P.I. ENVELOPES OF CLASSICAL SIMPLE LIE SUPERALGEBRAS

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Abstract. Let \( \mathfrak{g} \) be a classical simple Lie superalgebra. We describe the prime ideals \( P \) in the enveloping algebra \( U(\mathfrak{g}) \) such that \( U(\mathfrak{g})/P \) satisfies a polynomial identity. If the factor algebra \( U(\mathfrak{g})/P \) is not artinian, then it is an order in a matrix algebra over \( K(z) \).

Throughout this paper we work over an algebraically closed field \( K \) of characteristic zero. All unadorned tensor products are taken over \( K \). Let \( \mathfrak{g} \) be a finite dimensional classical simple Lie superalgebra over \( K \). A factor algebra of the enveloping algebra \( U(\mathfrak{g}) \) satisfying a polynomial identity is called a P.I. envelope of \( \mathfrak{g} \). Our aim is to describe all prime P.I. envelopes of \( \mathfrak{g} \). If \( \mathfrak{g} \) has a nonartinian prime P.I. envelope it is not hard to show that the center of \( \mathfrak{g}_0 \) must be nonzero (Lemma 1.3). Thus by the classification theorem in [K1], \( \mathfrak{g} = \mathfrak{sl}(m,n) \) with \( m > n \geq 1 \) or \( \mathfrak{g} = \mathfrak{osp}(2,2n) \).

It was shown by Bahturin and Montgomery that when \( \mathfrak{g} = \mathfrak{sl}(m,n) \) with \( m > n \geq 1 \), \( \mathfrak{g} \) has a nonartinian P.I. envelope. In fact the proof of [BM] Theorem 4.2 shows that this is true also when \( \mathfrak{g} = \mathfrak{osp}(2,2n) \) although these algebras are omitted from the statement of [BM] Theorems 1.5 and 4.2. If \( \mathfrak{g} = \mathfrak{sl}(m,n) \) with \( m > n \geq 1 \) or \( \mathfrak{g} = \mathfrak{osp}(2,2n) \), then \( \mathfrak{g}_0 = [\mathfrak{g}_0, \mathfrak{g}_0] \oplus Kz \) where \( z \) is central in \( \mathfrak{g}_0 \). Furthermore as a \( \mathfrak{g}_0 \)-module via the adjoint action, \( \mathfrak{g}_1 = \mathfrak{g}_1^+ \oplus \mathfrak{g}_1^- \), a direct sum of two simple submodules. If \( \mathfrak{p} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \), then \( U(\mathfrak{g}_0) \) is a homomorphic image of \( U(\mathfrak{p}) \) and thus any \( U(\mathfrak{g}_0) \)-module can be regarded as a \( U(\mathfrak{p}) \)-module. Choose a Cartan subalgebra \( \mathfrak{h} \) and a system of simple roots for \( \mathfrak{g}_0 \). Let \( P^+ \) denote the corresponding set of dominant integral weights. If \( \lambda \in P^+ \) let \( L_\lambda \) be the finite dimensional simple \( [\mathfrak{g}_0, \mathfrak{g}_0] \)-module with highest weight \( \lambda \) and set

\[ V(\lambda) = U(\mathfrak{g}) \otimes_{U(\mathfrak{p})} (L_\lambda \otimes K[z]). \]

Our main result is as follows.

Main Theorem. Set \( P_\lambda = \text{ann}_{U(\mathfrak{g})} V(\lambda), \) \( n = \dim_K L_\lambda \) and \( N = n^{2\dim \mathfrak{g}_1^-} \). Then \( P_\lambda \) is a prime ideal of \( U(\mathfrak{g}) \) such that \( U(\mathfrak{g})/P_\lambda \) is a subring of the matrix algebra \( M_N(K[z]) \) with Goldie quotient ring \( M_N(K(z)) \). In particular \( U(\mathfrak{g})/P_\lambda \) is a prime P.I. algebra.

Conversely, if \( P \) is a prime ideal in \( U(\mathfrak{g}) \) such that \( U(\mathfrak{g})/P \) satisfies a polynomial identity, then \( P = P_\lambda \) for a unique \( \lambda \in P^+ \).

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If \( C = K[z] \setminus \{0\} \), then \( C \) is an Ore set of regular elements in \( U(\mathfrak{g}) \). A key step in the proof that \( P_\lambda \) is prime is to show that the localized module \( V(\lambda)_C \) is in a natural way a \( U(\mathfrak{g}) - F \) bimodule where \( F = K(z) \), and then a simple Kac module over \( U(\mathfrak{g}) \otimes F \). For the converse we use some results of E.S. Letzter concerning prime ideals in finite extensions of Noetherian rings.

1.1. Let \( R \) and \( S \) be prime Noetherian rings. An \( R - S \) bimodule \( M \) is a bond from \( R \) to \( S \) if \( M \) is finitely generated and torsionfree both as a left \( R \)-module and as a right \( S \)-module.

Lemma. Suppose \( M \) is a bond from \( R \) to \( S \). Then:

(a) \( R \) is artinian if and only if \( S \) is artinian.
(b) \( R \) is a P.I. ring if and only if \( S \) is a P.I. ring.

Proof. (a) This is [J, Theorem 5.2.9].
(b) This is Remark (2) after [BS, Prop. 2.5]. We give some details for the convenience of the reader. Let \( D = Fract R \) and \( E = Fract S \). By [BS Prop. 2.5] there exists an integer \( t \) such that \( D \) embeds in the ring of \( t \times t \) matrices \( M_t(E) \) and \( E \) embeds in \( M_t(D) \). Thus if \( S \) is P.I., then \( R \) embeds in \( M_t(E) \) which is a central simple algebra by Posner’s Theorem [McR Theorem 13.6.5], so \( R \) is P.I. by the Amitsur-Levitzki Theorem [McR Corollary 13.3.5]. Similarly if \( R \) is P.I. so is \( S \).

1.2. Until the end of section 1.4 suppose that \( R \subseteq S \) is an extension of Noetherian \( K \)-algebras of finite Gel’fand-Kirillov dimension. Assume that \( S \) is finitely generated and free as a right \( R \)-module. The following definitions are due to Letzter [L2], [L3].

i) Suppose \( P \) is a prime ideal of \( S \) and set \( B = Fract(S/P) \). Let \( V_P \) be the set of prime ideals of \( R \) which are right annihilators of simple \( B - R \) factor bimodules of \( B \).

ii) Suppose \( Q \) is a prime ideal of \( R \) and set \( A = Fract(R/Q) \). Let \( W_Q \) be the set of prime ideals of \( S \) which are left annihilators of simple \( S - A \) factor bimodules of \( S \otimes_R A \).

In addition set \( J_Q = \ell - \text{ann}(S/SQ) \) and

\[
X_Q = \{ P \in \text{Spec } R | P \text{ is minimal over } J_Q \}.
\]

These definitions are related by the following results.

Theorem. (a) If \( Q \in \text{Spec } R \) and \( P \in \text{Spec } S \), then

\[
Q \in V_P \text{ if and only if } P \in W_Q.
\]

Furthermore if this condition holds there is a bond from \( S/P \) to \( R/Q \).

(b) \( W_Q \subseteq X_Q \).

Proof. (a) follows from [L2 Lemma 3.2] and [L1 Lemma 1.1], while (b) follows from the proof of [L2 Proposition 4.2].

1.3. Lemma. Let \( \mathfrak{g} \) be a classical simple Lie superalgebra such that there is a prime ideal \( P \) in \( U(\mathfrak{g}) \) with \( U(\mathfrak{g})/P \) a nonartinian P.I. algebra. Then the center of \( \mathfrak{g}_0 \) is nonzero.
Proof. We apply the results of the two previous subsections, with \( R = U(\mathfrak{g}_0) \) and \( S = U(\mathfrak{g}) \). Choose \( Q \in V_p \). Then there is a bond from \( S/P \) to \( R/Q \) by Theorem 1.2. Hence by Lemma 1.1, \( R/Q \) is a non-artinian P.I. algebra. The result follows from a result of Bahturin; see [BM, page 2837].

1.4. If the equivalent conditions of Theorem 1.2(a) hold we say that \( P \) lies directly over \( Q \). Recall that a module over a prime Noetherian ring is fully faithful if every nonzero submodule is faithful. We require another result of Letzer [L3, Lemma 2.6 (iv)].

Lemma. Suppose that \( P \) lies directly over \( Q \) and that \( M \) is a fully faithful \( S/P \)-module. Then there exists an \( R \)-submodule \( N \) of \( M \) such that \( Q = \text{ann}_R N \) and \( N \) is a fully faithful \( R/Q \)-module.

1.5. Again suppose that \( \mathfrak{g} \) is classical simple. We often write \( U \) for \( U(\mathfrak{g}) \). The simple artinian factor rings of \( U \) correspond to the finite dimensional simple \( \mathfrak{g} \)-modules and these have been classified [K1, Theorem 8].

For the remainder of this paper we assume therefore that \( P \) is a prime ideal of \( U \), such that \( U/P \) is a nonartinian P.I. algebra. By Lemma 1.3 this means that \( \mathfrak{g}_0 \) has a nonzero center.

As noted in the introduction we have \( \mathfrak{g}_0 = [\mathfrak{g}_0, \mathfrak{g}_0] \oplus K z \), and we can choose \( z \) in such a way that \([z, x] = \pm x \) for all \( x \in \mathfrak{g}_0^+ \). Set \( \mathfrak{h}' = \mathfrak{h} \oplus K z \), so that \( \mathfrak{h}' \) is a Cartan subalgebra of \( \mathfrak{g}_0 \) and \( \mathfrak{g} \). Fix a non-degenerate invariant bilinear form \((,\) on \((\mathfrak{h}')^*\). For \( \alpha \in (\mathfrak{h}')^* \), we write \( g^\alpha \) for the corresponding root space. There is a unique \( h_\alpha \in \mathfrak{h}' \) such that \((\mu, \alpha) = \mu(h_\alpha)\) for all \( \mu \in (\mathfrak{h}')^* \). Let \( \Delta_1^+ \) be the set of roots of \( \mathfrak{g}_1^+ \). For \( \alpha \in \Delta_1^+ \), choose \( e_{\pm \alpha} \) such that \( \mathfrak{g}_1^{\pm \alpha} = Ke_{\pm \alpha} \) and \( h_\alpha = [e_\alpha, e_{-\alpha}] \).

1.6. We construct a functor \( T \) between categories of left modules:

\[ T : U(\mathfrak{g}_0)\text{-mod} \longrightarrow U(\mathfrak{g})\text{-mod}. \]

First set \( \mathfrak{p} = \mathfrak{g}_0 \oplus \mathfrak{g}_1^+ \) and \( J = U(\mathfrak{p})\mathfrak{g}_0^+ \). Then \( J \) is a nilpotent ideal of \( U(\mathfrak{p}) \) with \( U(\mathfrak{p})/J \cong U(\mathfrak{g}_0) \). Thus we can regard \( U(\mathfrak{g}_0)\text{-mod} \) as a subcategory of \( U(\mathfrak{p})\text{-mod} \) and define \( T(\underline{\lambda}) = U(\mathfrak{g}) \otimes_{U(\mathfrak{p})} (\underline{\lambda}) \).

We use the functor \( T \) to construct some examples of P.I. envelopes of \( \mathfrak{g} \). If \( M \) is any \( K[z] \)-module and \( \lambda \in P^+ \) we regard \( L(\lambda, M) = L_\lambda \otimes M \) as a \( U(\mathfrak{g}_0) \)-module by allowing \([\mathfrak{g}_0, \mathfrak{g}_0] \) (resp. \( K z \)) to act on the first (resp. second) factor of the tensor product. For \( a \in K \), let \( \mathcal{O}_a = K[z]/(z - a) \) and set

\[ L(\lambda) = L(\lambda, K[z]), \quad L(\lambda, a) = L(\lambda, \mathcal{O}_a), \]

\[ V(\lambda) = T(L(\lambda)), \quad V(\lambda, a) = T(L(\lambda, a)). \]

The natural map \( K[z] \longrightarrow \mathcal{O}_a \) induces an epimorphism of \( U(\mathfrak{g}) \)-modules

\[ V(\lambda) \longrightarrow V(\lambda, a). \]

The pair \( \lambda' = (\lambda, a) \) can be viewed as the element of \((\mathfrak{h}')^* \) with \( \lambda'|_\mathfrak{h} = \lambda \) and \( \lambda(z) = a \). The module \( V(\lambda, a) \) is called the Kac module with highest weight \( \lambda' \). By [K2, Proposition 2.9] \( V(\lambda, a) \) is a simple \( U(\mathfrak{g}) \)-module if and only if \( \lambda' = (\lambda, a) \) satisfies \((\lambda' + \rho, \alpha) \neq 0 \) for all odd positive roots \( \alpha \). Here \( \rho = \rho_0 - \rho_1 \), where \( \rho_0 \) (resp. \( \rho_1 \)) is the half-sum of the positive even (resp. odd) roots. Since \( V(\lambda) \) maps onto any Kac module of the form \( V(\lambda, a) \) we call \( V(\lambda) \) the universal Kac-module with highest weight \( \lambda \in \mathfrak{h}'^* \).
1.7. The enveloping algebra $U$ has a $\mathbb{Z}$-grading, $U = \bigoplus_{n \in \mathbb{Z}} U(n)$ extending the $\mathbb{Z}$-grading on $g$ given by $\deg g_0 = 0$, $\deg g_1^\pm = \pm 1$. Henceforth the adjective “graded” refers to this grading. We use this grading to construct a useful localization of $U$.

Suppose that $M = \bigoplus M(n)$ is a graded $U$-module which is torsionfree as a $K[z]$-module. We can make $M$ into a $U - K[z]$-bimodule via the rule

\[ mf(z) = f(z - n)m \]

for $m \in M(n)$, $f(z) \in K[z]$. Let $F = K(z)$ and give $M^F = M \otimes_{K[z]} F$ the right $F$-module structure obtained by localization. In particular $U^F$ becomes a $U - F$-bimodule in this way and we can extend the algebra structure on $U$ to $U^F$ by

\[(u \otimes f_1(z))(v \otimes f_2(z)) = uv \otimes f_1(z + n)f_2(z)\]

for $u \in U$, $v \in U(n)$ and $f_1, f_2 \in F$. It is now easy to verify the following:

**Lemma.** The multiplicative set $C = K[z] \setminus \{0\}$ is Ore in $U$ and $U_C \cong U^F$ with the above algebra structure. If $M$ is a graded left $U$-module which is torsionfree as a $K[z]$-module, then $M_C \cong M \otimes_{K[z]} F$ as a $U - F$ bimodule via the map

\[ f^{-1}(z)m \rightarrow m \otimes f^{-1}(z + n) \]

for $m \in M(n)$, $f(z) \in K[z]$. In addition if $N$ is any graded $U$-submodule of $M_C$, then $N$ is a $U_C$-submodule if any only if it is a $U - F$ sub-bimodule of $M_C$.

1.8. If $V$ is a vector space over $K$ we write $V_F$ for $V \otimes F$. If $A$ is a $K$-algebra and $M$ is a left $A$-module, then $A_F$ is an $F$-algebra and $M_F$ an $A_F$-module by extension of scalars.

We apply these remarks to the universal Kac module $V(\lambda) = U(g) \otimes_{U(p)} L(\lambda)$. If $\Lambda = \bigoplus \Lambda^n$ is the exterior algebra on $g_1^-$, then as a left $U(g_0)$-module

\[ V(\lambda) \cong \bigoplus \Lambda^n \otimes L(\lambda). \]

By definition $L(\lambda) = L_{\lambda} \otimes K[z]$, and so $L(\lambda)$ and $V(\lambda)$ are in an obvious way right $K[z]$-modules. Since $\deg g_1^- = -1$ the gradings on $\Lambda$ and $U$ satisfy $\Lambda^n \subseteq U(-n)$. Observe that the $K[z]$-bimodule structure on $V(\lambda)$ satisfies (1) in section 1.7.

Note also that $V(\lambda)$ is torsionfree as a left (and right) $K[z]$-module. Thus $V(\lambda)_C$ is a $U - F$ bimodule or equivalently a left $U_F$-module and we have

\[ V(\lambda)_C \cong (U(g) \otimes_{U(p)} L(\lambda)) \otimes_{K[z]} F \]

\[ \cong U(g) \otimes_{U(p)} (L(\lambda) \otimes_{K[z]} F) \]

\[ \cong U(g)_F \otimes_{U(p)_F} (L(\lambda) \otimes_{K[z]} F). \]

Next we consider the $U(p)_F$-module $L(\lambda) \otimes F$. This is annihilated by $g_1^+_{U_F}$ and so is a $U(g_0)_F$-module. In fact it is the finite dimensional simple module over this algebra whose highest weight $\lambda'$ is the unique $F$-linear map $h'_p \rightarrow F$ such that $\lambda'_0 = \lambda$ and $\lambda'(z) = z$. Thus $V(\lambda)_C$ is a Kac module over $u(g) \otimes F$. In fact we have

**Proposition.** (a) The module $V(\lambda)_C$ is a simple Kac module with highest weight $\lambda'$ module over the algebra $U(g) \otimes F$.

(b) The module $V(\lambda)_C$ is a simple module over the algebra $U(g)_C$.

**Proof.** It remains to show simplicity in both cases.

(a) We extend $(\cdot, \cdot)$ to a bilinear form $(\cdot, \cdot)_F$ on $(h'_p)^+_F$. It suffices to show that $(\lambda' + \rho, \alpha) \neq 0$ for all odd positive roots $\alpha$. Since the highest exterior power of $g_1^+
is trivial as a $g_0$-module we have $(\rho_1, \beta) = 0$ for all even roots. Thus $h^1 = K\rho_1$. If $\alpha$ is an odd root, it follows that $(\rho_1, \alpha) = \rho_1(h_\alpha) \neq 0$. Hence $h_\alpha \notin (K\rho_1)^1 = h$, so $h_\alpha - bz \in h$ for some nonzero $b \in K$. Therefore

$$(\lambda' + \rho, \alpha)_F = \lambda'(h_\alpha - bz + bz)_F + (\rho, \alpha) = bz + \lambda(h_\alpha - bz) + (\rho, \alpha),$$

which is a linear polynomial in $z$. 

(b) Fix an order on $\Delta_1^+$ and for $I \subseteq \Delta_1^+$ set

$$e_I = \prod_{\alpha \in I} e_{-\alpha}$$

where the product is taken with respect to this order. Let $N$ be a nonzero $U(g)_C$-submodule of $V(\lambda)_C$ and suppose

$$n = \sum_I e_In_I$$

is nonzero with $n_I \in L(\lambda) \otimes_K [z] F$ for all $I$. Choose $m$ minimal such that $n_I \neq 0$ for some subset $I$ with $|I| = m$. Set $I' = \Delta_1^+ \setminus I$. Then

$$e_{I'}n = \pm e_{\Delta_1^+}n_I$$

is a nonzero homogeneous element of $N$, so generates a graded submodule. It follows from Lemma 1.7 that $N = V(\lambda)_C$.

**Corollary.** (a) If $N$ is any nonzero $U(g)$-submodule of $V(\lambda)$, then $N$ contains a submodule isomorphic to $V(\lambda)$.

(b) $P_\lambda = \text{ann}_{U(g)} V(\lambda)$ is a prime ideal of $U(g)$.

**Proof.** (a) By the Proposition $N_C = (V(\lambda))_C$. Hence $(L_\lambda \otimes 1) \subseteq N_C$ so $L_\lambda \otimes (f) \subseteq N$ for some nonzero $f$. The submodule of $V(\lambda)$ generated by $L_\lambda \otimes (f)$ is isomorphic to $T(L(\lambda, (f))) \cong V(\lambda)$.

(b) This follows since any nonzero submodule of $V(\lambda)$ has annihilator $P_\lambda$, by part (a).

**1.9. Lemma.** Identify $L(\lambda)$ with the $U(p)$-submodule $1 \otimes L(\lambda)$ of $V(\lambda)$ and let $J = U(p)g_+^1$. Then $\text{ann}_{V(\lambda)} J = L(\lambda)$.

**Proof.** We use the same notation as in the proof of Proposition 1.8(b).

For $0 \leq m \leq |\Delta_1^+|$ set

$$V(m) = \bigoplus_{|I| = m} e_IL(\lambda).$$

Then $V(\lambda) = \bigoplus_m V(m)$ and $g_+^1V(m) \subseteq V(m - 1)$. Therefore it suffices to show that if $m > 0$, then $g_+^1(\sum_{|I| = m} e_Iw_I) \neq 0$ provided the $w_I \in L(\lambda)$ are not all zero.

Now $L(\lambda) = L_\lambda \otimes_K [z]$ has a filtration given by setting $\text{deg}(L_\lambda \otimes z^n) = n$. Choose $I$ so that $w_I$ has maximum degree in this filtration, choose $\alpha \in I$ and set $H = I \setminus \{\alpha\}$. Note that

$$\text{deg}(h_\alpha w_I) = \text{deg}(w_I) + 1.$$ 

Using the formula

$$[e_\alpha, ab] = [e_\alpha, a]b \pm a[e_\alpha, b]$$

for homogeneous $a, b \in U(g)$ we see that

$$e_\alpha e_Iw_I = \pm e_Hh_\alpha w_I$$

plus a sum of terms of smaller degree. The result follows easily from this.
1.10. The next result is an easy consequence of the Artin-Wedderburn theorem.

**Lemma.** Let $U$ be a $K$-algebra and $L$ a finite dimensional simple $U$-module. Then for any field extension $K'$ of $K$ we have

$$\text{End}_{U \otimes_K K'}(L \otimes_K K') = K'.$$

1.11. **Lemma.** For any $\lambda \in P^+$,

$$\text{End}_{U(\mathfrak{g})}(V(\lambda)) \cong K[z].$$

**Proof.** By the adjoint isomorphism $f \in \text{End}_{U(\mathfrak{g})}(V(\lambda))$ is determined by

$$f_1 = f|_{L(\lambda)} \in \text{Hom}_{U(\mathfrak{p})}(L(\lambda), V(\lambda)).$$

If $J$ is as in Lemma 1.9, then $f_1(L(\lambda)) \subseteq \text{ann}_{V(\lambda)}J = L(\lambda)$, and hence

$$f_1 \in \text{End}_{U(\mathfrak{p})}(L(\lambda) \otimes K[z]) = \text{End}_{U(\mathfrak{g})}(L(\lambda) \otimes K[z]) \cong K[z],$$

using Lemma 1.10.

1.12. Suppose that $\mathfrak{t}$ is a semisimple Lie algebra over $K$ and $C$ is a commutative $K$-algebra. We describe the prime ideals $Q$ of $R = U(\mathfrak{t}) \otimes C$ such that $R/Q$ is P.I. Since $C$ is central, $q = Q \cap C$ is prime in $C$ and by replacing $R$ by the factor ring $R/Rq$ we can assume that $Q \cap C = 0$. There is a one-one correspondence between prime ideals $Q$ of $R$ such that $Q \cap C = 0$ and prime ideals of $U(\mathfrak{t}) \otimes \text{Fract}(C)$. Thus we may assume that $C$ is a field extension of $K$. By the argument on page 2837 of [BM] $Q$ is the annihilator of a finite dimensional simple module over $U(\mathfrak{t}) \otimes C$.

To apply this to our reductive algebra $\mathfrak{g}_0$ with center $Kz$ set $\mathfrak{t} = [\mathfrak{g}_0, \mathfrak{g}_0], R = U(\mathfrak{g}_0)$ and $C = K[z]$. If $q \neq 0$, then $R/Rq \cong U(\mathfrak{t})$ and $R/Q$ is artinian. Thus if $R/Q$ is nonartinian, then $q = 0$ and $Q$ corresponds to the annihilator of a finite dimensional simple module over $U(\mathfrak{t}) \otimes \text{Fract}(C)$. This gives the following result.

**Lemma.** Suppose $Q$ is a prime ideal of $U(\mathfrak{g}_0)$ such that $U(\mathfrak{g}_0)/Q$ is a nonartinian P.I. ring. Then for some uniquely determined $\lambda \in P^+, Q = \text{ann}_{U(\mathfrak{g}_0)}L(\lambda)$.

**Proof of the Main Theorem.** Suppose $\lambda \in P^+$ and let $n = \text{dim}_K L_\lambda$. By Corollary 1.8 $P_\lambda$ is a prime ideal in $U(\mathfrak{g})$. Set $U_\lambda = U(\mathfrak{g})/P_\lambda$. Note that $V(\lambda)$ is a torsionfree $K[z]$-module and thus $U_\lambda$ embeds in $(U_\lambda)_C$. Since $V(\lambda)$ is a $U(\mathfrak{g}) - K[z]$-bimodule which is free of rank $N = n2^{\dim F_\lambda^+}$ on the right, $U_\lambda$ embeds in $M_N(K[z])$. This embedding induces an embedding of $(U_\lambda)_C$ into $M_N(F)$ which is surjective since $V(\lambda)_C$ is a simple $(U_\lambda)_C$-module of dimension $N$ over its endomorphism ring $F$.

Conversely suppose $P$ is a prime ideal of $U(\mathfrak{g})$ with $U(\mathfrak{g})/P$ a nonartinian P.I. ring. We apply the results in sections 1.1 and 1.2 with $R = U(\mathfrak{p})$ and $S = U(\mathfrak{g})$. If $Q \in V_P$ there is a bond from $S/P$ to $R/Q$, so $R/Q$ is a nonartinian P.I. ring, by Lemma 1.1. Lemma 1.12 implies that $Q = Q_\lambda$ for some $\lambda \in P^+$. Since $P \in X_Q$, $P$ is minimal over $\text{ann}_S(S/QS)$ which equals $\text{ann}_S V(\lambda) = P_\lambda$ by [BGR] Satz 10.4. As $P_\lambda$ is prime we get $P = P_\lambda$. To show that $\lambda$ is uniquely determined by $P$ it suffices to show that $V_{P_\lambda} = \{Q_\lambda\}$. However if $Q' \in V_{P_\lambda}$, then by Lemma 1.4 $Q' = \text{ann}_R N$ for some $R$-submodule $N$ of $V(\lambda)$ which is fully faithful as an $R/Q'$-submodule. Since $J = U(\mathfrak{p})|_{Q'}$ is nilpotent $J \subseteq Q'$, so using Lemma 1.9 $N \subseteq \text{ann}_{V(\lambda)}J = L(\lambda)$. However every nonzero submodule of $L(\lambda)$ has annihilator $Q_\lambda$, so $Q' = Q_\lambda$ as desired.
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