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EXTENDED CESÀRO OPERATORS ON MIXED NORM SPACES

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Abstract. We define an extended Cesàro operator T_g with holomorphic symbol g in the unit ball B of C^n as

$$T_g(f)(z) = \int_0^1 f(tz)\Re g(tz)\frac{dt}{t}, \qquad f \in H(B), z \in B,$$

where $\Re g(z) = \sum_{j=1}^n z_j \frac{\partial f}{\partial z_j}$ is the radial derivative of g. In this paper we characterize those g for which T_g is bounded (or compact) on the mixed norm space $H_{p,q}(w)$.

1. Introduction

We denote by D the unit disk in the complex plane C. For a holomorphic function $f(z)$ on D with Taylor expansion $f(z) = \sum_{j=0}^{\infty} a_j z^j$, the Cesàro operator acting on f is

$$C[f](z) = \sum_{j=0}^{\infty}\left(\frac{1}{j+1}\sum_{k=0}^{j} a_k\right) z^j.$$

It is well known that $C[\cdot]$ acts as a bounded linear operator on various spaces of holomorphic functions (see [7], [8], [13], [15]) including the Hardy and Bergman spaces. In particular $C[H^p] \subset H^p$ for each $p \in (0,\infty)$. In [11] Shi and Ren proved $C[\cdot]$ is bounded on the mixed norm space.

A little calculation shows $C[f](z) = \frac{1}{z}\int_0^z f(t)\left(\log\frac{1}{1-t}\right)' dt$. Hence, on most holomorphic function spaces, $C[f]$ is bounded if and only if the integral operator $f \mapsto \int_0^z f(t)\left(\log\frac{1}{1-t}\right)' dt$ is bounded. From this point of view it is natural to consider the extended Cesàro operator T_g with holomorphic symbol g

$$T_g f(z) = \int_0^z f(t)g'(t)dt. \tag{1}$$

The boundedness of T_g on Hardy space and Bergman space are studied in [1] and [2].

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The purpose of this paper is to define the extended Cesàro operator on the unit ball B of C^n and to characterize those holomorphic symbols on B for which the induced operator is bounded (or compact) on mixed norm space. Our work will extend [11, 15].

The class of all holomorphic functions on the unit ball B of C^n will be denoted by $H(B)$. For $g \in H(B)$ having the homogeneous expansion $g = \sum_{j=0}^{\infty} G_j$, as in [9] we let $\Re g(z) = \sum_{j=0}^{\infty} jG_j(z)$ be the radial derivative of g. With a little calculation one can show that $\Re g(z) = \sum_{j=1}^{n} z_j \frac{\partial g}{\partial z_j}$. Given $g \in H(B)$, the extended Cesàro operator T_g with symbol g is defined on $H(B)$ as

$$T_g(f)(z) = \int_0^1 f(tz)\Re g(tz)\frac{dt}{t}, \qquad f \in H(B), z \in B. \tag{2}$$

It is trivial that when $n = 1$, (2) is just (1).

A positive continuous function φ on $[0, 1)$ is called normal if there are two constants $b > a > 0$ such that

$$\frac{\varphi(r)}{(1-r)^a} \downarrow 0, \qquad \frac{\varphi(r)}{(1-r)^b} \uparrow \infty \tag{3}$$

as $r \to 1^-$. Given φ normal, for $f \in H(B)$ we set

$$\|f\|_{p,q,\varphi} = \left\{\int_0^1 M_q^p(f,r)\frac{\varphi^p(r)}{1-r}dr\right\}^{\frac{1}{p}}, \qquad 0 < p < \infty,$$

and

$$\|f\|_{\infty,q,\varphi} = \sup_{0<r<1} M_q(f,r)\varphi(r).$$

Here

$$M_q(f,r) = \left\{\int_{\partial B} |f(r\zeta)|^q d\sigma(\zeta)\right\}^{\frac{1}{q}}, \qquad 0 < q < \infty,$$

$$M_\infty(f,r) = \sup_{\zeta\in\partial B} |f(r\zeta)|.$$

The mixed norm space $H_{p,q}(\varphi)$, $0 < p, q \le \infty$, consists of all $f \in H(B)$ such that $\|f\|_{p,q,w} < \infty$. When $0 < p = q < \infty, H_{p,q}(\varphi)$ is just the weighted Bergman space

$$A_a^p(\varphi) = \left\{f \in H(B) : \|f\|_{A_a^p} = \left\{\int_B |f(z)|^p \frac{\varphi^p(|z|)}{1-|z|}dm(|z|)\right\}^{\frac{1}{p}} < \infty\right\}.$$

As in [14] a function $f \in H(B)$ is called a Bloch function if $\sup\{Q_f : z \in B\} < \infty$, and f is called a little Bloch function if $\lim_{|z|\to 1} Q_f(z) = 0$. By Theorem 1.8 and Corollary 3.8 in [14] we know that a holomorphic function f is a Bloch function if and only if

$$\|f\|_{\mathcal{B}} = \sup\{|\Re f(z)|(1-|z|^2) : z \in B\} < \infty,$$

and f is a little Bloch function if and only if

$$\lim_{|z|\to 1} |\Re f(z)|(1-|z|^2) = 0.$$

The sets of all Bloch and little Bloch functions will be denoted by $\mathcal{B}$ and $\mathcal{B}_0$, respectively. We also know that $\|f\|_{\mathcal{B}}$ is an equivalent norm on $\mathcal{B}/C$.

In what follows C, C_1, C_2 will stand for positive constants whose value may change from line to line but not depend on the functions in $H(B)$. The expression $A \simeq B$ means $C_1 A \leq B \leq C_2 A$.

Our main result is the following.

Theorem 1. *Let $0 < p, q \leq \infty$, and let φ be normal. Then for $g \in H(B)$*

(I) *T_g is bounded on $H_{p,q}(\varphi)$ if and only if $g \in \mathcal{B}$. Moreover, when T_g is bounded,*

$$\|T_g\| \simeq \|g\|_{\mathcal{B}}. \tag{4}$$

(II) *T_g is compact on $H_{p,q}(\varphi)$ if and only if $g \in \mathcal{B}_0$.*

2. Equivalent norms on $H_{p,q}$

Recall that, for $f \in H(B)$, the radial derivative of f can be expressed as

$$\Re f(z) = \sum_{j=1}^{n} z_j \frac{\partial f(z)}{\partial z_j}.$$

For $m = 1, 2, \cdots$, write $\Re^m f(z) = \Re(\Re^{m-1} f(z))$, $|\mathrm{grad}_m f(z)| = \sum_{|\alpha|=m} |\frac{\partial^\alpha f}{\partial z^\alpha}|$.

Lemma 1. *Suppose $f \in H(B)$, $f(0) = 0$. Then*

$$M_q(f, r) \leq \frac{C}{r} \int_0^r M_q(\Re f, t) dt, \qquad \text{if } 1 \leq q \leq \infty,$$

$$M_q(f, r) \leq \frac{C}{r} \left\{ \int_0^r (r-t)^{q-1} M_q^q(\Re f, t) dt \right\}^{\frac{1}{q}}, \qquad \text{if } 0 < q < 1.$$

Proof. For $f \in H(B), f(0) = 0$, and $z \in B$ we know

$$f(z) = \int_0^1 \frac{\Re f(tz)}{t} dt. \tag{5}$$

By Proposition 1.4.7 of [10],

$$\begin{aligned} \int_{\partial B} \left| \frac{\Re f(tr\zeta)}{t} \right|^q d\sigma(\zeta) &= \int_{\partial B} d\sigma(\zeta) \int_0^{2\pi} \left| \frac{\Re f((te^{i\theta}) r\zeta)}{t} \right|^q \frac{d\theta}{2\pi} \\ &= \int_{\partial B} d\sigma(\zeta) \int_0^{2\pi} \left| \frac{\Re f((te^{i\theta}) r\zeta)}{te^{i\theta}} \right|^q \frac{d\theta}{2\pi}. \end{aligned} \tag{6}$$

Then we know $\int_{\partial B} \left| \frac{\Re f(tr\zeta)}{t} \right|^q d\sigma(\zeta)$ is increasing with $t \in [0, 1)$, because $\Re f(wr\zeta)/w$ is a holomorphic function of w in the unit disc D. Now for $1 \leq q \leq \infty$, by (5) and Minkowski's inequality,

$$\begin{aligned} M_q(f, r) &\leq \int_0^1 dt \left\{ \int_{\partial B} \left| \frac{\Re f(tr\zeta)}{t} \right|^q d\sigma(\zeta) \right\}^{\frac{1}{q}} \\ &\leq C \int_{\frac{1}{2}}^1 dt \left\{ \int_{\partial B} \left| \frac{\Re f(tr\zeta)}{t} \right|^q d\sigma(\zeta) \right\}^{\frac{1}{q}} \\ &\leq C \int_0^1 dt \left\{ \int_{\partial B} |\Re f(tr\zeta)|^q d\sigma(\zeta) \right\}^{\frac{1}{q}} \\ &= C \frac{1}{r} \int_0^r M_q(\Re f, t) dt. \end{aligned}$$

To deal with the case $0 < q < 1$ we denote $t_j = 1 - 2^{-j}, j = 0, 1, 2, \cdots$. From [3] (or Lemma 2 in [10]) we obtain

$$\int_{\partial B} \sup_{t_{j-1} \le t \le t_j} \left| \frac{\Re f(tr\zeta)}{t} \right|^q d\sigma(\zeta) \le C \int_{\partial B} \left| \frac{\Re f(t_j r\zeta)}{t_j} \right|^q d\sigma(\zeta).$$

Then by (5) and [4, p. 57],

$$\begin{aligned} M_q^q(f, r) &\le C \sum_{j=1}^{\infty} 2^{-jq} \int_{\partial B} \sup_{t_{j-1} \le t \le t_j} \left| \frac{\Re f(tr\zeta)}{t} \right|^q d\sigma(\zeta) \\ &\le C \sum_{j=1}^{\infty} 2^{-jq} \int_{\partial B} \left| \frac{\Re f(t_j r\zeta)}{t_j} \right|^q d\sigma(\zeta) \\ &\le C \int_0^1 (1-t)^{q-1} M_q^q(\Re f, rt) dt \\ &\le C \frac{1}{r^q} \int_0^r (r-t)^{q-1} M_q^q(\Re f, t) dt. \end{aligned}$$

The lemma is proved. □

The following theorem, which has its own interest, will be used in the proof of the main result.

Theorem 2. *Let $0 < p, q \le \infty$ and let m be a positive integer. Then for $f \in H(B)$,*

$$\|f\|_{p,q,\varphi} \simeq \sum_{j=0}^{m-1} |\mathrm{grad}_j f(0)| + \left\{ \int_0^1 M_q^p(\Re^m f, r)(1-r^2)^{mp} \frac{\varphi^p(r)}{1-r} dr \right\}^{\frac{1}{p}}. \tag{7}$$

Proof. First we consider $m = 1$. Then (7) becomes

$$\|f\|_{p,q,\varphi} \simeq |f(0)| + \left\{ \int_0^1 M_q^p(\Re f, r)(1-r^2)^p \frac{\varphi^p(r)}{1-r} dr \right\}^{\frac{1}{p}}. \tag{8}$$

To prove this we notice that, for $f \in H(B)$,

$$|f(0)| \le C \|f\|_{p,q,\varphi}. \tag{9}$$

Combine [4, p. 80] and [9, Proposition 1.4.7] to have

$$(1-r^2) M_q(\Re f, r) \le C M_q(f, \frac{1+r}{2}).$$

This and the fact that

$$\varphi(r) \simeq \varphi(\frac{1+r}{2}) \tag{10}$$

imply

$$\left\{ \int_0^1 M_q^p(\Re f, r)(1-r)^p \frac{\varphi^p(r)}{1-r} dr \right\}^{\frac{1}{p}} \le C \|f\|_{p,q,\varphi}. \tag{11}$$

Then, by (9) and (11),

$$\|f\|_{p,q,\varphi} \geq C\left\{|f(0)| + \left\{\int_0^1 M_p^q(\Re f, r)(1-r^2)^q \frac{\varphi^q(r)}{1-r} dr\right\}^{\frac{1}{q}}\right\}.$$

This gives one direction of (8). Notice that

$$\|f\|_{p,q,\varphi} \leq C[\|f(0)\|_{p,q,\varphi} + \|f - f(0)\|_{p,q,\varphi}].$$

Hence, to prove the other direction of (8) we need only prove

$$\|f\|_{p,q,\varphi} \leq C\left\{\int_0^1 M_q^p(\Re f, r)(1-r^2)^p \frac{\varphi^p(r)}{1-r} dr\right\}^{\frac{1}{p}} \tag{12}$$

provided $f(0) = 0$. For $0 < p < \infty$ and $0 < q \leq \infty$, (12) can be proved as the estimate (11) in [10] with the only attention that Lemma 1 and Theorem 3 in [10] should be replaced by Lemma 2 in [11] and our Lemma 1 above, respectively. We will omit the details here. Now we deal with the case $p = \infty, 0 < q \leq \infty$. If $p = \infty$ and $1 \leq q \leq \infty$, by Lemma 1 and (3) we have

$$\begin{aligned}
&\sup_{0\leq r<1} \varphi(r) M_q(f, r) \\
&\leq C \sup_{\frac{1}{2}\leq r<1} \varphi(r) M_q(f, r) \\
&\leq C \sup_{0\leq r<1} r\varphi(r) M_q(f, r) \\
&\leq C \sup_{0\leq r<1} \varphi(r) \int_0^r M_q(\Re f, t) dt \\
&\leq C \sup_{0\leq r<1} (1-r)^a \int_0^r \frac{\varphi(t)}{(1-t)^a} M_q(\Re f, t) dt \\
&\leq C \left[\sup_{0\leq r<1} (1-r)^a \int_0^r \frac{dt}{(1-t)^{a+1}}\right] \sup_{0\leq t<1} \varphi(t)(1-t) M_q(\Re f, t) \\
&\leq C \sup_{0\leq t<1} \varphi(t)(1-t) M_q(\Re f, t).
\end{aligned}$$

If $p = \infty$ and $0 < q < 1$, by Lemma 1 and (3) again,

$$\begin{aligned}
&\sup_{0\leq r<1} \varphi(r) M_q(f, r) \\
&\leq C \sup_{0\leq r<1} r\varphi(r) M_q(f, r) \\
&\leq C \sup_{0\leq r<1} (1-r)^a \left\{\int_0^r \frac{\varphi(t)^q}{(1-t)^{aq}} M_q^q(\Re f, t) dt\right\}^{\frac{1}{q}} \\
&\leq C \left[\sup_{0\leq r<1} (1-r)^a \left\{\int_0^r \frac{dt}{(1-t)^{aq+1}}\right\}^{\frac{1}{q}}\right] \sup_{0\leq t<1} \varphi(t)(1-t) M_q(\Re f, t) \\
&\leq C \sup_{0\leq t<1} \varphi(t)(1-t) M_q(\Re f, t).
\end{aligned}$$

These give (12) for $p = \infty$ and end the proof of (8).

For general m we see that $(1-r)^m \varphi(r)$ is still normal. Then (7) comes from (8) by induction. The proof of the theorem is completed. □

3. The proof of the main result

Lemma 2. *Given $0 < p, q \leq \infty$, take $\beta > b$ and*

$$f_\zeta(z) = \frac{(1-|\zeta|^2)^\beta}{\varphi(|\zeta|)(1-\langle z,\zeta\rangle)^{\frac{n}{q}+\beta}}, \qquad \zeta \in B. \tag{13}$$

Then $\|f_\zeta\|_{p,q,\varphi} \leq C$. Here, the constant C is independent of ζ.

Proof. By [9] we have

$$M_q(f_\zeta, r) \leq C \frac{(1-|\zeta|^2)^\beta}{\varphi(|\zeta|)(1-r|\zeta|)^\beta}.$$

Then for $0 < p < \infty$,

$$\begin{aligned}\|f_\zeta\|^p_{p,q,\varphi} &\leq C \int_0^1 \frac{(1-|\zeta|^2)^{p\beta}}{\varphi^p(|\zeta|)(1-r|\zeta|)^{p\beta}} \frac{\varphi^p(r)}{1-r} dr \\ &= C\left\{\int_0^{|\zeta|} + \int_{|\zeta|}^1\right\} \frac{(1-|\zeta|^2)^{p\beta}}{\varphi^p(|\zeta|)(1-r|\zeta|)^{p\beta}} \frac{\varphi^p(r)}{1-r} dr = I_1 + I_2.\end{aligned}$$

For I_1, by (3) and [12] we have

$$\begin{aligned}I_1 &\leq C \frac{(1-|\zeta|)^{p\beta}}{\varphi^p(|\zeta|)} \frac{\varphi^p(|\zeta|)}{(1-|\zeta|)^{pb}} \int_0^{|\zeta|} \frac{(1-r)^{pb-1}}{(1-r|\zeta|)^{p\beta}} dr \\ &\leq C \frac{(1-|\zeta|)^{p\beta}}{(1-|\zeta|)^{pb}} \int_0^1 \frac{(1-r)^{pb-1}}{(1-r|\zeta|)^{p\beta}} dr \\ &\leq C.\end{aligned}$$

Similarly,

$$I_2 \leq \frac{(1-|\zeta|)^{p\beta}}{(1-|\zeta|)^{pa}} \int_0^1 \frac{(1-r)^{pa-1}}{(1-r|\zeta|)^{p\beta}} dr \leq C.$$

For $p = \infty$,

$$\|f_\zeta\|_{p,q,\varphi} \leq \sup_{0 \leq r \leq |\zeta|} \varphi(r) M_q(f,r) + \sup_{|\zeta| \leq r < 1} \varphi(r) M_q(f,r) = J_1 + J_2.$$

For J_1,

$$J_1 \leq C \frac{(1-|\zeta|^2)^\beta}{(1-|\zeta|^2)^b} \sup_{0 \leq r \leq |\zeta|} \frac{(1-r)^b}{(1-r|\zeta|)^\beta} \leq C.$$

Also, $J_2 \leq C$. Therefore $\|f_\zeta\|_{p,q,\varphi} \leq C$. The lemma is proved. □

Proof of Theorem 1. (I). Suppose $g \in H(B)$, and T_g is bounded on $H_{p,q}(\varphi)$. First, we claim that, for $f, g \in H(B)$,

$$\Re\left(T_g f\right)(z) = \left(f \Re g\right)(z). \tag{14}$$

In fact, we may suppose the holomorphic function $f\Re g$ has the expansion $(f\Re g)(z) = \sum_{|\alpha|\geq 1} a_\alpha z^\alpha$. Then

$$\begin{aligned}\Re(T_g f)(z) &= \Re \int_0^1 \sum_{|\alpha|\geq 1} a_\alpha (tz)^\alpha \frac{dt}{t} \\ &= \Re\left[\sum_{|\alpha|\geq 1} \frac{a_\alpha z^\alpha}{|\alpha|}\right] = \sum_{|\alpha|\geq 1} a_\alpha z^\alpha.\end{aligned}$$

Notice also that $(T_g f_\zeta)(0) = 0$. Then for any $\zeta \in B$, by Theorem 1 and Lemma 2,

$$\begin{aligned}\|T_g\| &\geq C\|T_g f_\zeta\|_{p,q,\varphi} \\ &\geq C\left\{\int_0^1 M_q^p(\Re(T_g f_\zeta), r)(1-r^2)^p \frac{\varphi^p(r)}{1-r} dr\right\}^{\frac{1}{p}} \\ &\geq C\left\{\int_{\frac{1+|\zeta|}{2}}^{\frac{3+|\zeta|}{4}} M_q^p(\Re(T_g f_\zeta), r)(1-r^2)^p \frac{\varphi^p(r)}{1-r} dr\right\}^{\frac{1}{p}} \\ &\geq CM_q(\Re(T_g f_\zeta), \frac{1+|\zeta|}{2})(1-|\zeta|^2)\varphi(|\zeta|) \\ &\geq C|\Re(T_g f_\zeta)(\zeta)|(1-|\zeta|^2)^{\frac{n}{q}}(1-|\zeta|^2)\varphi(|\zeta|) \\ &= C|\Re g(\zeta)|(1-|\zeta|^2)\left[|f_\zeta(\zeta)|(1-|\zeta|^2)^{\frac{n}{q}}\varphi(|\zeta|)\right].\end{aligned} \tag{15}$$

Hence $g \in \mathcal{B}$ and

$$\|g\|_{\mathcal{B}} \leq C\|T_g\|. \tag{16}$$

Conversely, suppose $g \in \mathcal{B}$. Then by Theorem 2 and (14) we get, for any $f \in H(B)$,

(17)

$$\begin{aligned}\|T_g f\|_{p,q,\varphi} &\leq C\left\{\int_0^1 M_q^p(\Re(T_g f), r)(1-r^2)^p \frac{\varphi^p(r)}{1-r} dr\right\}^{\frac{1}{p}} \\ &= C\left\{\int_0^1 M_q^p(f, r) \sup\{|\Re g(z)|^p : |z| = r\}(1-r^2)^p \frac{\varphi^p(r)}{1-r} dr\right\}^{\frac{1}{p}} \\ &\leq C\|g\|_{\mathcal{B}}\|f\|_{p,q,\varphi}.\end{aligned}$$

This gives

$$\|g\|_{\mathcal{B}} \geq C\|T_g\|. \tag{18}$$

(16) and (18) end the proof of (I).

(II). Suppose T_g is compact. Since f_ζ weakly convergence to zero in $H_{p,q}(\varphi)$, by the compactness we have as (15)

$$|\Re g(\zeta)|(1-|\zeta|^2) \leq C\|T_g f_\zeta\|_{p,q,\varphi} \to 0$$

as $|\zeta| \to 1$. That is, $g \in \mathcal{B}_0$.

Now we let $g \in \mathcal{B}_0$. Given any sequence $\{f_m\}$ in $H_{p,q}(\varphi)$ satisfying

$$\|f_m\|_{p,q,\varphi} \leq 1, \qquad f_m(z) \to 0 \tag{19}$$

uniformly on compact subsets of B, we are going to prove

$$\lim_{m\to\infty} \|T(f_m)\|_{p,q,\varphi} = 0. \tag{20}$$

For any $\varepsilon > 0$ we can choose $\eta \in (0,1)$ such that

$$|\Re g(\zeta)|(1-|\zeta|^2) < \varepsilon \qquad \text{if } \eta \le |z| < 1.$$

By (17) and (19) we obtain

$$\begin{aligned}
\|T_g f_m\|_{p,q,\varphi} &\le C \left\{ \int_0^1 M_q^p(f,r) \sup\{|\Re g(z)|^p : |z| = r\} (1-r^2)^p \frac{\varphi^p(r)}{1-r} dr \right\}^{\frac{1}{p}} \\
&\le C \left\{ \left\{ \int_0^\eta M_q^p(f,r) \sup\{|\Re g(z)|^p : |z| = r\} (1-r^2)^p \frac{\varphi^p(r)}{1-r} dr \right\}^{\frac{1}{p}} + \right. \\
&\qquad \left. + \left\{ \int_\eta^1 M_q^p(f,r) \sup\{|\Re g(z)|^p : |z| = r\} (1-r^2)^p \frac{\varphi^p(r)}{1-r} dr \right\}^{\frac{1}{p}} \right\} \\
&\le C \left\{ \|g\|_{\mathcal{B}} \sup\{|f_m(z)| : |z| \le \eta\} \right. \\
&\qquad \left. + \|f_m\|_{p,q,\varphi} \sup\left\{ |\Re g(z)|(1-|z|^2) : \eta \le |z| < 1 \right\} \right\} \\
&\le C \left\{ \|g\|_{\mathcal{B}} \sup\{|f_m(z)| : |z| \le \eta\} + \varepsilon \right\}.
\end{aligned}$$

Here, the constant C is independent of f_m and ε. Therefore, (20) follows. The proof is completed. □

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