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A CORRECTION TO "ULTRADIFFERENTIABLE FUNCTIONS ON LINES IN $\mathbb{R}^n$"

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ABSTRACT. The proof of Theorem 1 in Proc. Amer. Math. Soc. **127** (1999), no. 7, 2099–2104, is revised.

The proof of Theorem 1 in [1] uses Lemma 3(ii) which turns out to be valid only for the case $n = 2$. In this note the proof of Theorem 1 is revised by replacing Lemma 3(ii) by the proposition stated below.

For $x \in \mathbb{R}^n$, $0 \neq \xi \in \mathbb{S}^{n-1}$, let $H_{x\xi}$ denote the hyperplane in $\mathbb{R}^n$ that passes through x with ξ as its normal vector. If $\varphi : \mathbb{R}^{n-1} \to \mathbb{R}^n$ is a linear map such that $H_{x\xi} = \varphi(\mathbb{R}^{n-1})$, then the restriction of a function $u \in C^\infty(\mathbb{R}^n)$ to $H_{x\xi}$ is defined by $u_{x\xi}(\mathbf{t}) = u(x + \varphi(\mathbf{t}))$, $\mathbf{t} \in \mathbb{R}^{n-1}$.

Proposition. *For any $u \in C^\infty(\mathbb{R}^n)$, the following inequality holds:*

$$\max_{|\alpha|=k} |\partial^\alpha u(x)| \leq \max_{\beta \in \mathbb{Z}_+^{n-1}, |\beta|=k} \max_{|\xi|=1} \left|\partial^\beta (u_{x\xi})(0)\right|, \forall k \geq 0. \tag{0.1}$$

Proof. Conforming to the notation in [1], let $a_1, \varepsilon \in \mathbb{R}, \varepsilon > 0$, $x \in \mathbb{R}^n, k \in \mathbb{Z}, k \geq 0$, and $(\alpha_1, \alpha_2, \ldots, \alpha_n) \in \mathbb{Z}_+^n, |\alpha| = k$, be fixed. Put $a_{1j} = a_1 + \frac{\varepsilon j}{k}, j = 0, ..., k$. Consider the hyperplanes given by the images of the linear maps

$$\begin{aligned} \varphi_j &: \mathbb{R}^{n-1} \to \mathbb{R}^n, \\ \varphi_j(t_2, ..., t_n) &= (x_1 + a_{1j} t_2, x_2 + t_2, ..., x_n + t_n), \forall j, 0 \leq j \leq k. \end{aligned}$$

Then,

$$\partial_2^{\alpha_1+\alpha_2} \partial^{\alpha''} (u \circ \varphi_j)(0) = \sum_{l=0}^{\alpha_1+\alpha_2} a_{1j}^{\alpha_1} \left(\partial_1^l \partial_2^{\alpha_1+\alpha_2-l} \partial^{\alpha''} u\right)(x),$$

where $\partial^{\alpha''} = \partial_3^{\alpha_3} \cdots \partial_n^{\alpha_n}$. By applying Lemma 3(i), we have for $\varepsilon = 8e^3$ and $l = \alpha_1$,

$$|\partial^\alpha u(x)| \leq \max_{|\beta|=|\alpha|} \max_{0 \leq r \leq k} \left|\partial^\beta (u \circ \varphi_r)(0)\right|.$$

□

Let V be an n-dimensional vector space. By using a linear isomorphism between V and $\mathbb{R}^n$, the classes $C^\infty(V)$ and $C^M(V)$ can be identified with $C^\infty(\mathbb{R}^n)$ and

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$C^M(\mathbb{R}^n)$, respectively. When $n = 2$, the inequality (0.1) reduces to the last inequality in the proof of Theorem 1. In particular, the conclusion of Theorem 1 is valid for any 2-dimensional vector space V. Hence, by the above proposition and by induction on n, we see that Theorem 1 holds for all n.

Reference

1. Tejinder Neelon, *Ultradifferentiable functions on lines in* $\mathbb{R}^n$, Proc. Amer. Math. Soc. **127** (1999), no. 7, 2099–2104. MR **99j:**46044

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