

ZERO PRODUCT PRESERVING MAPS OF OPERATOR-VALUED FUNCTIONS

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ABSTRACT. Let X, Y be locally compact Hausdorff spaces and \mathcal{M}, \mathcal{N} be Banach algebras. Let $\theta : C_0(X, \mathcal{M}) \rightarrow C_0(Y, \mathcal{N})$ be a zero product preserving bounded linear map with dense range. We show that θ is given by a continuous field of algebra homomorphisms from \mathcal{M} into \mathcal{N} if \mathcal{N} is irreducible. As corollaries, such a surjective θ arises from an algebra homomorphism, provided that \mathcal{M} is a W^* -algebra and \mathcal{N} is a semi-simple Banach algebra, or both \mathcal{M} and \mathcal{N} are C^* -algebras.

1. INTRODUCTION

We are interested in the question how the zero product structure of a Banach algebra determines its full algebraic structure. For example, two abelian C^* -algebras are $*$ -isomorphic if there exists a bijective linear map between them preserving zero products ([13], [11], [14]). On the other hand, *bounded* bijective linear zero product preservers of nonabelian C^* -algebras also provide algebraic $*$ -isomorphisms [7]. Recently, Araujo and Jarosz [2] showed that the existence of a bijective linear map of standard operator algebras preserving zero products in *both* ways also implies that they are isomorphic as Banach algebras. In this paper, we study bounded zero product preservers between Banach algebras of operator-valued functions. We show that such maps are given by continuous fields of algebra homomorphisms in many situations.

Let X be a locally compact Hausdorff space. Denote by $X_\infty = X \cup \{\infty\}$ the one-point compactification of X . In case X is already compact, ∞ is an isolated point in X_∞ . For a real or complex Banach algebra \mathcal{M} , let $C_0(X, \mathcal{M}) = \{f \in C(X, \mathcal{M}) : f(\infty) = 0\}$ be the Banach algebra of all continuous vector-valued functions from X into \mathcal{M} vanishing at infinity. Note that $C_0(X, \mathcal{M})$ is isometrically and algebraically isomorphic to the (projective) tensor product $C_0(X) \otimes \mathcal{M}$.

In this paper, we shall study those bounded linear maps θ from $C_0(X, \mathcal{M})$ into another such algebra $C_0(Y, \mathcal{N})$ preserving zero products. Namely, $fg = 0$ implies $\theta(f)\theta(g) = 0$. In other words,

$$f(x)g(x) = 0 \text{ in } \mathcal{M} \text{ for all } x \in X \implies \theta(f)(y)\theta(g)(y) = 0 \text{ in } \mathcal{N} \text{ for all } y \in Y.$$

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For example, let $\sigma : Y \rightarrow X$ be a continuous function, let h be a uniformly bounded norm continuous function from Y into the center of \mathcal{N} , and let φ be a uniformly bounded SOT continuous function from Y into $B(\mathcal{M}, \mathcal{N})$ such that each $\varphi_y = \varphi(y)$ is an algebra homomorphism. Then

$$(1.1) \quad \theta(f)(y) = h(y)\varphi_y(f(\sigma(y)))$$

defines a zero product preserving bounded linear map from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$. In particular, $\theta = h\varphi$ for a bounded central element h in the algebra $C(Y, \mathcal{N})$ and an algebra homomorphism φ from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$. We will investigate when zero product preserving bounded linear maps arise in this way.

For the scalar case, every zero product preserving bounded linear map θ from $C_0(X)$ into $C_0(Y)$ is of the expected form (1.1) ([13], [11], [14]). Recall that a subalgebra \mathcal{S} of the algebra $B(E)$ of all bounded linear operators on a Banach space E is said to be *standard* if \mathcal{S} contains the identity operator and all continuous finite rank operators. Using an interesting geometric approach, Araujo and Jarosz [2] showed that when X, Y are real compact and \mathcal{M} and \mathcal{N} are standard operator algebras, every bijective linear map from $C(X, \mathcal{M})$ onto $C(Y, \mathcal{N})$ preserving zero products in both directions is in the form of (1.1). However, in the non-bijective case, it becomes a very difficult task without assuming continuity. Even discontinuous algebra homomorphisms have complicated structure ([15], [19]). Finally, readers are referred to [1], [12], [6], and [21] for problems of similar interests.

2. RESULTS

A linear map θ from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$ is said to be *strictly separating* if

$$\|f(x)\|\|g(x)\| = 0 \text{ for all } x \in X \implies \|\theta f(y)\|\|\theta g(y)\| = 0 \text{ for all } y \in Y.$$

Denote by $\text{coz}(f) = \{x \in X : f(x) \neq 0\}$ the *cozero set* of an f in $C_0(X, \mathcal{M})$. Then θ is strictly separating if and only if it preserves the disjointness of cozeroes. We note that a subset U of X is the cozero of a continuous function in $C_0(X, \mathcal{M})$ if and only if U is σ -compact and open. For any σ -compact open subset U of X , denote by $C_0(U, \mathcal{M})$ the subalgebra of all f in $C_0(X, \mathcal{M})$ with $\text{coz}(f) \subseteq U$.

A Banach algebra \mathcal{N} is said to be *irreducible* if it has a faithful irreducible representation $\pi : \mathcal{N} \rightarrow B(E)$, where E is a Banach space. Recall that every irreducible representation of a Banach algebra is automatically bounded [15].

Theorem 1. *Let X and Y be locally compact Hausdorff spaces. Let \mathcal{M} and \mathcal{N} be Banach algebras such that \mathcal{N} is irreducible, and let θ be a continuous zero product preserving linear map from $C_0(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$ with dense range. Then θ is strictly separating.*

Indeed, there exists a continuous map $\sigma : Y \rightarrow X$, and for each y in Y a bounded zero product preserving linear map $H_y : \mathcal{M} \rightarrow \mathcal{N}$ with dense range such that

$$\theta(f)(y) = H_y(f(\sigma(y))) \quad \text{for all } f \in C_0(X, \mathcal{M}) \text{ and } y \in Y.$$

Moreover, the correspondence $y \mapsto H_y$ defines a uniformly bounded map $H : Y \rightarrow B(\mathcal{M}, \mathcal{N})$ continuous in the strong operator topology (SOT).

Proof. Let $\pi : \mathcal{N} \rightarrow B(E)$ be a faithful irreducible representation of \mathcal{N} . Composing θ with π , we can assume that \mathcal{N} is an irreducible subalgebra of $B(E)$ and that θ is again bounded and zero product preserving with dense range.

Fix y in Y , and denote by

$$S_y = \left\{ x \in X_\infty : \text{for each } \sigma\text{-compact open neighborhood } U \text{ of } x, \right. \\ \left. \text{there is an } f \text{ in } C_0(U, \mathcal{M}) \text{ such that } \theta(f)(y) \neq 0 \right\}.$$

Claim 1. $S_y \neq \emptyset$.

Suppose not, and for each x in X_∞ , suppose there is a σ -compact open neighborhood U of x such that $\theta|_{C_0(U, \mathcal{M})}$ is trivial at y . Write

$$X_\infty = U_0 \cup U_1 \cup \dots \cup U_n$$

for $x_0 = \infty$, and some x_1, \dots, x_n in X , with a σ -compact open neighborhood U_i for $i = 0, 1, \dots, n$, respectively. Let

$$1 = f_0 + f_1 + \dots + f_n$$

be a continuous partition of unity such that $\text{coz } f_i \subseteq U_i$ for $i = 0, 1, \dots, n$. Then for all f in $C_0(X, \mathcal{M})$,

$$\theta(f)(y) = \theta(f_0f + f_1f + \dots + f_nf)(y) = 0,$$

since $\text{coz}(f_i f) \subseteq U_i$ for each $i = 0, 1, \dots, n$. This is impossible.

Claim 2. $x_1, x_2 \in S_y \implies x_1 = x_2$.

Suppose $x_2 \neq x_1$ and $x_1 \neq \infty$. Let U_1 and U_2 be disjoint σ -compact open neighborhoods of x_1 and x_2 , respectively. We can assume that $\infty \notin U_1$. Since

$$f_1 f_2 = f_2 f_1 = 0 \quad \text{for all } f_i \in C_0(U_i, \mathcal{M}), \quad i = 1, 2,$$

we have

$$\theta(f_1)\theta(f_2) = \theta(f_2)\theta(f_1) = 0 \quad \text{in } C_0(Y, \mathcal{N}).$$

Let E_1 be the intersection of the kernels of all $\theta(f_1)(y)$ with f_1 in $C_0(U_1, \mathcal{M})$. Because both $\theta|_{C_0(U_1, \mathcal{M})}$ and $\theta|_{C_0(U_2, \mathcal{M})}$ are not trivial at y , we have that E_1 is a proper nontrivial subspace of E , that is, $\{0\} \neq E_1 \neq E$.

Let V be a nonempty open set in Y such that $\overline{V} \subseteq U_1$. Let g be in $C_0(X)$ such that $\text{coz } g \subseteq U_1$ and $g|_V = 1$. For each f in $C_0(X, \mathcal{M})$, write

$$f = fg + f(1 - g).$$

Since $\text{coz}(fg) \subseteq U_1$, we have

$$\theta(fg)(y)|_{E_1} = 0.$$

Hence

$$\theta(f)(y)|_{E_1} = \theta(f(1 - g))(y)|_{E_1}.$$

For any k in $C_0(X, \mathcal{M})$ with $\text{coz } k \subseteq V$, we have $k(f(1 - g)) = 0$. This implies

$$\theta(k)(y)\theta(f)(y)|_{E_1} = \theta(k)(y)\theta(f(1 - g))(y)|_{E_1} = 0 \quad \text{for all } f \in C_0(X, \mathcal{M}).$$

However, $\{\theta(f)(y) : f \in C_0(X, \mathcal{M})\}$ is dense in \mathcal{N} , which is irreducible on E . Therefore,

$$\theta(k)(y) = 0 \quad \text{for all } k \in C_0(X, \mathcal{M}) \text{ with } \text{coz } k \subseteq V.$$

Since V is an arbitrary nonempty open set with closure contained in U_1 , we have

$$\theta(k)(y) = 0 \quad \text{for all } k \in C_0(U_1, \mathcal{M}).$$

This conflict establishes Claim 2.

By Claims 1 and 2, S_y is a singleton.

Claim 3. If $S_y = \{x\}$, then

$$f(x) = 0 \implies \theta(f)(y) = 0.$$

By Urysohn's Lemma, we can assume that f vanishes in a neighborhood of x . Now $x \notin \overline{\text{coz } f}$, which is compact in X_∞ . For each x' in $\overline{\text{coz } f}$, there is a σ -compact open neighborhood U' of x' such that $\theta|_{C_0(U', \mathcal{M})}$ is trivial at y . By a compactness argument similar to the one proving Claim 1, we see that $\theta(f)(y) = 0$.

It follows from Claim 3 that $S_y \neq \{\infty\}$ for all y in Y since θ has dense range. Denote by $\sigma(y) = x$ if $S_y = \{x\}$. Then there is a linear map $H_y : \mathcal{M} \rightarrow \mathcal{N}$ such that

$$\theta(f)(y) = H_y(f(\sigma(y))) \quad \text{for all } f \in C_0(X, \mathcal{M}) \text{ and } y \in Y.$$

In particular, θ is strictly separating.

The rest of the proof follows in a straightforward manner, or one can quote the standard results about strictly separating maps in [6], [12]. \square

The following lemma might be known, although we do not find a proof from the literature. Remark that it is shown in [17] that every nonzero Banach algebra homomorphism from $B(H)$ into $B(K)$ is injective if both H and K are separable Hilbert spaces. However, there is an example in [18] of a nonzero homomorphism from $B(H)$ into $B(H)$ with compact operators as its kernel, where H is inseparable. Moreover, it is known that every irreducible representation of a Banach algebra is norm continuous [15] and every algebra isomorphism between C^* -algebras is a $*$ -isomorphism [20, Theorem 4.1.20].

Lemma 2. *Let H, K be real or complex Hilbert spaces of arbitrary dimension. Let $B(H)$ and $B(K)$ be the algebras of all bounded linear operators on H and K , respectively. Then every surjective algebra homomorphism from $B(H)$ onto $B(K)$ is an isomorphism.*

Proof. The case is trivial when H is of finite dimension since $B(H)$ is then a simple algebra. Suppose the (Hilbert space) dimension of H is an infinite cardinal number \aleph_H . For each infinite cardinal number $\aleph \leq \aleph_H$, let I_\aleph be the closed two-sided ideal of $B(H)$ consisting of operators T such that all closed subspaces contained in the range of T are of dimension less than \aleph . In case H is separable, $I_{\aleph_H} = \mathcal{K}(H)$, the ideal of compact operators on H . In general, as indicated in [5], I_{\aleph_H} is the largest two-sided ideal of $B(H)$. In fact, every closed two-sided ideal of $B(H)$ is in the form of I_\aleph for some $\aleph \leq \aleph_H$ [9, Section 17].

Let θ be an algebra homomorphism from $B(H)$ onto $B(K)$. Then the kernel I of θ is a closed two-sided ideal of $B(H)$. Since the quotient algebra $B(H)/I$ is isomorphic to $B(K)$, there is an e in $B(H)$ such that $(e + I)B(H)(e + I) = eB(H)e + I$ is of dimension one modulo I . Assume I is nonzero. Let \aleph be the infinite cardinal number such that $I = I_\aleph$. Then the range of e contains a closed subspace of dimension \aleph . By halving this subspace into two, each of dimension \aleph , we see that $eB(H)e$ contains two elements linearly independent modulo I_\aleph , a contradiction. This completes our proof. \square

Corollary 3. *Let X, Y be locally compact Hausdorff spaces. Let \mathcal{M}, \mathcal{N} be either the Banach algebras $B(H), B(K)$ of all bounded operators or the algebras $\mathcal{K}(H), \mathcal{K}(K)$ of compact operators on real or complex Hilbert spaces H, K , respectively. Let $\theta : C_0(X, \mathcal{M}) \rightarrow C_0(Y, \mathcal{N})$ be a continuous surjective zero product preserving linear map. Then there exist a continuous function σ from Y into X , a continuous scalar*

function h on Y , and a SOT continuous map $y \mapsto S_y$ from Y into $B(K, H)$ such that S_y is invertible and

$$(2.1) \quad \theta(f)(y) = h(y)S_y^{-1}f(\sigma(y))S_y, \quad \forall f \in C(X, \mathcal{M}), \forall y \in Y.$$

Proof. It follows from Theorem 1 that for each fixed y in Y , θ induces a bounded zero product preserving linear map $H(y)$ from \mathcal{M} onto \mathcal{N} . By either [10, Theorem 2.1] or [7, Corollary 3.2], $H(y)$ is a scalar multiple of a bounded algebra homomorphism from \mathcal{M} onto \mathcal{N} . Since $\mathcal{K}(H)$ is simple, this algebra homomorphism is indeed an isomorphism if \mathcal{M} and \mathcal{N} are $\mathcal{K}(H)$ and $\mathcal{K}(K)$, respectively. On the other hand, by Lemma 2, the algebra homomorphism above is again an isomorphism in case \mathcal{M} and \mathcal{N} are $B(H)$ and $B(K)$, respectively. Thus, by either [3, Theorem 4] or [8, Corollary 3.2], there exist a scalar $h(y)$ and a bounded invertible operator S_y on K to implement (2.1). It is then routine to check the continuity of h and the map $y \mapsto S_y$. \square

The following corollary holds, for example, when \mathcal{M} is a W^* -algebra, or a unital C^* -algebra of real rank zero [4].

Corollary 4. *Let X and Y be locally compact Hausdorff spaces such that X is compact. Let \mathcal{M} be a unital Banach algebra such that the subalgebra of \mathcal{M} generated by its idempotents is norm dense in \mathcal{M} , and let \mathcal{N} be a semi-simple Banach algebra. Let θ be a continuous zero product preserving linear map from $C(X, \mathcal{M})$ into $C_0(Y, \mathcal{N})$ with dense range. Then $\theta(1)$ is in the center of $C_0(Y, \mathcal{N})$, and*

$$(2.2) \quad \theta(1)\theta(fg) = \theta(f)\theta(g) \quad \text{for all } f, g \in C(X, \mathcal{M}).$$

Suppose, in addition, that Y is compact and \mathcal{N} is unital. If $\theta(1)$ is invertible or θ is surjective, then $\theta = \theta(1)\varphi$ for an algebra homomorphism φ .

Proof. Let $\pi : \mathcal{N} \rightarrow B(E)$ be an irreducible representation of \mathcal{N} . Then $\theta_\pi = \pi \circ \theta$ is again a continuous zero product preserving linear map from $C(X, \mathcal{M})$ into $C_0(Y, \pi(\mathcal{N}))$ with dense range. By Theorem 1, we find that θ_π carries a weighted composition operator form

$$\theta_\pi(f)(y) = H_y(f(\sigma(y))) \quad \text{for all } f \in C(X, \mathcal{M}) \text{ and } y \in Y.$$

In particular, each H_y is a continuous zero product preserving linear map from \mathcal{M} into $\pi(\mathcal{N})$ with dense range.

By results in [10] (see also [7]), for each y in Y we have that $\theta_\pi(1)(y) = H_y(1)$ is in the center of \mathcal{N} and

$$H_y(1)H_y(ab) = H_y(a)H_y(b) \quad \text{for all } a, b \in \mathcal{M}.$$

Hence

$$\pi(\theta(1)\theta(f) - \theta(f)\theta(1)) = 0$$

and

$$\pi(\theta(1)\theta(fg) - \theta(f)\theta(g)) = 0$$

for all f, g in $C(X, \mathcal{M})$. Being semi-simple, \mathcal{N} has a faithful family of irreducible representations. Thus $\theta(1)$ is in the center of $C_0(Y, \mathcal{N})$ and (2.2) holds.

Now, we assume that Y is compact and \mathcal{N} is unital. If θ is surjective, $1 = \theta(f)$ for some f in $C(X, \mathcal{M})$. It follows from $\theta(1)\theta(f^2) = \theta(f)^2 = 1$ that $\theta(1)$ is invertible. Assume $\theta(1)$ is invertible. Then $\theta(1)^{-1}\theta$ is again a bounded zero product preserving linear map with dense range, and sends 1 to 1. Suppose now that $\theta(1) = 1$. Then (2.2) ensures that θ is an algebra homomorphism. \square

A recent result in [7] states that every surjective zero product preserving bounded linear map θ between unital C^* -algebras is a product $\theta = \theta(1)\varphi$ of the invertible central element $\theta(1)$ and an algebra homomorphism φ . Since $C(X, \mathcal{A})$ (resp. $C(Y, \mathcal{B})$) is $*$ -isomorphic to the (projective) tensor product $C(X) \otimes \mathcal{A}$ (resp. $C(Y) \otimes \mathcal{B}$) as C^* -algebras (see, e.g., [16]), we have the following.

Corollary 5. *Let X and Y be compact Hausdorff spaces, and \mathcal{A}, \mathcal{B} be unital C^* -algebras. Let θ be a continuous zero product preserving linear map from $C(X, \mathcal{A})$ onto $C(Y, \mathcal{B})$. Then $\theta(1)$ is an invertible element in the center of $C(Y, \mathcal{B})$, and $\theta = \theta(1)\varphi$ for an algebra homomorphism φ .*

The following example shows that the irreducibility condition on \mathcal{N} cannot be dropped in Theorem 1, and the map θ in the Corollaries 4 and 5 cannot be written as a weighted composition operator in the form of (1.1) in general.

Example 6. Let $X = \{0\}$ and $\mathcal{M} = \mathbb{C} \oplus \mathbb{C}$ be the two-dimensional C^* -algebra, and let $Y = \{1, 2\}$ and $\mathcal{N} = \mathbb{C}$ be the one-dimensional C^* -algebra. Define $\theta : C(X, \mathcal{M}) \rightarrow C(Y, \mathcal{N})$ by $\theta(a \oplus b) = g$ with $g(1) = a$ and $g(2) = b$. Then θ is bijective and preserves zero products in both directions.

Remark that $\theta : C(X, \mathcal{M}) \rightarrow C(Y, \mathcal{N})$ satisfies the condition stated in Theorem 1. In fact, let $h_1(a \oplus b) = a$ and $h_2(a \oplus b) = b$ be the canonical projection of $\mathbb{C} \oplus \mathbb{C}$ onto its summands, and set $\sigma(1) = \sigma(2) = 0$. Then

$$\theta(f)(y) = h_y(f(\sigma(y))), \quad \forall f \in C(X, \mathcal{M}), \forall y \in Y.$$

However, \mathcal{M} is not irreducible and $T^{-1} : C(Y, \mathcal{N}) \rightarrow C(X, \mathcal{M})$ does not carry a weighted composition operator form. Note also that X and Y are not homeomorphic although both $C(X, \mathcal{M})$ and $C(Y, \mathcal{N})$ are isomorphic to $\mathbb{C} \oplus \mathbb{C}$ as C^* -algebras and θ implements an algebra isomorphism between them.

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REFERENCES

- [1] Y. A. Abramovich, *Multiplicative representation of disjointness preserving operators*, Nederl. Akad. Wetensch. Indag. Math. **45** (1983), 265–279. MR **85f**:47040
- [2] J. Araujo and K. Jarosz, *Biseparating maps between operator algebras*, J. Math. Anal. Appl. **282** (2003), no. 1, 48–55.
- [3] B. H. Arnold, *Rings of operators on vector spaces*, Ann. of Math. (2) **45** (1944), 24–49. MR **5**:147c
- [4] L. G. Brown and G. K. Pedersen, *C^* -algebras of real rank zero*, J. Funct. Anal. **99** (1991), 131–149. MR **92m**:46086
- [5] J. W. Calkin, *Two-sided ideals and congruences in the ring of bounded operators in Hilbert space*, Ann. of Math. **42** (1941), 839–873. MR **3**:208c
- [6] J. T. Chan, *Operators with the disjoint support property*, J. Operator Theory **24** (1990), 383–391. MR **93c**:47035
- [7] M. A. Chebotar, W.-F. Ke, P.-H. Lee and N.-C. Wong, *Mappings preserving zero products*, Studia Math. **155**, no. 1 (2003), 77–94. MR **2003m**:47066
- [8] P. R. Chernoff, *Representations, automorphisms, and derivations of some operator algebras*, J. Funct. Anal. **12** (1973), 275–289. MR **50**:2934
- [9] J. B. Conway, *A course in operator theory*, Graduate Studies in Math. **21**, American Mathematical Society, Providence, Rhode Island, 2000. MR **2001d**:47001

- [10] J. Cui and J. Hou, *Linear maps on von Neumann algebras preserving zero products or TR-rank*, Bull. Austral. Math. Soc. **65** (2002), 79–91. MR **2002m**:46092
- [11] J. J. Font and S. Hernández, *On separating maps between locally compact spaces*, Arch. Math. (Basel) **63** (1994), 158–165. MR **95k**:46083
- [12] J. E. Jamison and M. Rajagopalan, *Weighted composition operator on $C(X, E)$* , J. Operator Theory **19** (1988), 307–317. MR **90b**:47052
- [13] K. Jarosz, *Automatic continuity of separating linear isomorphisms*, Canad. Math. Bull. **33** (1990), 139–144. MR **92j**:46049
- [14] J.-S. Jeang and N.-C. Wong, *Weighted composition operators of $C_0(X)$'s*, J. Math. Anal. Appl. **201** (1996), 981–993. MR **97f**:47029
- [15] B. E. Johnson, *Continuity of homomorphisms of algebras of operators*, J. London Math. Soc. **42** (1967), 537–541. MR **35**:5953
- [16] B.-R. Li, *Introduction to operator algebras*, World Scientific, Singapore, 1992.
- [17] H. Porta and J. T. Schwartz, *Representations of the algebra of all operators in Hilbert space, and related analytic function algebras*, Comm. Pure and Applied Math. **20** (1967), 457–492. MR **35**:2157
- [18] H. Porta, *A note on homomorphisms of operator algebras*, Colloq. Math. **20** (1969), 117–119. MR **39**:1988
- [19] V. Runde, *The structure of discontinuous homomorphisms from non-commutative C^* -algebras*, Glasgow Math. J. **36** (1994), 209–218. MR **95i**:46091
- [20] S. Sakai, *C^* -algebras and W^* -algebras*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 60, Springer-Verlag, New York, 1971. MR **56**:1082
- [21] M. Wolff, *Disjointness preserving operators on C^* -algebras*, Arch. Math. (Basel) **62** (1994), 248–253. MR **94k**:46122

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