

## NILPOTENCY DEGREE OF COHOMOLOGY RINGS IN CHARACTERISTIC $p$

PHAM ANH MINH

(Communicated by Stephen D. Smith)

*Dedicated to Professor Huỳnh Mùi on his sixtieth birthday*

ABSTRACT. Let  $p$  be an odd prime number. The purpose of this paper is to provide a  $p$ -group  $\mathcal{G}$  whose mod- $p$  cohomology ring has a nilpotent element  $\xi \in H^*(\mathcal{G})$  satisfying  $\xi^p \neq 0$ .

### 1. STATEMENT OF THE MAIN RESULT

For every  $p$ -group  $\mathcal{G}$ , denote by  $H^*(\mathcal{G})$  the mod- $p$  cohomology algebra of  $\mathcal{G}$ . We are now interested in the nilpotency degrees of elements of  $H^*(\mathcal{G})$ . For the case  $p = 2$ , in [1], [4], it was shown that, given any positive integer  $n$ , there exists a 2-group whose cohomology ring has elements of nilpotency degree  $n + 1$ . It was also noted in [1] that, for  $p$  odd, we did not have any example of elements of  $H^*(\mathcal{G})$  having nilpotency degrees greater than  $p$ .

Recently, such an example was given in [8] for  $p = 3$ . In this case,  $\mathcal{G}$  is chosen to be an extension of  $\mathbb{E} \times \mathfrak{A}$  by  $\mathbb{Z}/p$ , where  $\mathbb{E}$  is the extraspecial 3-group of order  $3^3$  and of exponent 3, and  $\mathfrak{A}$  the elementary abelian 3-group of rank 4.

The purpose of this paper is to generalize the result in [8] to the case of any odd prime  $p$ . Let  $\mathfrak{S}_{p^2,p}$  be the Sylow subgroup of the symmetric group on  $p^2$  letters (so  $\mathfrak{S}_{p^2,p} = \mathbb{Z}/p \wr \mathbb{Z}/p$ , the wreath product of  $\mathbb{Z}/p$  and  $\mathbb{Z}/p$ ) and let  $Z$  be the center of  $\mathfrak{S}_{p^2,p}$ . Define  $\mathfrak{R} = \mathfrak{S}_{p^2,p}/Z$  (so  $\mathfrak{R} = \mathbb{E}$  for  $p = 3$ ) and let  $\mathfrak{A}_k$  be the elementary abelian  $p$ -group of rank  $2k$ . We shall prove

**Theorem.** *For  $k \geq 2p - 2$ , there exists a  $p$ -group  $\mathcal{G}$  given by a central extension*

$$0 \rightarrow \mathbb{Z}/p \rightarrow \mathcal{G} \rightarrow \mathfrak{R} \times \mathfrak{A}_k \rightarrow 1$$

*whose mod- $p$  cohomology ring has a nilpotent element  $\xi \in H^2(\mathcal{G})$  satisfying  $\xi^p \neq 0$ .*

We shall use the following notation. For homogeneous elements  $X, Y, \dots$  of a graded ring  $R$ ,  $|X|$  denotes the degree of  $X$  and  $(X, Y, \dots)$  the ideal of  $R$  generated by  $X, Y, \dots$ .  $p$  is assumed from now on to be an arbitrary odd prime number. If  $S$  is a subset of a group  $G$ , then  $\langle S \rangle$  denotes the subgroup of  $G$  generated by  $S$ . With some abuse of notation, for every  $\zeta \in H^*(G)$  and for every subgroup  $K$  of  $G$ , we consider  $\zeta$  as an element of  $H^*(K)$  via the restriction map; also, for every extension  $T$  of  $G$ ,  $\zeta$  is considered as an element of  $H^*(T)$  via the inflation maps.

---

Received by the editors August 15, 2001 and, in revised form, September 18, 2001.  
2000 *Mathematics Subject Classification.* Primary 20J06; Secondary 20D15, 55R40.

2. THE GROUPS  $\mathfrak{R}$  AND  $\mathfrak{U}$

Let  $\mathfrak{U}$  be the  $p$ -group of order  $p^{p+2}$  given by

$$\begin{aligned} \mathfrak{U} &= \langle a, a_1, \dots, a_{p+1} \mid a^p = a_3^p = \dots = a_{p+1}^p = [a, a_{p+1}] = [a_i, a_j] = a_1^p a_2^{p(p-1)/2} a_p \\ &= a_2^p a_{p+1} = 1, [a, a_\ell] = a_{\ell+1}, 1 \leq i, j \leq p+1, 1 \leq \ell \leq p \rangle. \end{aligned}$$

Then  $\mathfrak{U}$  is a 2-generator  $p$ -group of maximal class and  $\mathfrak{R} = \mathfrak{U}/\langle a_p, a_{p+1} \rangle$  (see e.g. [2, Section 4]). Set  $\mathfrak{T} = \mathfrak{U}/\langle a_{p+1} \rangle, g_i = a_i \langle a_p, a_{p+1} \rangle \in \mathfrak{R}, 1 \leq i \leq p-1$ , and  $K = \langle g_1, \dots, g_{p-1} \rangle$ . We then have extensions of groups

$$\begin{aligned} (\mathfrak{U}) \quad & 0 \rightarrow \mathbb{Z}/p \rightarrow \mathfrak{U} \xrightarrow{p_1} \mathfrak{T} \rightarrow 1, \\ (\mathfrak{T}) \quad & 0 \rightarrow \mathbb{Z}/p \xrightarrow{j} \mathfrak{T} \xrightarrow{p_2} \mathfrak{R} \rightarrow 1, \\ (\mathfrak{R}) \quad & 0 \rightarrow (\mathbb{Z}/p)^{p-1} \cong K \rightarrow \mathfrak{R} \rightarrow \langle a \rangle \rightarrow 1. \end{aligned}$$

Let  $x \in H^1(\langle a \rangle)$  be the dual of  $a$  and consider  $x$  as an element of  $H^1(\mathfrak{R})$  via the inflation map. Define  $x_1 \in H^1(\mathfrak{R})$  satisfying  $x_1(g_1) = 1, x_1(a) = 0$  ( $x_1$  is well-defined, as  $\mathfrak{R}$  is a 2-generator  $p$ -group and  $\mathfrak{R} = \langle a, g_1 \rangle$ ).  $x, x_1$  is then a basis of  $H^1(\mathfrak{R})$ . Set  $y = \beta x, y_1 = \beta x_1$  with  $\beta$  the Bockstein homomorphism. Let  $w$  be the element of  $H^2(\mathfrak{R})$  classifying the extension  $(\mathfrak{T})$  and set  $U = w + y_1$ . We have

- Lemma 1.** (i)  $U|_K = 0$ ;  
 (ii)  $U^p + xy^{p-2} \cdot \beta U - y^{p-1} \cdot U = 0$ ;  
 (iii)  $x \cdot U = xy_1$ ;  
 (iv)  $U^p = y^{p-1}y_1$ .

*Proof.* Set  $f = p_1(a_1)$ . Since  $f^p = j(-1)$  and  $p_2^{-1}(K)$  is abelian,  $w|_K = -y_1$ . Hence  $U|_K = 0$ . (ii) follows from [6, Remark 1] (see also [7, Theorem 1.1]) and the fact that  $U|_K = 0$ .

Note that the term  $E_2(\mathfrak{T})$  of the Hochschild-Serre spectral sequence corresponding to the extension  $(\mathfrak{T})$  is of form

$$E_2(\mathfrak{T}) = H^*(\mathfrak{R}) \otimes H^*(j(\mathbb{Z}/p)).$$

Let  $w'$  be the element of  $H^2(\mathfrak{T})$  classifying the extension  $(\mathfrak{U})$  and let  $u$  be the element of  $H^1(j(\mathbb{Z}/p))$  satisfying  $d_2(u) = w$ . Since  $\langle a_p, a_{p+1} \rangle = p_1^{-1}(\text{Im } j)$  is of exponent  $p$ ,  $w'$  restricts trivially on  $j(\mathbb{Z}/p)$ . So  $w'$  represents a non-zero element of  $E_\infty^{2,0}(\mathfrak{T}) \oplus E_\infty^{1,1}(\mathfrak{T})$ . Note that  $X|_{\langle a, j(\mathbb{Z}/p) \rangle}$  (resp.  $X|_{\langle a_1, j(\mathbb{Z}/p) \rangle}$ ) is a scalar multiple of  $y$  (resp.  $y_1$ ), for any  $X \in E_\infty^{2,0}(\mathfrak{T}) \subset \text{Im Inf}_{\mathfrak{T}}^{\mathfrak{R}}$ . As  $[a, a_p] = a_{p+1}$  and  $[a_1, a_p] = 1$ , it follows that  $w'$  represents a non-zero element  $\theta$  of  $E_2^{1,1}(\mathfrak{T})$  and  $w'|_{\langle a, j(\mathbb{Z}/p) \rangle} \neq 0, w'|_{\langle f, j(\mathbb{Z}/p) \rangle} = 0$ . So  $\theta$  is a non-zero scalar multiple of  $x \otimes u$ . This means that  $xd_2(u) = xw = 0$ . Hence  $x \cdot U = x(w + y_1) = xy_1$ .

Finally, as

$$\begin{aligned} y \cdot U - x \cdot \beta U &= \beta(x \cdot U) \\ &= \beta(xy_1) \quad \text{by (iii)} \\ &= yy_1, \end{aligned}$$

we have, by (ii),

$$\begin{aligned} 0 &= U^p + xy^{p-2} \cdot \beta U - y^{p-1} \cdot U \\ &= U^p + y^{p-2}(x \cdot \beta U - y \cdot U) \\ &= U^p - y^{p-2} \cdot yy_1 \\ &= U^p - y^{p-1}y_1. \end{aligned}$$

The lemma is proved. □

*Remarks.* 1. It is known that  $\text{Inf}_{\mathfrak{S}_{p^2,p}}^{\mathfrak{A}} : H^2(\mathfrak{A}) \rightarrow H^2(\mathfrak{S}_{p^2,p})$  is surjective (this comes from the fact that  $\mathfrak{S}_{p^2,p}$  is a terminal  $p$ -group; see [3]). Furthermore, as  $H^*(\mathfrak{S}_{p^2,p})$  is detected by elementary abelian subgroups ([9]) and  $U|_{\langle a \rangle} = 0, U|_K = 0$ , it follows that  $\text{Inf}_{\mathfrak{S}_{p^2,p}}^{\mathfrak{A}}(U) = 0$ . Hence  $U$  is nothing other than the cohomology class classifying the central extension  $1 \rightarrow Z \rightarrow \mathfrak{S}_{p^2,p} \rightarrow \mathfrak{A} \rightarrow 1$ .

2. The group  $\mathfrak{U}$  can be obtained from algebraic number theory, as follows. Let  $\vartheta$  be a primitive  $p$ th root of unity and let  $\mathbb{Z}_p$  be the ring of  $p$ -adic integers. Then

$$\mathbb{Z}_p[\vartheta] = \bigoplus_{j=0}^{p-2} \vartheta^j \mathbb{Z}_p.$$

Let  $\langle a \rangle$  be the cyclic group of order  $p$ . Multiplication by  $\vartheta$  induces an action of  $\langle a \rangle$  on  $\mathbb{Z}_p[\vartheta]$ , hence on  $\mathfrak{Z} = \mathbb{Z}_p[\vartheta]/((\vartheta - 1)^{p+1})$ . We can check that the map

$$\{a, a_1, \dots, a_{p+1}\} \rightarrow \langle a \rangle \times \mathfrak{Z}$$

which maps  $a$  to  $a$ ,  $a_i$  to  $(\vartheta - 1)^{i-1}$ ,  $1 \leq i \leq p + 1$ , can be extended to a homomorphism of groups from  $\mathfrak{U}$  to  $\langle a \rangle \times \mathfrak{Z}$  which is bijective. So  $\mathfrak{U} \cong \langle a \rangle \times \mathfrak{Z}$ .

### 3. PROOF OF THE THEOREM

Pick a basis  $c_1, \dots, c_{2k}$  of the elementary abelian  $p$ -group  $\mathfrak{A}_k$  of rank  $2k$ . Let  $u_1, \dots, u_{2k}$  be the basis of  $H^1(\mathfrak{A}_k)$ , dual to that of  $\mathfrak{A}_k$ . Define  $v_i = \beta u_i$ ,  $1 \leq i \leq 2k$ . Set  $G = \mathfrak{A} \times \mathfrak{A}_k$ . By Künneth formula,

$$\begin{aligned} H^*(G) &= H^*(\mathfrak{A}) \otimes H^*(\mathfrak{A}_k) \\ &= H^*(\mathfrak{A}) \otimes \Lambda[u_1, \dots, u_{2k}] \otimes \mathbb{F}_p[v_1, \dots, v_{2k}]. \end{aligned}$$

Let

$$\begin{aligned} z &= y + u_1 u_2 + \dots + u_{2k-1} u_{2k}, \\ \alpha &= u_1 u_2 + \dots + u_{2k-1} u_{2k}, \\ \gamma &= u_3 u_4 + \dots + u_{2k-1} u_{2k} \end{aligned}$$

be elements of  $H^*(G)$ . Set  $\Lambda = \Lambda[u_1, \dots, u_{2k}]$  and  $\Lambda' = \Lambda[u_3, \dots, u_{2k}]$ . For  $i \geq 0$ , let  $\Lambda_i$  be the degree  $i$ -part of  $\Lambda$ . We have

**Lemma 2.** *Let  $0 \neq X$  be a homogeneous element of  $H^*(G)$  and let  $n, k$  be positive integers.*

- (i) *If  $n \leq \min(p - 1, k)$  and  $X \cdot \alpha^n = 0$ , then  $|X| \geq \min(2p - 2n, k - n + 1)$ .*
- (ii) *If  $X \cdot z = 0$ :*
  - (iia)  *$|X| \geq \min(2p - 2, k)$ ;*
  - (iib)  *$|X| > 2p - 1$ , provided that  $k \geq 2p - 2$  and  $X \in (v_1, \dots, v_{2k})$ .*

*Proof.* (i) By induction on  $k$ . It obviously holds for  $k = 1$ . Assume that it holds for  $k - 1$ . Without loss of generality, we may assume that  $X \in \Lambda$ . If  $n = k \leq p - 1$ , then  $0 = X \cdot \alpha^n = k!X \cdot u_1 \dots u_{2k}$  implies  $|X| \geq 1$ . We may then suppose that  $n \leq k - 1$ .

Write  $X = X_1 + X_2 \cdot u_1 u_2 + X_3 \cdot u_1 + X_4 \cdot u_2$  with  $X_i$  homogeneous in  $\Lambda'$ . So  $0 = X \cdot \alpha^n = X \cdot (\gamma + u_1 u_2)^n$  implies

$$(1) \quad X_1 \cdot \gamma^n = 0,$$

$$(2) \quad (nX_1 + X_2 \cdot \gamma) \cdot \gamma^{n-1} = 0,$$

$$(3) \quad X_3 \cdot \gamma^n = X_4 \cdot \gamma^n = 0.$$

If  $X_3 \neq 0$  or  $X_4 \neq 0$ , it follows from (3) and the inductive hypothesis that  $\min(|X_3|, |X_4|) \geq \min(2p - 2n, k - n)$ , hence

$$|X| = \min(|X_3|, |X_4|) + 1 \geq \min(2p - 2n + 1, k - n + 1).$$

So we may suppose that  $X_3 = X_4 = 0$ . By (2) and by the inductive hypothesis,  $|nX_1 + X_2 \cdot \gamma| \geq \min(2p - 2n + 2, k - n + 1)$ . Hence  $|X| = |nX_1 + X_2 \cdot \gamma| \geq \min(2p - 2n, k - n + 1)$  if  $nX_1 + X_2 \cdot \gamma \neq 0$ . If  $nX_1 + X_2 \cdot \gamma = 0$ , then  $nX_1 = -X_2 \cdot \gamma$ . By (1),  $X_2 \cdot \gamma^{n+1} = 0$ . By the inductive hypothesis,  $|X_2| \geq \min(2p - 2n - 2, k - n - 1)$ . Hence  $|X| = |X_2| + 2 \geq \min(2p - 2n, k - n + 1)$ . (i) is then proved.

(ii) Write  $X = \sum_{i=0}^m X_i \cdot Y_i$  with  $Y_i \in \Lambda_i, X_i \in H^*(\mathfrak{R}) \otimes \mathbb{F}_p[v_1, \dots, v_{2k}]$  and  $X_m \cdot Y_m \neq 0$ . Then  $X_m \cdot Y_m \cdot \alpha = 0$ . By (i), it follows that  $|X| = |X_m \cdot Y_m| \geq \min(2p - 2, k)$ . (iia) is proved.

Suppose that  $k \geq 2p - 2, |X| \leq 2p - 1$  and  $X \in (v_1, \dots, v_{2k})$ . Fix  $i$  with  $1 \leq i \leq 2k$  and write  $X = v_i X' + X''$  with  $X''$  free of  $v_i$ . Then  $X' \cdot z = 0$ . As  $|X'| \leq 2p - 3$ , it follows from (iia) that  $X' = 0$ .  $X$  is then free of  $v_i$ . Hence by repeating the argument, we see that  $X$  is free of  $v_1, \dots, v_{2k}$ , a contradiction.

The lemma is proved. □

Suppose from now on that  $k \geq 2p - 2$ . We have

**Lemma 3.** *If  $X$  is homogeneous in  $H^*(G)$  of degree  $\leq 2p - 5$  and  $X \cdot \beta z \in (z)$ , then  $X \in (z, \beta z)$ .*

*Proof.* Write  $X \cdot \beta z = A \cdot z$  with  $A$  homogeneous in  $H^*(G)$ . Then  $A \cdot \beta z \cdot z = 0$ . As  $|A \cdot \beta z| = |X| + 4 \leq 2p - 1$  and  $A \cdot \beta z \in (v_1, \dots, v_{2k})$ , it follows from Lemma 2(ii) that  $A \cdot \beta z = 0$ . By [5, Lemma 2.1],  $A \in (\beta z, u_1 \dots u_{2k})$ . So  $A \in (\beta z)$ , as  $|A| \leq 2p - 4 < 2k$ . Write  $A = B \cdot \beta z$ . We then have  $(X - B \cdot z)\beta z = 0$ . Again, by [5, Lemma 2.1],  $X - B \cdot z \in (\beta z)$ . Therefore  $X \in (z, \beta z)$ . The lemma is proved. □

For every  $1 \leq j \leq p - 1$ , let  $B_j$  be a set of elements of  $\Lambda_{2j}$  such that the disjoint union  $B_j \sqcup \{\alpha^j\}$  forms a basis of  $\Lambda_{2j}$ . We then have a decomposition

$$(4) \quad H^*(\mathfrak{R}) \otimes \Lambda_{2j} = H^*(\mathfrak{R}) \otimes (\langle B_j \rangle \oplus \langle \alpha^j \rangle), 1 \leq j \leq p - 1.$$

For  $1 \leq i < p - 1$  and for  $0 \neq b \in \langle B_i \rangle, \mu \in \mathbb{Z}/p$ , as  $|b - \mu \alpha^i| = 2i < 2p - 2$ , it follows from Lemma 2 that  $(b - \mu \alpha^i) \cdot \alpha \neq 0$ . The  $B_j$ 's can then be chosen such that, for every  $1 \leq i < p - 1$ ,

$$(5) \quad \{b \cdot \alpha \mid 0 \neq b \in \langle B_i \rangle\} \subset \langle B_{i+1} \rangle.$$

**Lemma 4.**  $y^{p-1}y_1 \notin (z, \beta z)$ .

*Proof.* Note that, as  $\mathfrak{R}$  is of exponent  $p$ ,  $y^{p-1}y_1|_{\langle ag_1 \rangle} \neq 0$ . So  $y^{p-1}y_1 \neq 0$ .

Suppose that  $y^{p-1}y_1 = A \cdot z + B \cdot \beta z$  with  $A, B$  homogeneous in  $H^*(G)$ . Write  $A = A' + A''$  with  $A' \in H^*(\mathfrak{R}) \otimes \Lambda, A'' \in (v_1, \dots, v_{2k})$ . It follows that  $y^{p-1}y_1 = A' \cdot z$  and  $A'' \cdot z + B \cdot \beta z = 0$ . Hence, without loss of generality, we may assume that  $y^{p-1}y_1 = A \cdot z$  with  $A \in H^*(\mathfrak{R}) \otimes \Lambda$ . By the decomposition (4), we have

$$A = A_{2p-2} + \sum_{i=1}^{p-1} (A_{2p-2i-2} \cdot \alpha^i + \sum_{b \in B_i} A_{2p-2i-2,b} \cdot b)$$

with  $A_j, A_{j,b} \in H^*(\mathfrak{R})$ . By (5), it follows that

$$\begin{aligned} A_{2p-2}y &= y^{p-1}y_1, \\ (A_{2p-2} + A_{2p-4}y) \cdot \alpha &= 0, \\ &\dots \\ (A_2 + A_0y) \cdot \alpha^{p-1} &= 0. \end{aligned}$$

Therefore, for every  $i \geq 1, A_{2i} = -A_{2i-2}y$ . So  $y^{p-1}y_1 = A_0y^p$ . As  $y^{p-1}y_1|_{\langle a \rangle} = 0$ , this implies  $A_0 = 0$ . Hence  $y^{p-1}y_1 = 0$ , a contradiction. Thus  $y^{p-1}y_1 \notin (z, \beta z)$ . The lemma is proved.  $\square$

Let  $\{E_r, d_r\}$  be the Hochschild-Serre spectral sequence corresponding to the central extension

$$0 \rightarrow \mathbb{Z}/p \xrightarrow{i} \mathcal{G} \rightarrow G \rightarrow 1$$

classified by  $z \in H^2(G)$ . Pick  $s$  (resp.  $t$ ) in  $H^1(i(\mathbb{Z}/p))$  (resp.  $H^2(i(\mathbb{Z}/p))$ ) satisfying  $d_2(s) = z$  (resp.  $d_3(t) = \beta z$ ). It is known that

$$\begin{aligned} E_2 &= H^*(G) \otimes H^*(i(\mathbb{Z}/p)), \\ E_3 &= H^*(G)/(z) \otimes \mathbb{F}_p[t] \oplus \text{Ann}_{H^*(G)}(z) \otimes \mathbb{F}_p[t]s. \end{aligned}$$

We have

**Lemma 5.** *There exists no element  $\eta \in E_n^{2p-n, n-1}, n \geq 3$ , satisfying  $d_n(\eta) = y^{p-1}y_1$ .*

*Proof.* Suppose that  $\eta$  is an element of  $E_{2r}^{2p-2r, 2r-1}, r \geq 2$ , represented by  $X \otimes t^{r-1}s \in E_2^{2p-2r, 2r-1}$ . Then  $0 = d_2(X \otimes t^{r-1}s) = X \cdot z \otimes t^{r-1}$ . So  $X \cdot z = 0$ . Since  $|X| = 2p - 2r \leq 2p - 4, X = 0$  by Lemma 2. Hence  $0 = d_{2r}(\eta) \neq y^{p-1}y_1$ .

Suppose now that  $\eta \in E_{2r+1}^{2p-2r-1, 2r}$  is represented by  $X \otimes t^r \in E_3^{2p-2r-1, 2r}, p-1 \geq r \geq 1$ . If  $r = 1$ , then  $y_1^{p-1}y_2 \neq X \cdot \beta z = -d_3(X \otimes t)$  by Lemma 4. If  $r \geq 2$ , then  $-rX \cdot \beta z \otimes t^{r-1} = d_3(X \otimes t^r) = 0$  in  $E_3$ . So  $X \cdot \beta z \in (z)$ . As  $|X| = 2p - 2r - 1 \leq 2p - 5, X \in (z, \beta z)$  by Lemma 3. This implies  $X \otimes t^r = 0$  in  $E_4$  unless  $r = p - 1$  and  $X \in (\beta z)$ . But  $r = p - 1$  also implies  $|X| = 2p - 2r - 1 = 1$  and  $X \in (\beta z)$ , hence  $X = 0$ . So  $0 = d_{2r+1}(\eta) \neq y^{p-1}y_1$ . The lemma is proved.  $\square$

*Proof of the Theorem.* Set  $\xi = \text{Inf}_G^G(U)$ . By Lemma 1,  $\xi^p = y^{p-1}y_1$ . Since  $z$  vanishes in  $H^*(\mathcal{G}), y^p = -\alpha^p = 0$  in  $H^*(\mathcal{G})$ . So  $\xi^{2p} = y^{2p-2}y_1^2 = 0$ . Hence  $\xi$  is nilpotent. Besides, Lemmas 4 and 5 show that  $y^{p-1}y_1 \notin \text{Ker } \text{Inf}_G^G$ , which means that  $\xi^p = y^{p-1}y_1 \neq 0$ . The Theorem is proved.  $\square$

## REFERENCES

1. G.S. Avrunin, J.F. Carlson, *Nilpotency degree of cohomology rings in characteristic two*, Proc. Amer. Math. Soc. **118** (1993), 339-443. MR **93g**:20098
2. N. Blackburn, *On a special class of  $p$ -groups*, Acta Math. **100** (1958), 45-92. MR **21**:1349
3. L. Evens, *Terminal  $p$ -groups*, Illinois J. Math. **12** (1968), 682-699. MR **39**:2875
4. K. Inoue, A. Kono, *Nilpotency of a kernel of the Quillen map*, J. Math. Kyoto Univ. **33** (1993), 1047-1055. MR **95a**:20053
5. P. A. Minh, *On the mod  $p$  cohomology groups of extra-special  $p$ -groups*, Japan. J. Math. **18** (1992), 139-154.
6. ———, *Essential mod- $p$  cohomology classes of  $p$ -groups: an upper bound for the nilpotency degree*, Bull. London Math. Soc. **32** (2000), 285-291. MR **2001c**:20106
7. ———, *Evens norm and restriction maps in mod- $p$  cohomology of  $p$ -groups*, Math. Proc. Camb. Phil. Soc. **129** (2000), 253-262. MR **2001f**:20119
8. ———, *Nilpotency degree of cohomology rings in characteristic 3*, Proc. Amer. Math. Soc. **130** (2002), 307-310.
9. D. Quillen, *The Adams conjecture*, Topology **10** (1971), 67-80. MR **43**:5525

DEPARTMENT OF MATHEMATICS, COLLEGE OF SCIENCES, UNIVERSITY OF HUE, DAI HOC KHOA HOC, HUE, VIETNAM

*E-mail address:* `paminh@dng.vnn.vn`