

## ERRATUM TO “A RELATION BETWEEN HOCHSCHILD HOMOLOGY AND COHOMOLOGY FOR GORENSTEIN RINGS”

MICHEL VAN DEN BERGH

(Communicated by Lance W. Small)

The paper [5] contains an error in the sense that Theorem 1 (the “duality theorem”) is false in the generality stated. As a result the same is true for its corollaries: Proposition 3 and Corollary 6. The main conclusion, which is an affirmative answer to a question by Patrick Polo, remains valid however (see below).

That Theorem 1 is false as stated was pointed out in [2]. In general it can be seen as follows. If the conclusion of Theorem 1 is true, then the Hochschild dimension (the cohomological dimension of  $HH^*$ ) of the ring  $A$  is finite. So Theorem 1 must be false for every ring of infinite Hochschild dimension, and hence in particular for every ring of infinite global dimension.

Thus to save Theorem 1 we must assume that  $A$  has finite Hochschild dimension (let us say that  $A$  is *smooth* in this case). It is easy to see that in that case the proof becomes valid. The smoothness hypothesis is automatically satisfied in Proposition 2 (see [4]) but it must be added in Proposition 3 and Corollary 6.

The following lemma shows that smoothness is a reasonable condition.

- Lemma.** (1) *If  $A$  is commutative of finite type over the ground field  $k$ , then  $A$  is smooth in the above sense if and only if it is smooth in the classical sense (the “only if” part is related to the Hochschild-Kostant-Rosenberg theorem).*
- (2) *If  $A$  and  $B$  are smooth, then so is  $A \otimes_k B$  (this is [1, Prop. 2(2)]).*
- (3) *The following are equivalent:*
- (a)  *$A$  is smooth.*
  - (b)  *$A^e$  has finite global dimension.*

*Remark.* If  $A$  and  $B$  have finite global dimension, then this is not necessarily the case for  $A \otimes_k B$ . The standard counterexample is given by two fields of infinite transcendence degree.

*Remark.* In practice  $A$  will often be a DG-algebra or an  $A_\infty$ -algebra. In this case the correct notion of smoothness is that  $A$  should be a *compact object* in  $D(A^e)$  ( $\text{Hom}_{D(A^e)}(A, -)$  should commute with direct sums). The author learnt this from a talk by Kontsevich.

It follows that in order to answer Patrick Polo’s question we need to show additionally that if  $A$  is a regular minimal quotient of a semi-simple enveloping algebra

---

Received by the editors December 5, 2001.

1991 *Mathematics Subject Classification.* Primary 16E40.

*Key words and phrases.* Hochschild homology, Gorenstein rings.

The author is a senior researcher at the FWO.

$U(\mathfrak{g})$ , then  $A$  is smooth. This follows from the result by Soergel [3] that the category of  $A$ -bimodules is equivalent to the category of left modules over some regular minimal primitive quotient of  $U(\mathfrak{g} \oplus \mathfrak{g})$ . Thus  $A^e$  has finite global dimension and hence  $A$  is smooth.

I wish to thank Andrea Solotar for bringing this error to my attention and for commenting on the present erratum.

## REFERENCES

- [1] S. Eilenberg, A. Rosenberg, and D. Zelinsky, *On the dimension of modules and algebras, VIII*, Nagoya Math. Journal **12** (1957), 71–93. MR **20**:5229
- [2] M. A. Farinati, A. Solotar, and M. Suarez-Alvarez, *Hochschild homology and cohomology of generalized Weyl algebras*, to appear.
- [3] W. Soergel, *The Hochschild cohomology of regular maximal primitive quotients of enveloping algebras of semisimple Lie algebras*, Ann. Sci. École Norm. Sup. (4) **29** (1996), 535–538. MR **97e**:17016
- [4] M. Van den Bergh, *Non-commutative homology of some three dimensional quantum spaces*, J. K-theory **8** (1994), no. 3, 213–230. MR **95i**:16009
- [5] ———, *A relation between Hochschild homology and cohomology for Gorenstein rings*, Proc. Amer. Math. Soc. **126** (1998), no. 5, 1345–1348. MR **99m**:16013

DEPARTEMENT WNI, LIMBURGS UNIVERSITAIR CENTRUM, UNIVERSITAIRE CAMPUS, BUILDING D, 3590 DIEPENBEEK, BELGIUM  
*E-mail address*: `vdbergh@luc.ac.be`