PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 130, Number 9, Pages 2809–2810
S 0002-9939(02)06684-4
Article electronically published on April 16, 2002

ERRATUM TO "A RELATION BETWEEN HOCHSCHILD HOMOLOGY AND COHOMOLOGY FOR GORENSTEIN RINGS"

MICHEL VAN DEN BERGH

(Communicated by Lance W. Small)

The paper [5] contains an error in the sense that Theorem 1 (the "duality theorem") is false in the generality stated. As a result the same is true for its corollaries: Proposition 3 and Corollary 6. The main conclusion, which is an affirmative answer to a question by Patrick Polo, remains valid however (see below).

That Theorem 1 is false as stated was pointed out in [2]. In general it can be seen as follows. If the conclusion of Theorem 1 is true, then the Hochschild dimension (the cohomological dimension of HH^*) of the ring A is finite. So Theorem 1 must be false for every ring of infinite Hochschild dimension, and hence in particular for every ring of infinite global dimension.

Thus to save Theorem 1 we must assume that A has finite Hochschild dimension (let us say that A is *smooth* in this case). It is easy to see that in that case the proof becomes valid. The smoothness hypothesis is automatically satisfied in Proposition 2 (see [4]) but it must be added in Proposition 3 and Corollary 6.

The following lemma shows that smoothness is a reasonable condition.

Lemma. (1) *If A is commutative of finite type over the ground field k, then A is smooth in the above sense if and only it is smooth in the classical sense (the "only if" part is related to the Hochschild-Kostant-Rosenberg theorem).*
(2) *If A and B are smooth, then so is $A \otimes_k B$ (this is* [1, Prop. 2(2)]*).*
(3) *The following are equivalent:*
  (a) *A is smooth.*
  (b) *A^e has finite global dimension.*

Remark. If A and B have finite global dimension, then this is not necessarily the case for $A \otimes_k B$. The standard counterexample is given by two fields of infinite transcendence degree.

Remark. In practice A will often be a DG-algebra or an A_∞-algebra. In this case the correct notion of smoothness is that A should be a *compact object* in $D(A^e)$ ($\mathrm{Hom}_{D(A^e)}(A, -)$ should commute with direct sums). The author learnt this from a talk by Kontsevich.

It follows that in order to answer Patrick Polo's question we need to show additionally that if A is a regular minimal quotient of a semi-simple enveloping algebra

Received by the editors December 5, 2001.
1991 *Mathematics Subject Classification.* Primary 16E40.
Key words and phrases. Hochschild homology, Gorenstein rings.
The author is a senior researcher at the FWO.

$U(\mathfrak{g})$, then A is smooth. This follows from the result by Soergel [3] that the category of A-bimodules is equivalent to the category of left modules over some regular minimal primitive quotient of $U(\mathfrak{g} \oplus \mathfrak{g})$. Thus A^e has finite global dimension and hence A is smooth.

I wish to thank Andrea Solotar for bringing this error to my attention and for commenting on the present erratum.

References

[1] S. Eilenberg, A. Rosenberg, and D. Zelinsky, *On the dimension of modules and algebras, VIII*, Nagoya Math. Journal **12** (1957), 71–93. MR **20:**5229

[2] M. A. Farinati, A. Solotar, and M. Suarez-Alvarez, *Hochschild homology and cohomology of generalized Weyl algebras*, to appear.

[3] W. Soergel, *The Hochschild cohomology of regular maximal primitive quotients of enveloping algebras of semisimple Lie algebras*, Ann. Sci. École Norm. Sup. (4) **29** (1996), 535–538. MR **97e:**17016

[4] M. Van den Bergh, *Non-commutative homology of some three dimensional quantum spaces*, J. K-theory **8** (1994), no. 3, 213–230. MR **95i:**16009

[5] ———, *A relation between Hochschild homology and cohomology for Gorenstein rings*, Proc. Amer. Math. Soc. **126** (1998), no. 5, 1345–1348. MR **99m:**16013

Departement WNI, Limburgs Universitair Centrum, Universitaire Campus, Building D, 3590 Diepenbeek, Belgium

E-mail address: vdbergh@luc.ac.be