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BOUNDING THE FITTING HEIGHT OF A SOLVABLE GROUP BY THE NUMBER OF ZEROS IN A CHARACTER TABLE

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Abstract. In this paper, we bound the Fitting height of a solvable group by the number of zeros in a character table.

1. Introduction

In this paper, all characters are complex characters and $\mathrm{Irr}(G)$ is the set of irreducible characters of a finite group G. An old theorem of Burnside asserts that any nonlinear $\chi \in \mathrm{Irr}(G)$ must vanish at some element of G. In this paper, we consider the following problem: given the number of zeros in a character table of a finite group G, what can be said about the structure of G? Although there are many results about zeros of irreducible characters (see [1], [2], [5], [7]), our result is that the Fitting height of a finite solvable group can be bounded by the number of zeros in a character table.

For a finite solvable group G, we define characteristic subgroups $F_i(G)$ by letting $F_1(G) = F(G)$, the unique largest nilpotent normal subgroup of G, and $F_{i+1}(G)/F_i(G) = F(G/F_i(G))$. The Fitting height of group G, denoted by $nl(G)$, is the smallest number l for which $F_l(G) = G$. For a subset T of G, let $k_G(T)$ be the number of G-conjugacy classes contained in T. For $\chi \in \mathrm{Irr}(G)$, $v_G(\chi)$ denotes the set of elements contained in $G - F(G)$ on which χ vanishes. It is clear that $v_G(\chi)$ is a union of some conjugacy classes of G. Let $nv_G(\chi) = k_G(v_G(\chi))$, i.e. the number of G-conjugacy classes outside $F(G)$ on which χ vanishes, and let $nv(G) = max\{nv_G(\chi) | \chi \in \mathrm{Irr}(G)\}$. Now our result can be stated as follows.

Theorem 1. *Let G be a solvable group. Then $nl(G) \leq (2nv(G) + 5)/3$.*

Note that if we let $m(G)$ be the maximal number of G-classes on which some $\chi \in \mathrm{Irr}(G)$ vanishes, then we see that $nv(G) \leq m(G)$ and hence we also get the bound $nl(G) \leq (2m(G) + 5)/3$.

Throughout this paper, we use the following notations:

$cl_G(g)$:= the G-conjugacy class containing $g \in G$. Also if we say "class", we always mean "conjugacy class".

$\mathrm{Irr}(G|N) := \mathrm{Irr}(G) - \mathrm{Irr}(G/N)$ where $N \lhd G$. For a character σ, $\mathrm{Irr}(\sigma)$ denotes the set of irreducible constituents of σ.

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$\Phi(G) := \cap\{M|M$ is a maximal subgroup of $G\}$; p always denotes a prime number.

2. Proof of Theorem 1

In this section, we assume that the letter G always denotes a finite solvable group, and we shall freely use the following known results:

(1) Suppose $G = MN$ is a Frobenius group with kernel N and complement M. Then N is nilpotent and $(|M|, |N|) = 1$. Furthermore, if M is abelian, then M is cyclic; if $2||M|$, then N is abelian.

(2) Let $N \lhd G$ and $\chi \in \mathrm{Irr}(G)$. If $e\chi = \lambda^G$ for some $\lambda \in \mathrm{Irr}(N)$ and some positive integer e, then χ vanishes on $G - N$.

(3) Let $N < M$ be two normal subgroups of G. If $\psi \in \mathrm{Irr}(M)$ vanishes on $M - N$, then by Clifford's theorem, each member of $\mathrm{Irr}(\psi^G)$ also vanishes on $M - N$.

Lemma 1 ([6, Lemma 18.1]). *If $G/F(G)$ is abelian, then there exists $\chi \in Irr(G)$ such that $\chi(1) = |G : F(G)|$.*

Lemma 2. *Let M,N be two normal subgroups of G with $1 < N < M$. Then:*

(1) *If $k_G(M - N) = 1$ and $(|M : N|, |N|) = 1$, then M is a Frobenius group with kernel N and prime order complement.*

(2) *If $G = M$ and $k_G(G - N) = 1$, then $|G : N| = 2$ and G is a Frobenius group with abelian kernel N of odd order.*

(3) *If $N = F(G)$ and M/N is abelian, then there exists $\psi \in Irr(M|N)$ of degree $|M : N|$ vanishing on $M - N$, and every $\chi \in Irr(\psi^G)$ ($\subseteq Irr(G|N)$) also vanishes on $M - N$.*

Proof. (1) Write $M = HN$, where $H \cong M/N$. Since $k_G(M - N) = 1$, then M/N is a principal factor of G, and hence is an elementary abelian group with prime power order p^r. Also, we see that all elements in $M - N$ have the same order p since $(|M : N|, |N|) = 1$. It implies that for any element $h \in H - \{1\}$, $C_M(h) = H$. Therefore, M is a Frobenius group with abelian complement H so that H is a cyclic group of prime order p.

(2) Since $k_G(G - N) = 1$, G/N is a principal factor of G and hence $|G/N| = p$. Let $x \in G - N$. Observe that the length of class $cl_G(x)$ is less than or equal to the order of derived group G'. It is easy to see that $N = G'$, $|G : N| = 2$ and $C_G(x) = \langle x\rangle$ with order 2. So, $G = \langle x\rangle N$ is a Frobenius group with a complement $\langle x\rangle$ of order 2, and hence the Frobenius kernel N is an abelian group of odd order.

(3) Since $F(G/\Phi(G)) = F(G)/\Phi(G)$, we may assume that $\Phi(G) = 1$ and hence $F(G)$ is abelian. Since $F(M) = F(G)$, by Lemma 1, there exists $\psi \in \mathrm{Irr}(M|F(G))$ such that $\psi(1) = |M : F(G)|$. It is clear that $\psi = \lambda^M$ for some non-principal linear $\lambda \in \mathrm{Irr}(F(G))$ and hence ψ vanishes on $M - F(G)$. Now for any $\chi \in \mathrm{Irr}(\psi^G)$, we see that $\chi \in \mathrm{Irr}(G|F(G))$ and that χ also vanishes on $M - F(G)$ by Clifford's theorem. □

Lemma 3 is somewhat trivial, but is useful and is used over and over in the rest of this paper. Note that Lemma 3 is true for any finite group.

Lemma 3. *Let $N \lhd G$ and $\psi \in Irr(G/N)$. Write $\bar{G} = G/N$. Then:*

(1) *$cl_{\bar{G}}(\bar{g})$(viewed as a subset of G) is again a G-class if and only if for every $\chi \in Irr(G|N), \chi(g) = 0$; similarly, if A is a union of some cosets of N, then $k_G(A) = 1$ if and only if $k_{\bar{G}}(\bar{A}) = 1$ and each $\chi \in Irr(G|N)$ vanishes on A.*

(2) *If* $A = v_{\bar{G}}(\psi)$ *contains* m *classes of* G/N, *then* $nv_{\bar{G}}(\psi) \leq nv_G(\psi)$; *with equality if and only if* χ *vanishes on* A *for every* $\chi \in Irr(G|N)$; *and if* $nv_{\bar{G}}(\psi)+1 = nv_G(\psi)$, *then there exists a subset* A_1 *of* A *such that* $m-1 = k_G(A_1) = K_{\bar{G}}(A_1)$, *and hence* χ *vanishes on* A_1 *for every* $\chi \in Irr(G|N)$.

(3) $nv(\bar{G}) \leq nv(G)$.

Proof. (1) It follows that $|C_G(g)| = |C_{G/N}(gN)| + \sum_{\chi \in \mathrm{Irr}(G|N)} |\chi(g)|^2$ and that $cl_{G/N}(gN) = cl_G(g)$ if and only if $|C_{G/N}(gN)| = |C_G(g)|$.

Statement (2) is immediate from statement (1), and (3) follows from (2). □

Lemma 4. *Suppose* $nl(G) \geq 2$. *Write* $F(G) = F$ *and* $F_2(G) = F_2$. *Then:*

(1) *There exists* $T_0 \lhd G$ *such that* $T_0 < F$, $nl(G/T_0) = nl(G)$ *and* F/T_0 *is a principal factor of* G.

(2) $nv(G) \geq nv(G/F) + 1$.

(3) *Let* $T \leq F_2$ *be a normal subgroup of* G *such that* $F \leq T \leq F_2$ *and* F_2/T *is abelian. If* $|F_2 : F|$ *is not prime, then there exists* $\chi \in Irr(G|T)$ *such that* χ *vanishes on at least two* G-*classes contained in* $F_2 - F$. *In particular, if* $nl(G) \geq 4$ *and* F_2/T *is a principal factor, then the same conclusion holds.*

Proof. (1) Let $F/\Phi(G) = D_1/\Phi(G) \times \cdots \times D_s/\Phi(G)$ be a direct product of minimal normal subgroups $D_j/\Phi(G)$. Let $T_j = \prod_{k \neq j} D_k$. It is clear that there exists some $T_j = T_0$ such that $nl(G/T_0) = nl(G)$ as desired.

Now we prove conclusions (2) and (3). Let $L \lhd G$ maximal such that L/F is an abelian group. By Lemma 2(3), there exists $\chi_0 \in \mathrm{Irr}(G|F)$ such that χ_0 vanishes on $L - F$.

(2) It is easy to see that if $nl(G) = 2$, then $nv(G) \geq 1$. Suppose $nl(G) \geq 3$ and let $\psi \in \mathrm{Irr}(G/F)$ such that $nv_{G/F}(\psi) = m = nv(G/F)$ and let $A = v_{G/F}(\psi)$. If $nv(G) = nv(G/F) = m$, then $k_{G/F}(A) = m = k_G(A)$, and hence each $\chi \in \mathrm{Irr}(G|F)$ vanishes on A by Lemma 3. Thus χ_0, as a member of $\mathrm{Irr}(G|F)$, vanishes not only on $L - F$ but also on A. Observe that $A \subseteq G - F_2$. It forces that $nv_G(\chi_0) \geq m+1$, a contradiction.

(3) Let G be a counterexample. Since χ_0 vanishes on $L - F$ and $\chi_0 \in \mathrm{Irr}(G|F) \subseteq \mathrm{Irr}(G|T)$, then $k_G(L - F) = 1$ and hence L/F is a p-principal factor of G. In particular, F_2/F is a p-group because $Z(F_2/F) \leq L/F$. Let $N = O_p(G)$. Then by Lemma 3(1), $k_{G/N}(L/N - F/N) = 1$. Now Lemma 2(1) implies that L/N is a Frobenius group with cyclic complement L/F of prime order p.

Since $|F_2/F|$ is not a prime, we may choose M/L to be a p-principal factor of G with $M \leq F_2$. By the assumption of L, it is easy to see that M/F is an extra-special p-group. Then there exists $\psi \in \mathrm{Irr}(M/F)$ such that ψ vanishes on $M - L$. Now let $\chi_1 \in \mathrm{Irr}(\psi^G)$; then χ_1 also vanishes on $M - L$. Since $ker(\chi_1) = F$ and $F < T$ (note that we assume F_2/T is abelian), we have $\chi_1 \in \mathrm{Irr}(G|T)$. But since G is a counterexample, it forces that $M - L$ is just a class of G. Now applying Lemma 3(1), we see that χ_0 (as a member of $\mathrm{Irr}(G|L)$) vanishes not only on $L - F$ but also on $M - L$, so χ_0 vanishes off two classes contained in $F_2 - F$, a contradiction.

In particular, if $nl(G) \geq 4$ and F_2/T is a principal factor of G, then $|F_2 : F|$ is not prime because the alternative would force $G/F_2 \leq Aut(F_2/F)$ to be abelian, and so that the same conclusion holds. □

Lemma 5. *Suppose* $nl(G) \geq 3$. *Then* $nv(G) \geq 2$ *and with equality if and only if* $G \cong S_4$.

Proof. Write $F(G) = F, F_2(G) = F_2$. By Lemma 4(2), we have $nv(G) \geq 1 + nv(G/F) \geq 2$. It is clear that $nv(S_4) = 2$. We now prove $G \cong S_4$ provided that $nv(G) = 2$ and $nl(G) \geq 3$.

By Lemma 4(2), we see that $nl(G) = 3$ and $nv(G/F) = 1$.

Step 1. If F is a minimal normal subgroup of G, then $G \cong S_4$.

We claim that G/F_2 is abelian. Assume this is not the case. Let $N \lhd G$ maximal such that $F_2 \leq N$ and G/N is not abelian, and let $Z > N$ with $Z/N = Z(G/N)$. By [4, Lemma 12.3], there exists $\phi \in \mathrm{Irr}(G/N)$ such that ϕ vanishes on $G - Z$. Since $1 = nv(G/F) \geq nv_{G/F}(\phi)$, we have $k_{G/F}(G/F - Z/F) = 1$. Now applying Lemma 2(2), we have $|G : Z| = 2$. But it is impossible because $|G/Z| = \phi(1)^2 \neq 2$.

Now G/F_2 is abelian; by Lemma 2(3), there exists $\chi_1 \in \mathrm{Irr}(G/F|F_2/F)$ such that χ_1 vanishes on $G - F_2$. Since $nv(G/F) = 1$ and then $k_{G/F}(G/F - F_2/F) = 1$, Lemma 2(2) yields that G/F is a Frobenius group with complement of order 2 and abelian kernel F_2/F of odd order. If $k_G(G - F_2) = 1$, then Lemma 2(2) yields that G is a Frobenius group with kernel F_2 and thus F_2 is nilpotent, a contradiction. Thus $k_G(G - F_2) = 2$ and then we easily deduce that $|C_G(g)| = 4$ for every $g \in G - F_2$. It follows that F is an elementary abelian 2-group.

Let $P \in syl_2(G)$ and let $g \in P - F$. Observe that $|P/P'| = |C_{P/P'}(gP')| = \sum_{\chi \in \mathrm{Irr}(P/P')} |\chi(g)|^2 \leq \sum_{\chi \in \mathrm{Irr}(P)} |\chi(g)|^2 = |C_P(g)| \leq |C_G(g)| = 4$. It follows that $|P/P'| = 4$, and hence by O. Taussky's Theorem [3, III, Theorem 11.9], P has a cyclic subgroup with index 2. Since F is elementary abelian, it forces that $|F| \leq 4$. Now observe that $2'$-group F_2/F acts faithfully on 2-group F. It is easy to check that $|F_2/F| = 3$ and $|F| = 4$, so that $G \cong S_4$, as desired.

Step 2. $G \cong S_4$.

By Lemma 4(1), we may choose $T \lhd G$, $T < F$ such that $nl(G/T) = nl(G) = 3$ and F/T is a principal factor of G. By Lemma 4(2) and Lemma 3(3), we have $nv(G/T) = 2$, and then $G/T \cong S_4$ by induction and step 1. Now it suffices to show $T = 1$. Suppose $T \neq 1$. To see a contradiction, we may assume that T is a minimal normal subgroup of G. Write $\bar{G} = G/T$. It is clear that $F(\bar{G}) = \bar{F}$, $F_2(\bar{G}) = \bar{F}_2$. Applying Lemma 2(3), we may choose $\chi_1, \chi_2 \in \mathrm{Irr}(\bar{G})$ such that χ_1 vanishes on $G - F_2$ and χ_2 vanishes on $F_2 - F$. Since $2 = k_{\bar{G}}(\bar{G} - \bar{F}_2) = nv_{\bar{G}}(\chi_1) \leq nv_G(\chi_1) \leq 2$, it follows that $k_{\bar{G}}(\bar{G} - \bar{F}_2) = k_G(G - F_2) = 2$. By Lemma 3, each member χ of $\mathrm{Irr}(G|T)$ vanishes on two G-classes: $G - F_2$, and hence $v_G(\chi) = G - F_2$ because $nv_G(\chi) \leq 2$.

Now we conclude that $|C_G(g)| = |C_{\bar{G}}(\bar{g})| = 4$ for $g \in G - F_2$. Note that each member χ of $\mathrm{Irr}(G|T)$ does not vanish on $F_2 - F$; we have $k_{\bar{G}}(\bar{F}_2 - \bar{F}) < k_G(F_2 - F)$ by Lemma 3. It follows that $2 = k_G(F_2 - F)$ because $2 = 1 + k_{\bar{G}}(\bar{F}_2 - \bar{F}) \leq k_G(F_2 - F) \leq nv_G(\chi_2) \leq 2$.

Since $k_G(F_2 - F) = 2$, it is easy to check that for every $x \in F_2 - F$, $|C_G(x)| = 6$. Observe that $|C_{\bar{G}}(\bar{x})| = 3$. It forces that T is a 2-group. Now let $P \in syl_2(G)$. Arguing as in Step 1, we see that $|P/P'| = 4$ and P has an element of order $|P|/2$. But P has no such element in $P - F$ (note that for every $g \in P - F$, $|C_G(g)| = 4$) or in F, a contradiction. □

Theorem 1. *Let G be a solvable group. Then $nl(G) \leq (2nv(G) + 5)/3$.*

Proof. By Lemma 4 and Lemma 5, we may assume $nl(G) \geq 4$. Let G be a minimal counterexample with $nl(G) = m + 2$. By Lemma 3(3) and Lemma 4(1), we may assume that $\Phi(G) = 1$ and $F(G)$ is the unique minimal normal subgroup of G and hence $(|F_2(G):F(G)|, |F(G)|) = 1$. For the sake of simplicity, write $F_3 = F_3(G)$, $F_2 = F_2(G)$ and $F = F(G)$.

First we claim that $nl(G) \geq 5$. Suppose $nl(G) = 4$ with

$$4 = nl(G) > (2nv(G) + 5)/3.$$

By Lemma 4(2) and Lemma 5, we see that $nv(G) = 3$, $nv(G/F) = 2$ and $G/F \cong S_4$. Also, there exists $\chi_0 \in \mathrm{Irr}(G/F_2)$ such that χ_0 vanishes on $G-F_3$. Since $nv(G) = 3$ and $2 = k_{G/F}(G/F - F_3/F) \leq k_G(G-F_3) \leq 3$, by Lemma 3, there exists $g_0 \in G-F_3$ such that $cl_{G/F}(g_0F)$ is again a class of G and then $\chi(g_0) = 0$ for all $\chi \in \mathrm{Irr}(G|F)$. Since F_2/F is an abelian group of order 4, by Lemma 2(3), there exists $\chi \in \mathrm{Irr}(\psi^G)$ such that χ vanishes on $F_2 - F$, where $\psi \in \mathrm{Irr}(F_2)$ with $\psi(1) = 4$. Now we see that χ vanishes on g_0 and $F_2 - F$. If $k_G(F_2 - F) = 1$, then Lemma 2(1) implies that F_2 is a Frobenius group (note that $(|F_2/F|, |F|) = 1$ as assumed in above paragraph), and so its abelian complement F_2/F is cyclic, a contradiction. Now $k_G(F_2 - F) > 1$. Since $nv_G(\chi) \leq 3$, it follows that $v_G(\chi) = cl_G(g_0) \cup (F_2 - F)$ and that χ is an extension of ψ (otherwise, it is easy to check that χ vanishes on $G - F_3$ or $F_3 - F_2$). Now $\chi_0\chi \in \mathrm{Irr}(G)$ by [4, Corollary 6.17], and $\chi_0\chi$ vanishes on $\{G - F_3\} \cup \{F_2 - F\}$ and then $nv(G) \geq nv_G(\chi_0\chi) \geq 4$, a contradiction.

By Lemma 4(1), we can choose $T_1 \lhd G$, $F \leq T_1 < F_2$ such that $nl(G/T_1) = m+1$ and F_2/T_1 is a principal factor of G. Similarly, let $K, K_1 \lhd G$ such that $K/T_1 = F(G/T_1)$, $K_1/T_1 = F_2(G/T_1)$, and let $T_2 \lhd G$ with $K \leq T_2 < K_1$ such that K_1/T_2 is a principal factor and $nl(G/T_2) = m$. Write $s = nv(G/T_2)$. By induction, we have $m = nl(G/T_2) \leq (2s+5)/3$. Then the hypothesis that G is a counterexample implies $(2nv(G)+5)/3 < nl(G) = m+2 \leq (2s+11)/3$, so that $nv(G) \leq s+2$. Now applying Lemma 3(3) and Lemma 4(2), we conclude $s = nv(G/T_2) \leq nv(G/F_2) < nv(G/F) < nv(G) \leq s+2$, which forces that $nv(G) = s+2$, $nv(G/F) = s+1$ and $nv(G/F_2) = s$. Because K/T_1 is the Fitting subgroup of G/T_1, by Lemma 3(3) and Lemma 4(2), we have $s \leq nv(G/K) < nv(G/T_1) \leq nv(G/F) = s+1$, thus $nv(G/T_1) = s+1$.

Choose $\chi_2 \in \mathrm{Irr}(G/T_2)$ such that $nv_{G/T_2}(\chi_2) = nv(G/T_2) = s$ and let $A = v_{G/T_2}(\chi_2)$. Clearly, A (viewed as a subset of G/T_1) is the union of s or $s+1$ G/T_1-classes because $s+1 = nv(G/T_1)$.

Case 1. Suppose $k_{G/T_1}(A) = s$.

By Lemma 3, for any $\chi \in \mathrm{Irr}(G/T_1|T_2/T_1), \chi$ vanishes on A. Since $nl(G/T_1) = m+1 \geq 4$, by Lemma 4(3), there exists $\chi_1 \in \mathrm{Irr}(G/T_1|T_2/T_1)$ such that χ_1 vanishes on at least two G/T_1-classes contained in $K_1/T_1 - K/T_1$. Observe that $A \cap K_1 = \emptyset$. Since χ_1 also vanishes on A, we see that $nv(G/T_1) \geq nv_{G/T_1}(\chi_1) \geq s+2$, a contradiction.

Case 2. Suppose $k_{G/T_1}(A) = s+1$.

Then by Lemma 3(2), there exists a subset A_1 of A such that $k_{G/T_1}(A_1) = k_{G/T_2}(A_1) = s-1$, and hence for every $\chi \in \mathrm{Irr}(G/T_1|T_2/T_1)$, χ vanishes on A_1. By Lemma 4(3), there exists $\chi_1 \in \mathrm{Irr}(G/T_1|T_2/T_1)$ which vanishes on C: two G/T_1-classes contained in $K_1/T_1 - K/T_1$. Note that $C \cap A = \emptyset$.

Now χ_1 vanishes on $s+1$ G/T_1-classes: $B = A_1 \cup C$; χ_2 vanishes on $s+1$ G/T_1-classes: A. Since $nv_G(\chi_1) \leq nv(G) = s+2$, by Lemma 3(2), there exists a subset B_1 of B such that $k_G(B_1) = s = k_{G/T_1}(B_1)$, and for every $\chi \in \mathrm{Irr}(G|T_1), \chi$ vanishes on B_1. Similarly, there exists $A_2 \subset A$ which contains s classes of G, and for every $\chi \in \mathrm{Irr}(G|T_1), \chi$ vanishes on A_2. Thus for every $\chi \in \mathrm{Irr}(G|T_1), \chi$ vanishes on $A_2 \cup B_1$, which contains at least $s+1$ classes of G. By Lemma 4(3), there exists

$\chi_3 \in \mathrm{Irr}(G|T_1)$ which vanishes on at least two G-classes D contained in $F_2 - F$. Note that $(A_2 \cup B_1) \cap (F_2 - F) = \emptyset$. Now χ_3 vanishes on at least $s + 3$ G-classes: $A_2 \cup B_1 \cup D$ and then $nv(G) \geq nv_G(\chi_3) \geq s + 3$, a contradiction. The proof is complete. □

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