P.I. ENVELOPES OF CLASSICAL SIMPLE LIE SUPERA LGBRAS

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Abstract. Let \( g \) be a classical simple Lie superalgebra. We describe the prime ideals \( P \) in the enveloping algebra \( U(g) \) such that \( U(g)/P \) satisfies a polynomial identity. If the factor algebra \( U(g)/P \) is not artinian, then it is an order in a matrix algebra over \( K(z) \).

Throughout this paper we work over an algebraically closed field \( K \) of characteristic zero. All unadorned tensor products are taken over \( K \). Let \( g \) be a finite dimensional classical simple Lie superalgebra over \( K \). A factor algebra of the enveloping algebra \( U(g) \) satisfying a polynomial identity is called a P.I. envelope of \( g \). Our aim is to describe all prime P.I. envelopes of \( g \). If \( g \) has a nonartinian prime P.I. envelope it is not hard to show that the center of \( g_0 \) must be nonzero (Lemma 1.3). Thus by the classification theorem in [K1], \( g = \mathfrak{sl}(m, n) \) with \( m > n \geq 1 \) or \( g = \mathfrak{osp}(2, 2n) \).

It was shown by Bahturin and Montgomery that when \( g = \mathfrak{sl}(m, n) \) with \( m > n \geq 1 \), \( g \) has a nonartinian P.I. envelope. In fact the proof of [BM] Theorem 4.2 shows that this is true also when \( g = \mathfrak{osp}(2, 2n) \) although these algebras are omitted from the statement of [BM] Theorems 1.5 and 4.2. If \( g = \mathfrak{sl}(m, n) \) with \( m > n \geq 1 \) or \( g = \mathfrak{osp}(2, 2n) \), then \( g_0 = [g_0, g_0] \oplus Kz \) where \( z \) is central in \( g_0 \). Furthermore as a \( g_0 \)-module via the adjoint action, \( g_1 = g_1^+ \oplus g_1^- \), a direct sum of two simple submodules. If \( p = g_0 \oplus g_1^+ \), then \( U(g_0) \) is a homomorphic image of \( U(p) \) and thus any \( U(g_0) \)-module can be regarded as a \( U(p) \)-module. Choose a Cartan subalgebra \( h \) and a system of simple roots for \( g_0 \). Let \( P^+ \) denote the corresponding set of dominant integral weights. If \( \lambda \in P^+ \) let \( L_\lambda \) be the finite dimensional simple \( [g_0, g_0] \)-module with highest weight \( \lambda \) and set

\[
V(\lambda) = U(g) \otimes_{U(p)} (L_\lambda \otimes K[z]).
\]

Our main result is as follows.

Main Theorem. Set \( P_\lambda = \operatorname{ann}_{U(g)}V(\lambda), n = \dim_K L_\lambda \) and \( N = n^{2\dim g_1^+} \). Then \( P_\lambda \) is a prime ideal of \( U(g) \) such that \( U(g)/P_\lambda \) is a subring of the matrix algebra \( M_N(K[z]) \) with Goldie quotient ring \( M_N(K(z)) \). In particular \( U(g)/P_\lambda \) is a prime P.I. algebra.

Conversely, if \( P \) is a prime ideal in \( U(g) \) such that \( U(g)/P \) satisfies a polynomial identity, then \( P = P_\lambda \) for a unique \( \lambda \in P^+ \).
If \( C = K[z]\{0\} \), then \( C \) is an Ore set of regular elements in \( U(g) \). A key step in the proof that \( P_\lambda \) is prime is to show that the localized module \( V(\lambda)C \) is in a natural way a \( U(g) - F \) bimodule where \( F = K(z) \), and then a simple Kac module over \( U(g) \otimes F \). For the converse we use some results of E.S. Letzter concerning prime ideals in finite extensions of Noetherian rings.

1.1. Let \( R \) and \( S \) be prime Noetherian rings. An \( R - S \) bimodule \( M \) is a bond from \( R \) to \( S \) if \( M \) is finitely generated and torsionfree both as a left \( R \)-module and as a right \( S \)-module.

**Lemma.** Suppose \( M \) is a bond from \( R \) to \( S \). Then:
(a) \( R \) is artinian if and only if \( S \) is artinian.
(b) \( R \) is a P.I. ring if and only if \( S \) is a P.I. ring.

**Proof.** (a) This is [J, Theorem 5.2.9].
(b) This is Remark (2) after [BS, Prop. 2.5]. We give some details for the convenience of the reader. Let \( D = \text{Fract } R \) and \( E = \text{Fract } S \). By [BS Prop. 2.5] there exists an integer \( t \) such that \( D \) embeds in the ring of \( t \times t \) matrices \( M_t(E) \) and \( E \) embeds in \( M_t(D) \). Thus if \( S \) is P.I., then \( R \) embeds in \( M_t(E) \) which is a central simple algebra by Posner’s Theorem [McR Theorem 13.6.5], so \( R \) is P.I. by the Amitsur-Levitzki Theorem [McR Corollary 13.3.5]. Similarly if \( R \) is P.I. so is \( S \).

1.2. Until the end of section 1.4 suppose that \( R \subseteq S \) is an extension of Noetherian \( K \)-algebras of finite Gel’fand-Kirillov dimension. Assume that \( S \) is finitely generated and free as a right \( R \)-module. The following definitions are due to Letzter [L2], [L3].

i) Suppose \( P \) is a prime ideal of \( S \) and set \( B = \text{Fract } S/P \). Let \( V_P \) be the set of prime ideals of \( R \) which are right annihilators of simple \( B - R \) factor bimodules of \( B \).

ii) Suppose \( Q \) is a prime ideal of \( R \) and set \( A = \text{Fract } R/Q \). Let \( W_Q \) be the set of prime ideals of \( S \) which are left annihilators of simple \( S - A \) factor bimodules of \( S \otimes_R A \).

In addition set \( J_Q = \ell \) - \( \text{ann}(S/SQ) \) and
\[
X_Q = \{ P \in \text{Spec } R | P \text{ minimal over } J_Q \}.
\]

These definitions are related by the following results.

**Theorem.** (a) If \( Q \in \text{Spec } R \) and \( P \in \text{Spec } S \), then
\[
Q \in V_P \text{ if and only if } P \in W_Q.
\]
Furthermore if this condition holds there is a bond from \( S/P \) to \( R/Q \).

(b) \( W_Q \subseteq X_Q \).

**Proof.** (a) follows from [L2 Lemma 3.2] and [L1 Lemma 1.1], while (b) follows from the proof of [L2 Proposition 4.2].

1.3. **Lemma.** Let \( g \) be a classical simple Lie superalgebra such that there is a prime ideal \( P \) in \( U(g) \) with \( U(g)/P \) a nonartinian P.I. algebra. Then the center of \( g_0 \) is nonzero.
Proof. We apply the results of the two previous subsections, with $R = U(g_0)$ and $S = U(g)$. Choose $Q \subseteq V_L$. Then there is a bond from $S/P$ to $R/Q$ by Theorem 1.2. Hence by Lemma 1.1, $R/Q$ is a non-artinian P.I. algebra. The result follows from a result of Balthurin; see [BM] page 2837.

1.4. If the equivalent conditions of Theorem 1.2(a) hold we say that $P$ lies directly over $Q$. Recall that a module over a prime Noetherian ring is fully faithful if every nonzero submodule is faithfil. We require another result of Letzter [L3, Lemma 2.6 (iv)].

Lemma. Suppose that $P$ lies directly over $Q$ and that $M$ is a fully faithful $S/P$-module. Then there exists an $R$-submodule $N$ of $M$ such that $Q = \text{ann}_R N$ and $N$ is a fully faithful $R/Q$-module.

1.5. Again suppose that $g$ is classical simple. We often write $U$ for $U(g)$. The simple artinian factor rings of $U$ correspond to the finite dimensional simple $g$-modules and these have been classified [K1, Theorem 8].

For the remainder of this paper we assume therefore that $P$ is a prime ideal of $U$, such that $U/P$ is a nonartinian P.I. algebra. By Lemma 1.3 this means that $g_0$ has a nonzero center.

As noted in the introduction we have $g_0 = [g_0, g_0] \oplus Kz$, and we can choose $z$ in such a way that $[z, x] = \pm x$ for all $x \in g^\pm$. Set $h' = h \oplus Kz$, so that $h'$ is a Cartan subalgebra of $g_0$ and $g$. Fix a non-degenerate invariant bilinear form $(,)$ on $(h')^*$. For $\alpha \in (h')^*$, we write $g^\alpha$ for the corresponding root space. There is a unique $h_\alpha \in h'$ such that $(\mu, \alpha) = \mu(h_\alpha)$ for all $\mu \in (h')^*$. Let $\Delta_1^+$ be the set of roots of $g_1^+$. For $\alpha \in \Delta_1^+$, choose $c_{\pm\alpha}$ such that $g_{1}^{\pm\alpha} = Ke_{\pm\alpha}$ and $h_\alpha = [e_\alpha, e_{-\alpha}]$.

1.6. We construct a functor $T$ between categories of left modules:

$$T : U(g_0)\text{-mod} \longrightarrow U(g)\text{-mod}.$$  

First set $p = g_0 \oplus g_1^+$ and $J = U(p)g_1^+$. Then $J$ is a nilpotent ideal of $U(p)$ with $U(p)/J \cong U(g_0)$. Thus we can regard $U(g_0)$-mod as a subcategory of $U(p)$-mod and define $T(\underline{L}) = U(g) \otimes_{U(p)} \underline{L}$.

We use the functor $T$ to construct some examples of P.I. envelopes of $g$. If $M$ is any $K[z]$-module and $\lambda \in P^+$ we regard $L(\lambda, M) = L_\lambda \otimes M$ as a $U(g_0)$-module by allowing $[g_0, g_0]$ (resp. $Kz$) to act on the first (resp. second) factor of the tensor product. For $a \in K$, let $O_a = K[z]/(z - a)$ and set

$$L(\lambda) = L(\lambda, K[z]), \quad L(\lambda, a) = L(\lambda, O_a),$$

$$V(\lambda) = T(L(\lambda)), \quad V(\lambda, a) = T(L(\lambda, a)).$$

The natural map $K[z] \longrightarrow O_a$ induces an epimorphism of $U(g)$-modules

$$V(\lambda) \longrightarrow V(\lambda, a).$$

The pair $\lambda' = (\lambda, a)$ can be viewed as the element of $(h)^*$ with $\lambda'|_h = \lambda$ and $\lambda(z) = a$. The module $V(\lambda, a)$ is called the Kac module with highest weight $\lambda'$. By [K2] Proposition 2.9 $V(\lambda, a)$ is a simple $U(g)$-module if and only if $\lambda' = (\lambda, a)$ satisfies $(\lambda' + \rho, \alpha) \neq 0$ for all odd positive roots $\alpha$. Here $\rho = \rho_0 - \rho_1$, where $\rho_0$ (resp. $\rho_1$) is the half-sum of the positive even (resp. odd) roots. Since $V(\lambda)$ maps onto any Kac module of the form $V(\lambda, a)$ we call $V(\lambda)$ the universal Kac-module with highest weight $\lambda \in h^*$. 

1.7. The enveloping algebra $U$ has a $\mathbb{Z}$-grading, $U = \bigoplus_{n \in \mathbb{Z}} U(n)$ extending the $\mathbb{Z}$-grading on $\mathfrak{g}$ given by $\deg g_0 = 0$, $\deg \mathfrak{g}_1^\pm = \pm 1$. Henceforth the adjective “graded” refers to this grading. We use this grading to construct a useful localization of $U$.

Suppose that $M = \bigoplus M(n)$ is a graded $U$-module which is torsionfree as a $K[z]$-module. We can make $M$ into a $U - K[z]$-bimodule via the rule

$$mf(z) = f(z - n)m$$

for $m \in M(n), f(z) \in K[z]$. Let $F = K(z)$ and give $M^F = M \otimes_{K[z]} F$ the right $F$-module structure obtained by localization. In particular $U^F$ becomes a $U - F$-bimodule in this way and we can extend the algebra structure on $U$ to $U^F$ by

$$(u \otimes f_1(z))(v \otimes f_2(z)) = uv \otimes f_1(z + n)f_2(z)$$

for $u \in U$, $v \in U(n)$ and $f_1, f_2 \in F$. It is now easy to verify the following:

**Lemma.** The multiplicative set $\mathcal{C} = K[z]\setminus\{0\}$ is Ore in $U$ and $U_\mathcal{C} \cong U^F$ with the above algebra structure. If $M$ is a graded left $U$-module which is torsionfree as a $K[z]$-module, then $M_\mathcal{C} \cong M \otimes_{K[z]} F$ as a $U - F$ bimodule via the map

$$f^{-1}(z)m \rightarrow m \otimes f^{-1}(z + n)$$

for $m \in M(n), f(z) \in K[z]$. In addition if $N$ is any graded $U$-submodule of $M_\mathcal{C}$, then $N$ is a $U_\mathcal{C}$-submodule if any only if it is a $U - F$ sub-bimodule of $M_\mathcal{C}$.

1.8. If $V$ is a vector space over $K$ we write $V_F$ for $V \otimes F$. If $A$ is a $K$-algebra and $M$ is a left $A$-module, then $A_F$ is an $F$-algebra and $M_F$ an $A_F$-module by extension of scalars.

We apply these remarks to the universal Kac module $V(\lambda) = U(\mathfrak{g}) \otimes_{U(\mathfrak{g})} L(\lambda)$. If $\Lambda = \bigoplus \Lambda^n$ is the exterior algebra on $\mathfrak{g}_1^\perp$, then as a left $U(\mathfrak{g}_0)$-module

$$V(\lambda) \cong \bigoplus \Lambda^n \otimes L(\lambda).$$

By definition $L(\lambda) = L_\lambda \otimes K[z]$, and so $L(\lambda)$ and $V(\lambda)$ are in an obvious way right $K[z]$-modules. Since $\deg \mathfrak{g}_1^\perp = -1$ the gradings on $\Lambda$ and $U$ satisfy $\Lambda^n \subseteq U(-n)$. Observe that the $K[z]$-bimodule structure on $V(\lambda)$ satisfies (1) in section 1.7.

Note also that $V(\lambda)$ is torsionfree as a left (and right) $K[z]$-module. Thus $V(\lambda)_\mathcal{C}$ is a $U - F$ bimodule or equivalently a left $U_F$-module and we have

$$V(\lambda)_\mathcal{C} \cong (U(\mathfrak{g}) \otimes_{U(\mathfrak{g})} L(\lambda)) \otimes_{K[z]} F \cong U(\mathfrak{g}) \otimes_{U(\mathfrak{g})} (L(\lambda) \otimes_{K[z]} F) \cong U(\mathfrak{g})_F \otimes_{U(\mathfrak{g})_F} (L(\lambda) \otimes_{K[z]} F).$$

Next we consider the $U(\mathfrak{p})_F$-module $L(\lambda) \otimes F$. This is annihilated by $\mathfrak{g}_1^\perp F$ and so is a $U(\mathfrak{g}_0)_F$-module. In fact it is the finite dimensional simple module over this algebra whose highest weight $\lambda'$ is the unique $F$-linear map $h'_F \rightarrow F$ such that $\lambda'|_\mathfrak{h} = \lambda$ and $\lambda'(z) = z$. Thus $V(\lambda)_\mathcal{C}$ is a Kac module over $U(\mathfrak{g}) \otimes F$. In fact we have

**Proposition.** (a) The module $V(\lambda)_\mathcal{C}$ is a simple Kac module with highest weight $\lambda'$ module over the algebra $U(\mathfrak{g}) \otimes F$.

(b) The module $V(\lambda)_\mathcal{C}$ is a simple module over the algebra $U(\mathfrak{g})_\mathcal{C}$.

**Proof.** It remains to show simplicity in both cases.

(a) We extend $(\ ,\ )$ to a bilinear form $(\ ,\ )_F$ on $(\mathfrak{h}')_F$. It suffices to show that $(\lambda' + \rho, \alpha) \neq 0$ for all odd positive roots $\alpha$. Since the highest exterior power of $\mathfrak{g}_1^+$
is trivial as a \( \mathfrak{g}_0 \)-module we have \( (\rho_1, \beta) = 0 \) for all even roots \( \beta \). Thus \( \mathfrak{h}^+ = K \rho_1 \). If \( \alpha \) is an odd root, it follows that \( (\rho_1, \alpha) = \rho_1(h_\alpha) \neq 0 \). Hence \( h_\alpha \not\in (K \rho_1)^\perp = \mathfrak{h} \), so \( h_\alpha - bz \in \mathfrak{h} \) for some nonzero \( b \in K \). Therefore

\[
(\lambda' + \rho, \alpha)_F = \lambda'(h_\alpha - bz + bz)_F + (\rho, \alpha) = bz + \lambda(h_\alpha - bz) + (\rho, \alpha),
\]

which is a linear polynomial in \( z \).

(b) Fix an order on \( \Delta_1^+ \) and for \( I \subseteq \Delta_1^+ \) set

\[
e_I = \prod_{\alpha \in I} e_{-\alpha}
\]

where the product is taken with respect to this order. Let \( N \) be a nonzero \( U(\mathfrak{g})_c \)-submodule of \( V(\lambda)_c \) and suppose

\[
n = \sum_I e_I n_I
\]

is nonzero with \( n_I \in L(\lambda) \otimes_K [z] F \) for all \( I \). Choose \( m \) minimal such that \( n_I \neq 0 \) for some subset \( I \) with \( |I| = m \). Set \( I' = \Delta_1^+ \setminus I \). Then

\[
e_{I'} n = \pm e_{I\uparrow} n_I
\]

is a nonzero homogeneous element of \( N \), so generates a graded submodule. It follows from Lemma 1.7 that \( N = V(\lambda)_c \).

**Corollary.** (a) If \( N \) is any nonzero \( U(\mathfrak{g})_c \)-submodule of \( V(\lambda)_c \), then \( N \) contains a submodule isomorphic to \( V(\lambda)_c \).

(b) \( P_\lambda = \text{ann}_{U(\mathfrak{g})} V(\lambda)_c \) is a prime ideal of \( U(\mathfrak{g}) \).

**Proof.** (a) By the Proposition \( N_c = (V(\lambda))_c \). Hence \( (L_\lambda \otimes 1) \subseteq N_c \) so \( L_\lambda \otimes (f) \subseteq N \) for some nonzero \( f \). The submodule of \( V(\lambda) \) generated by \( L_\lambda \otimes (f) \) is isomorphic to \( T(L(\lambda), (f)) \cong V(\lambda) \).

(b) This follows since any nonzero submodule of \( V(\lambda)_c \) has annihilator \( P_\lambda \), by part (a).

**1.9. Lemma.** Identify \( L(\lambda) \) with the \( U(\mathfrak{p}) \)-submodule \( 1 \otimes L(\lambda) \) of \( V(\lambda)_c \) and let \( J = U(\mathfrak{p})g_\lambda^+ \). Then \( \text{ann}_{V(\lambda)} J = L(\lambda) \).

**Proof.** We use the same notation as in the proof of Proposition 1.8(b).

For \( 0 \leq m \leq |\Delta_1^+| \) set

\[
V(m) = \bigoplus_{|I|=m} e_I L(\lambda).
\]

Then \( V(\lambda) = \bigoplus_m V(m) \) and \( g_\lambda^+ V(m) \subseteq V(m-1) \). Therefore it suffices to show that if \( m > 0 \), then \( g_\lambda^+ (\sum_{|I|=m} e_I w_I) \neq 0 \) provided the \( w_I \in L(\lambda) \) are not all zero. Now \( L(\lambda) = L_\lambda \otimes K[z] \) has a filtration given by setting \( \text{deg}(L_\lambda \otimes z^n) = n \). Choose \( I \) so that \( w_I \) has maximum degree in this filtration, choose \( \alpha \in I \) and set \( H = I \setminus \{\alpha\} \). Note that

\[
\text{deg}(h_\alpha w_I) = \text{deg}(w_I) + 1.
\]

Using the formula

\[
[e_\alpha, ab] = [e_\alpha, a]b \pm a[e_\alpha, b]
\]

for homogeneous \( a, b \in U(\mathfrak{g}) \) we see that

\[
e_\alpha e_I w_I = \pm e_H h_\alpha w_I
\]

plus a sum of terms of smaller degree. The result follows easily from this.
1.10. The next result is an easy consequence of the Artin-Wedderburn theorem.

Lemma. Let $U$ be a $K$-algebra and $L$ a finite dimensional simple $U$-module. Then for any field extension $K'$ of $K$ we have

$$\text{End}_{U \otimes_K K'}(L \otimes_K K') = K'.$$

1.11. Lemma. For any $\lambda \in P^+$,

$$\text{End}_{U(\mathfrak{g})}(V(\lambda)) \cong K[z].$$

Proof. By the adjoint isomorphism $f \in \text{End}_{U(\mathfrak{g})}(V(\lambda))$ is determined by

$$f_1 = f|_{L(\lambda)} \in \text{Hom}_{U(\mathfrak{g})}(L(\lambda), V(\lambda)).$$

If $J$ is as in Lemma 1.9, then $f_1(L(\lambda)) \subseteq \text{ann}_{V(\lambda)}J = L(\lambda)$, and hence

$$f_1 \in \text{End}_{U(\mathfrak{g})}(L(\lambda) \otimes K[z]) = \text{End}_{U(\mathfrak{g})}(L(\lambda) \otimes K[z]) \cong K[z],$$

using Lemma 1.10.

1.12. Suppose that $\mathfrak{t}$ is a semisimple Lie algebra over $K$ and $C$ is a commutative $K$-algebra. We describe the prime ideals $Q$ of $R = U(\mathfrak{t}) \otimes C$ such that $R/Q$ is P.I. Since $C$ is central, $q = Q \cap C$ is prime in $C$ and by replacing $R$ by the factor ring $R/Rq$ we can assume that $Q \cap C = 0$. There is a one-one correspondence between prime ideals $Q$ of $R$ such that $Q \cap C = 0$ and prime ideals of $U(\mathfrak{t}) \otimes \text{Fract}(C)$. Thus we may assume that $C$ is a field extension of $K$. By the argument on page 2837 of [BM] $Q$ is the annihilator of a finite dimensional simple module over $U(\mathfrak{t}) \otimes C$.

To apply this to our reductive algebra $\mathfrak{g}_0$ with center $Kz$ set $\mathfrak{t} = [\mathfrak{g}_0, \mathfrak{g}_0], R = U(\mathfrak{g}_0)$ and $C = K[z]$. If $q \neq 0$, then $R/Rq \cong U(\mathfrak{t})$ and $R/Q$ is artinian. Thus if $R/Q$ is nonartinian, then $q = 0$ and $Q$ corresponds to the annihilator of a finite dimensional simple module over $U(\mathfrak{t}) \otimes \text{Fract}(C)$. This gives the following result.

Lemma. Suppose $Q$ is a prime ideal of $U(\mathfrak{g}_0)$ such that $U(\mathfrak{g}_0)/Q$ is a nonartinian P.I. ring. Then for some uniquely determined $\lambda \in P^+, Q = \text{ann}_{U(\mathfrak{g}_0)}L(\lambda)$.

Proof of the Main Theorem. Suppose $\lambda \in P^+$ and let $n = \dim_K L_\lambda$. By Corollary 1.8 $P_\lambda$ is a prime ideal in $U(\mathfrak{g})$. Set $U_\lambda = U(\mathfrak{g})/P_\lambda$. Note that $V(\lambda)$ is a torsionfree $K[z]$-module and thus $U_\lambda$ embeds in $(U_\lambda)_C$. Since $V(\lambda)$ is a $U(\mathfrak{g}) - K[z]$-bimodule which is free of rank $N = n2^{\dim \mathfrak{g}}$, $U_\lambda$ embeds in $M_N(K[z])$. This embedding induces an embedding of $(U_\lambda)_C$ into $M_N(F)$ which is surjective since $V(\lambda)_C$ is a simple $(U_\lambda)_C$-module of dimension $N$ over its endomorphism ring $F$.

Conversely suppose $P$ is a prime ideal of $U(\mathfrak{g})$ with $U(\mathfrak{g})/P$ a nonartinian P.I. ring. We apply the results in sections 1.1 and 1.2 with $R = U(\mathfrak{p})$ and $S = U(\mathfrak{g})$. If $Q \in V_P$ there is a bond from $S/P$ to $R/Q$, so $R/Q$ is a nonartinian P.I. ring, by Lemma 1.1. Lemma 1.12 implies that $Q = Q_\lambda$ for some $\lambda \in P^+$. Since $P \in X_Q$, $P$ is minimal over $\text{ann}_S(S/SQ)$ which equals $\text{ann}_S V(\lambda) = P_\lambda$ by [BGR] Satz 10.4. As $P_\lambda$ is prime we get $P = P_\lambda$. To show that $\lambda$ is uniquely determined by $P$ it suffices to show that $V_{P_\lambda} = \{Q_\lambda\}$. However if $Q' \in V_{P_\lambda}$, then by Lemma 1.4 $Q' = \text{ann}_R N$ for some $R$-submodule $N$ of $V(\lambda)$ which is fully faithful as an $R/Q'$-submodule. Since $J = U(\mathfrak{p}) \mathfrak{g}_0^{+\mathfrak{p}}$ is nilpotent $J \subseteq Q'$, so using Lemma 1.9 $N \subseteq \text{ann}_{V(\lambda)}J = L(\lambda)$. However every nonzero submodule of $L(\lambda)$ has annihilator $Q_\lambda$, so $Q' = Q_\lambda$ as desired.
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