \{ u + J \in T/J \mid J \text{ is an ideal of } T \text{ with } J \not\subset p \} \).

Since \( T \) is Noetherian, if \( J \) is an ideal of \( T \), then \( J \) is finitely generated. Hence, \(|\{J \text{ ideal of } T/J \not\subset p\}| \leq |T| \). But since \(|T/M| \geq c \), we have \(|T| = |T/M| \). Also, \(|T/M| \leq |T/J| \leq |T| \), and so \(|T/J| = |T/M| \) for every ideal \( J \) of \( T \). It follows that \( |\Omega| = |T/M| \). Well-order \( \Omega \) so that each element has fewer than \( |\Omega| \) predecessors.

Let \( 0 \) denote the first element of \( \Omega \). We define \( R_0 \) to be the prime subring of \( T \) and \( R_0 \) to be \( R_0 \) localized at \( R_0 \cap M \). Note that \( R_0 \) is a \( p \)-subring.

We recursively define a family of \( p \)-subrings as follows. \( R_0 \) is already defined.

Let \( \lambda \in \Omega \) and assume \( R_\beta \) has been defined for every \( \beta < \lambda \). Then \( \gamma(\lambda) = u + J \) for some ideal \( J \) of \( T \) with \( J \not\subset p \). If \( \gamma(\lambda) < \lambda \), use Lemma 4 to obtain a \( p \)-subring \( R_\lambda \) so that \( R_\gamma(\lambda) \subset R_\lambda \subset T \), \( u + J \in I \text{mage}(R_\lambda \to T/J) \) and for every finitely generated ideal \( I \) of \( R_\lambda \) we have \( IT \cap R_\lambda = I \). Moreover, if \( u + J = 0 + J \), we have \( R_\lambda \cap J \neq (0) \). If \( \gamma(\lambda) = \lambda \), define \( R_\lambda = \bigcup_{\beta < \lambda} R_\beta \). Then, \( R_\lambda \) is a \( p \)-subring for every \( \lambda \in \Omega \). We claim \( A = \bigcup_{\lambda \in \Omega} R_\lambda \) is the desired domain.

Now, as each \( R_\lambda \) is a \( p \)-subring, we have \( R_\lambda \cap p = (0) \). Hence, \( A \cap p = (0) \). Also, if \( J \) is an ideal of \( T \) with \( J \not\subset p \), then \( 0 + J \in \Omega \). So, \( \gamma(\lambda) = 0 + J \) for some \( \lambda \in \Omega \)
with $\gamma(\lambda) < \lambda$. By construction, $R_\lambda \cap J \neq (0)$. It follows that $J \cap A \neq (0)$. Hence, the generic formal fiber of $A$ is local with maximal ideal $p$.

We will now show that the completion of $A$ is $T$. To do this, we make use of Proposition 6. Note that as $p$ is not the maximal ideal of $T$, we have $M^2 \not\subset p$. Hence, by the construction, the map $A \to T/M^2$ is surjective. Let $I$ be a finitely generated ideal of $A$ with $I = (y_1, \ldots, y_k)$. Let $c \in IT \cap A$. Then $\{c, y_1, \ldots, y_k\} \subset R_\lambda$ for some $\lambda \in \Omega$ with $\gamma(\lambda) < \lambda$. By construction, $(y_1, \ldots, y_k)T \cap R_\lambda = (y_1, \ldots, y_k)R_\lambda$. As $c \in (y_1, \ldots, y_k)T \cap R_\lambda$, we have $c \in (y_1, \ldots, y_k)R_\lambda \subset I$. Hence, $IT \cap A = I$. It follows by Proposition 6 that $A$ is Noetherian and the completion of $A$ is $T$.

Now suppose $I$ is a nonzero ideal of $A$. Let $J = IT$. If $J \not\subset p$, then $I \not\subset J \cap A \subset p \cap A = (0)$, a contradiction. Hence, $J \not\subset p$. It follows by the construction that the map $A \to T/J$ is surjective. Hence, $A/I \to T/J$ is surjective and hence $A/I$ is complete.

We now claim that if $J \not\subset p$, then $J = (A \cap J)T$. To see this, let $J \not\subset p$ and note that since $M \not\subset p$, we have $JM \not\subset p$. So, $A \to T/JM$ is onto by our construction. Consider the $T$-modules $J$, $M$, and $(A \cap J)T$. We will show that $J = MJ + (A \cap J)T$. Note that $MJ + (A \cap J)T \subset J$ is clear. Now, let $x \in J$. Then $x \in J$. Since $A \to T/JM$ is onto, there is an $a \in A$ such that $a + JM = x + JM$. So, $x = a + jm$ where $j \in J$ and $m \in M$. Also, $a = x - jm \in J$, so $a \in (A \cap J)T$. Hence, $x \in MJ + (A \cap J)T$ and it follows that $MJ + (A \cap J)T = J$. By Nakayama’s Lemma, we have $J = (A \cap J)T$. So, our claim that if $J \not\subset p$, then $J = (A \cap J)T$ holds.

Now, suppose $P$ is a nonzero prime ideal of $A$ and $q$ is a prime ideal of $T$ such that $q \cap A = P$. Then $q \not\subset p$ implies that $q \cap A = p \cap A = (0)$, a contradiction. So, we must have that $q \not\subset p$. Hence, $q = (q \cap A)T = PT$ and it follows that the only prime ideal of $T$ that lies over $P$ is $PT$. Therefore, there is only one prime ideal in the ring $T \otimes_A k(P)$. Since $PT$ is a prime ideal of $T$, we must have that $T \otimes_A k(P)$ is a domain. It follows that $T \otimes_A k(P)$ is a field and hence property (ii) of the theorem holds. (In fact, one can show that $T \otimes_A k(P)$ is isomorphic to $k(P)$.)

It is interesting to note that we have also shown that there is a one-to-one correspondence between nonzero prime ideals of $A$ and prime ideals of $T$ that are not in the generic formal fiber of $A$.

Theorem 7 is the desired result.

**Theorem 7.** Let $(T, M)$ be a complete local domain containing the integers and such that $|T/M| \geq c$. Let $p$ be a nonmaximal prime ideal of $T$ such that $T_p$ is a regular local ring. Then there exists an excellent local domain $A$ such that the completion of $A$ is $T$, the generic formal fiber of $A$ is local with maximal ideal $p$ and if $I$ is a nonzero ideal of $A$, then $A/I$ is complete.

**Proof.** Use Lemma 6 to construct $A$. The Theorem is clear except for the fact that $A$ is excellent. Note that $T \otimes_A k((0)) \cong T_p$ is a regular local ring by assumption. By Lemma 6, the ring $T \otimes_A k(P)$ is a field. Since $T$ contains the integers, $T \otimes_A k(P)$ is a field of characteristic zero. Hence, it follows that $A$ is excellent.

We do not know if Theorem 7 holds when $T$ contains a finite field.

It is interesting to note that the proof of Lemma 6 shows that for the rings we constructed in Lemma 6 and Theorem 7, there is actually a one-to-one correspondence between nonzero prime ideals of $A$ and prime ideals of $T$ that are not in the generic formal fiber of $A$.
Also, note in particular that the ring \( A \) from Lemma 6 and Theorem 7 satisfies the following property: For all prime ideals \( q \) of \( T \) such that \( p \subset q \), the ring \( A/(q \cap A) \) is complete. Therefore, by Theorem 2.7 in [3], there is exactly one analytically irreducible normal local Noetherian domain \( C \) that birationally dominates \( A \) satisfying \( \alpha(C) = 0 \) and satisfying \( C/\mathfrak{m}C \) is a finite \( A \)-module. (Here, \( \mathfrak{m} \) denotes the maximal ideal of \( A \).

In fact, one could modify the proofs in this paper by replacing the ideal \( p \) with a countable set of nonmaximal, incomparable prime ideals of \( T \). Doing this, one would see that all the results in this paper hold when \( p \) is replaced with such a set. In particular, one could show that the following result holds: Let \( (T, M) \) be a complete local domain containing the integers and such that \( |T/M| \geq c \). Let \( L \) be a countable set of nonmaximal incomparable prime ideals of \( T \) such that \( T_p \) is a regular local ring for all \( p \in L \). Then there exists an excellent local domain \( A \) such that the completion of \( A \) is \( T \), the maximal ideals in the generic formal fiber of \( A \) is exactly the set \( L \) and if \( I \) is a nonzero ideal of \( A \), then \( A/I \) is complete. Then, by Theorem 2.7 in [3], there is a one-to-one correspondence between elements of \( L \) and analytically irreducible normal local Noetherian domains \( C \) that birationally dominate \( A \), have \( \alpha(C) = 0 \) and satisfy that \( C/\mathfrak{m}C \) is a finite \( A \)-module where \( \mathfrak{m} \) denotes the maximal ideal of \( A \).

Acknowledgement

We would like to thank the referee for many helpful suggestions. For example, in the paper we originally submitted, the conditions on the complete local ring in Theorem 7 were stronger. We are grateful to the referee for pointing out how to weaken these conditions.

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