PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 133, Number 1, Pages 51–56
S 0002-9939(04)07539-2
Article electronically published on August 20, 2004

THE VALENTINER GROUP AS GALOIS GROUP

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(Communicated by Lance W. Small)

ABSTRACT. We obtain the complete set of solutions to the Galois embedding problem given by the Valentiner group as a triple cover of the alternating group A_6.

The Valentiner group is a primitive subgroup of the special linear group of order 3 over a field k of characteristic 0 containing the roots of unity of order 15. It can also be seen as the unique nontrivial triple cover of the alternating group A_6 in 6 letters. In this paper, we consider the Galois embedding problem given by the Valentiner group and a Galois realization K of A_6 over a field k containing $\mathbb{Q}(\mu_{15})$. We give a correspondence between the solutions of this embedding problem and the k-defined points of a certain algebraic variety and obtain all possible solutions to this embedding problem. This is achieved by determining all possible dimension 10 irreducible $k[A_6]$-submodules of K and characterizing among them the ones that are a symmetric cube. In the case when the field k is a differential field we determine the homogeneous linear differential equations of order 3 with Galois group the Valentiner group and Picard-Vessiot extension containing K.

Let F denote a field containing $\mathbb{Q}(\mu_{15})$. The Valentiner group is the subgroup of the special linear group $\mathrm{SL}(3, F)$ generated by the matrices

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \zeta^4 & 0 \\ 0 & 0 & \zeta \end{pmatrix} \quad , \quad E_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \quad ,$$

$$E_3 = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 & 2 \\ 1 & s & t \\ 1 & t & s \end{pmatrix} \quad , \quad E_4 = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2\lambda_2 & 2\lambda_2 \\ \lambda_1 & s & t \\ \lambda_1 & t & s \end{pmatrix} \quad ,$$

where ζ is a primitive 5th root of unity, $s = \zeta^2 + \zeta^3, t = \zeta + \zeta^4, \sqrt{5} = t - s$, $\lambda_1 = \dfrac{-1 \pm \sqrt{-15}}{4}, \lambda_2 = \dfrac{-1 \mp \sqrt{-15}}{4}$. Under the projection of $\mathrm{SL}(3, F)$ onto the projective group $\mathrm{PGL}(3, F)$, the Valentiner group is mapped onto the alternating group A_6 in 6 letters. The projection of the Valentiner group onto A_6 can be obtained by mapping the matrices E_1, E_2, E_3, E_4 to the permutations $(12345), (14)(23), (12)(34), (14)(56)$ of A_6 respectively (cf. [6]). The Valentiner

Received by the editors June 3, 2003 and, in revised form, October 9, 2003.
2000 *Mathematics Subject Classification.* Primary 12F12; Secondary 11F80, 12H05.
The first author was supported in part by BFM2000-0794-C02-01, Spanish Ministry of Education.

group is the unique nontrivial triple cover of the alternating group A_6. In the sequel we shall denote the Valentiner group by $3A_6$. By a result of Mestre [5], it is known that the group $3A_6$ appears as the Galois group of $\mathbb{Q}$-regular extensions of $\mathbb{Q}(t)$. More precisely, Mestre gives an explicit construction of $\mathbb{Q}$-regular extensions of $\mathbb{Q}(t)$ with Galois group A_6 that are embeddable in a $3A_6$-extension. However, no explicit construction of a $3A_6$-extension is known. In this work, we shall give an explicit construction of $3A_6$-extensions valid over a field of characteristic zero containing the roots of unity of order 15. The main tool for our construction is representation theory, and we are using a generalization of the method given in [2, 3].

We shall consider the faithful representation $\widetilde{\rho}$ of dimension 3 of the group $3A_6$ given by its presentation as a subgroup of $\mathrm{SL}(3, F)$ and the third symmetric power $\rho = \widetilde{\rho}^{(3)}$ of $\widetilde{\rho}$. We recall that A_6 is the image of $3A_6$ under the projection of $\mathrm{SL}(3, F)$ onto the projective group $\mathrm{PGL}(3, F)$ and that the kernel of $\mathrm{SL}(3, F) \twoheadrightarrow \mathrm{PGL}(3, F)$ is the subgroup of the homotheties of ratio a third root of unity. This implies that the representation ρ factors through A_6. By looking at the character table of the group A_6 (for example in [4], p. 289), we see that ρ corresponds to the unique irreducible representation of dimension 10 of the group A_6. The character of this representation is given by:

1	(123)	(12)(34)	(12345)	(13452)	(123)(456)	(1234)(56)
10	1	-2	0	0	1	0

In the case when k is a differential field (of characteristic 0 with algebraically closed field of constants), we shall use the connection between differential equations and group representations. We recall in particular the notion of symmetric power of a homogeneous linear differential equation (see [7, 8]).

Definition 1. Let $L(y) = 0$ be a homogeneous linear differential equation of order n over the differential field k. Let $\{y_1, \ldots, y_n\}$ be a fundamental set of solutions of $L(y) = 0$. We call the symmetric power of order m of $L(y) = 0$ the differential equation $L^{(m)}(y) = 0$ whose solution space is spanned by $\{y_1^{i_1} \ldots y_n^{i_n} | i_1 + \cdots + i_n = m\}$.

Let k be a differential field of characteristic 0 with algebraically closed field of constants $\mathcal{C}$ and $L(y) = 0$ a homogeneous linear irreducible differential equation of order 3 over k with Galois group $3A_6$. Then the third symmetric power $L^{(3)}(y) = 0$ of $L(y) = 0$ is an irreducible linear differential equation of order 10 with Galois group A_6, and the representation of A_6 obtained from the action of A_6 on the solution space is the representation ρ above.

We shall use the following lemma on representations.

Lemma 1. *Let V be a k-vector space of dimension n and $\varphi : G \to \mathrm{GL}(V)$ an absolutely irreducible representation. We consider:*

$$\varphi^m = \overbrace{\varphi \oplus \cdots \oplus \varphi}^{m} : G \to \mathrm{GL}(V^m)$$

where $V^m = \overbrace{V \oplus \cdots \oplus V}^{m}$, and we fix monomorphisms $f_j : V \to V^m$ such that $\pi_j \circ f_j : V \to V$, where π_j is the projection on the j-component, is an isomorphism of G-modules, $1 \leq j \leq m$.

Then every invariant k-subspace of V^m isomorphic to V as a G-module is of the form

$$\langle (\sum_j a_j f_j(v_i))_{1\le i\le n}\rangle$$

for some $(a_1,\dots,a_m)\in k^m\setminus\{(0,\dots,0)\}$ and $(v_1,\dots,v_n)$ a k-basis of V.

Proof. Let W be an invariant k-subspace of V^m, G-isomorphic to V. The thesis of the lemma is equivalent to the composition

$$V\simeq W\hookrightarrow V\oplus\cdots\oplus V \quad\overset{\pi_j}{\longrightarrow}\quad V$$

being a homothety for each j.

This composition is a G-endomorphism of V and, since φ is irreducible, either 0 or a G-automorphism of V. Now, using again the irreducibility of φ, we obtain that each G-automorphism of V is a homothety. □

We state now our main result.

Theorem 1. *Let k be a field of characteristic 0, containing the 15th roots of unity. Let $P(X)\in k[X]$ be a polynomial of degree 6 with Galois group A_6, K a splitting field of the polynomial $P(X)$.*

There exists an algebraic variety Q in the dimension 9 projective space defined over k, such that the Galois embedding problem

(GEP) $$3A_6\to A_6\simeq \mathrm{Gal}(K|k)$$

is solvable if and only if Q has a point defined over k.

Let V_j, $1\le j\le 10$, be ten k-vector subspaces of K such that the action of A_6 on each of them corresponds to the unique irreducible dimension 10 representation ρ of A_6 and such that the sum of the V_j is a direct sum. Let F_{ij}, $1\le i\le 10$, be a basis of V_j, $1\le j\le 10$, such that $F_{ij}\mapsto F_{ik}$ defines an isomorphism of A_6-modules from V_j onto V_k. The vectors F_{ij} can be chosen such that:

a) The extension $\widetilde{K}=K(\sqrt[3]{G_1})$, where $G_1=\sum_j a_jF_{1j}$, with $(a_1,\dots,a_{10})$ in $Q(k)$, is a solution to (GEP).

b) When k is a differential field with algebraically closed field of constants $\mathcal{C}$, then, for $(a_1,\dots,a_{10})\in Q(k)$, $\{H_1,H_2,H_3\}$, where $H_i=\sqrt[3]{\sum_j a_jF_{ij}}, 1\le i\le 3$, is a basis of the solution space of a differential equation of order 3 with Galois group $3A_6$ over k.

The elements F_{ij} and the variety Q can be obtained explicitly. From a point in $Q(k)$ we obtain then explicitly the element G_1 in a) and the differential equation in b) with Galois group $3A_6$.

Proof. Assume (GEP) is solvable and let $\widetilde{K}$ be a solution. We consider $\widetilde{K}$ as a k-vector space and the representation

$$\phi: 3A_6\to \mathrm{GL}(\widetilde{K})$$

given by the Galois action. By the normal basis theorem, ϕ is the regular representation, and so $\widetilde{K}$ contains an invariant k-vector space $U=\langle u_1,u_2,u_3\rangle$ of dimension 3 such that $\phi_{|U}$ is equivalent to the faithful unimodular representation $\widetilde{\rho}$ of $3A_6$. The symmetric cube of the representation $\phi_{|U}$ provides a k-vector space of dimension 10 contained in K, which is an irreducible invariant subspace of the representation of A_6 on K given by the Galois action. Moreover $\widetilde{K}=K(u_1)$.

Now let $P(X) \in k[X]$ be a polynomial of degree 6, with Galois group A_6, and let K denote a splitting field of $P(X)$. We consider the representation

$$A_6 \to \mathrm{GL}(K)$$

given by the Galois action. Again by the normal basis theorem, this representation is the regular one and so contains the dimension 10 irreducible representation $\rho = \widetilde{\rho}^{(3)}$ ten times. We shall determine explicitly ten k-subspaces V_i of dimension 10 of K such that their sum is direct and such that the Galois action on V_i, $i = 1, \dots, 10$, corresponds to ρ. To this end, we apply the following lemma, proven in [1], to the permutation representation

$$\sigma : A_6 \to \mathcal{S}_{\{e_1,\dots,e_6\}} \to \mathrm{GL}(6, k).$$

Lemma 2. *If $\sigma : G \hookrightarrow \mathrm{GL}(V)$ is a faithful permutation representation of a finite group G, then any irreducible representation of G is contained in some symmetric power $\sigma^{(N)}$ of σ.*

By computing the characters of the symmetric powers, we obtain that ρ is contained in the third symmetric power of σ. The corresponding invariant subspace V of dimension 10 is contained in the dimension 15 invariant subspace $\langle e_{ij} = e_i^2 e_j - e_i e_j^2 \rangle_{1 \le i < j \le 6}$.

The following vectors written in the basis e_{ij} (taken in lexicographic order) form a basis of V:

$v_1 = (0, 1, -1, 1, -1, -1, 1, -1, 1, 0, 0, 0, 0, 0, 0)$
$v_2 = (-1, 1, 0, 0, 0, 0, -1, 1, -1, 1, -1, 1, 0, 0, 0)$
$v_3 = (0, -1, 1, 0, 0, 1, -1, 0, 0, 0, 1, -1, -1, 1, 0)$
$v_4 = (0, 0, -1, 0, 1, 0, 1, 0, -1, -1, 0, 1, -1, 0, -1)$
$v_5 = (0, 0, 1, 0, -1, 0, -1, 0, 1, 0, 0, 0, 0, 0, 0)$
$v_6 = (0, 1, 0, 0, -1, -1, 0, 0, 1, 0, 0, 0, 0, 0, 0)$
$v_7 = (0, 0, 0, 0, 0, 0, -1, 0, 1, 1, 0, -1, 0, 0, 0)$
$v_8 = (0, 0, 0, 0, 0, 0, 0, 1, -1, 0, -1, 1, 0, 0, 0)$
$v_9 = (0, 0, 0, 0, 0, 1, 0, -1, 0, 1, 0, 0, 1, 0, 0)$
$v_{10} = (0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 1, 0, -1, 0, 0)$.

The restriction to V of the k-morphisms

$$e_{ij} \mapsto x_i^m x_j^n - x_i^n x_j^m$$

for $1 \le n < m \le 5$, where $x_1, \dots, x_6$ are the roots of the polynomial $P(X)$ in K, are monomorphisms, which we denote by $f_j, 1 \le j \le 10$, and their images are ten subspaces of dimension 10, isomorphic as A_6-modules to V and such that their sum is direct. In order to check this last statement, by using Lemma 1, it is enough to check that the ten vectors $f_j(v_i)$, for a fixed i, are linearly independent.

By Lemma 1, every invariant subspace of K isomorphic to V as an A_6-module is of the form

$$\langle (\sum_j a_j f_j(v_i))_{1 \le i \le 10} \rangle$$

for some $(a_1, \dots, a_{10}) \in \mathbb{P}_9(k)$.

Now let (u_1, u_2, u_3) be a basis in which the representation $\widetilde{\rho}$ is given by the matrices E_1, E_2, E_3, E_4 above. We take in V a basis $F_1, \dots, F_{10}$ such that assigning the vectors in the basis

$$(u_1^3, u_2^3, u_3^3, u_1^2 u_2, u_1 u_2^2, u_1^2 u_3, u_1 u_3^2, u_2^2 u_3, u_2 u_3^2, u_1 u_2 u_3)$$

of the third symmetric power $U^{(3)}$ of the $3A_6$-module $U = \langle u_1, u_2, u_3 \rangle$ to the vectors

F_i is a morphism of A_6-modules. By computation, we obtain that the basis change matrix from the basis $(F_i)_{1\leq i\leq 10}$ to the basis $(v_i)_{1\leq i\leq 10}$ is:

$$C = \tfrac{1}{8}(C_1 + \sqrt{-15}\,C_2)$$

where C_1 and C_2 are the following matrices written down by rows:

$C_1 = [[15(3\zeta^2-\zeta+3), 120(-3s+1), 120(3s+\zeta^2+3\zeta), 10(17s+26\zeta^2+43\zeta+17), 20(-11s-7\zeta^2-11\zeta), 10(-17s+17\zeta^2+26\zeta+26), 20(11s-7), 40(7s+16\zeta^2+23\zeta+7), 40(-7s+7\zeta^2+16\zeta+16), 20(9\zeta^2+12\zeta+9)], [15(3\zeta^2-\zeta+3), 120(s-z^2-3\zeta-3), 120(-s-3\zeta^2-4\zeta-1), 10(26s-17), 20(7s+11\zeta^2+18\zeta+7), -10(26s+17\zeta^2+26\zeta), 20(-7s+7\zeta^2+11\zeta+11), 40(16s-7), -40(16s+7\zeta^2+16\zeta), 20(9\zeta^2+12\zeta+9)], [0, 240(4s+4\zeta^2+7\zeta+1), 240(-4s+\zeta^2+3\zeta+4), 80(s+6\zeta^2+8\zeta+4), 80(4s-\zeta^2-3\zeta-4), 80(-s+4\zeta^2+7\zeta+6), -80(4s+4\zeta^2+7\zeta+1), -80(s+6\zeta^2+8\zeta+4), 80(s-4\zeta^2-7\zeta-6), 0], [0, 240(6s+\zeta^2+3\zeta-1), 240(-6s-\zeta^2-3\zeta+1), 80(-6s-\zeta^2-3\zeta+1), 80(6s+\zeta^2+3\zeta-1), 80(6s+\zeta^2+3\zeta-1), 80(-6s-\zeta^2-3\zeta+1), 80(6s+\zeta^2+3\zeta-1), 80(-6s-\zeta^2-3\zeta+1), 0], [30(16\zeta^2+23\zeta+16), 120(s-2\zeta^2-\zeta+1), 120(-s+\zeta^2-2\zeta-2), -10(39s+23\zeta^2+39\zeta), 20(-3s+3\zeta^2+14\zeta+14), 10(39s-23), 20(3s+14\zeta^2+17\zeta+3), 40(21s+17\zeta^2+21\zeta), 40(-21s+17), 20(21\zeta^2+33\zeta+21)], [-25(21\zeta^2+27\zeta+21), 120(6s+6\zeta^2+13\zeta+4), 120(-6s+4\zeta^2+7\zeta+6), 10(-34s+21\zeta^2+28\zeta+39), 20(22s-8\zeta^2-19\zeta-22), 10(34s+39\zeta^2+62\zeta+21), -20(22s+22\zeta^2+41\zeta+8), -40(14s+39\zeta^2+52\zeta+21), 40(14s-21\zeta^2-38\zeta-39), -300(2\zeta^2+3\zeta+2)], [15(-3\zeta^2+\zeta-3), 120(-5s-3\zeta^2-4\zeta+2), 120(5s+2\zeta^2+\zeta-3), 10(-25s-14\zeta^2-12\zeta+11), -20(5s+14\zeta^2+17\zeta+9), 10(25s+11\zeta^2+13\zeta-14), 20(5s-9\zeta^2-12\zeta-14), 40(-5s+26\zeta^2+48\zeta+31), 40(5s+31\zeta^2+53\zeta+26), -60(3\zeta^2+4\zeta+3), 30(-3\zeta^2+\zeta-3), 40(-36s-24\zeta^2-27\zeta+3), 40(36s+3\zeta^2+9\zeta-24), 10(-27s+50\zeta^2+100\zeta+84), -20(24s+17\zeta^2+31\zeta+4), 10(27s+84\zeta^2+127\zeta+50), 20(24s-4\zeta^2-7\zeta-17), 40(-27s+10\zeta^2+20\zeta+24), 40(27s+24\zeta^2+47\zeta+10), -120(3\zeta^2+4\zeta+3)], [-25(21\zeta^2+27\zeta+21), -600(s+\zeta^2+\zeta), 600(s-1), 250(-s+\zeta^2+2\zeta+2), 100(-s+1), 250(s+2\zeta^2+3\zeta+1), 100(s+\zeta^2+\zeta), 200(-s+\zeta^2+2\zeta+2), 200(s+2\zeta^2+3\zeta+1), -300(2\zeta^2+3\zeta+2)], [-30(16\zeta^2+23\zeta+16), -240(11s+7\zeta^2+11s), 240(11s-7), 20(-s+\zeta^2+18\zeta+18), 40(-17s+9), 20(1+18\zeta^2+19\zeta+1), 40(17s+9\zeta^2+17\zeta), 80(-11s+11\zeta^2+18\zeta+18), 80(11s+18\zeta^2+29\zeta+11), -60(7\zeta^2+11\zeta+7)]];$

$C_2 = [[15(5\zeta^2+9\zeta+5, 120(-s+1), 120(s+\zeta^2+\zeta), -10(s+2\zeta^2+3\zeta+1), 60(s+\zeta^2+\zeta), 10(s-\zeta^2-2\zeta-2), 60(-s+1), -40(s+2\zeta^2+3\zeta+1), 40(s-\zeta^2-2\zeta-2), -20(5\zeta^2+8\zeta+5)], [15(5\zeta^2+9\zeta+5), 120(s-\zeta^2-\zeta-1), -120(s+\zeta^2+2\zeta+1), 10(-2s+1), -60(s+\zeta^2+2\zeta+1), 10(2s+\zeta^2+2\zeta), 60(s-\zeta^2-\zeta-1), 40(-2s+1), 40(2s+\zeta^2+2\zeta), -20(5\zeta^2+8\zeta+5)], [0,0,0,0,0,0,0,0,0,0], [0,0,0,0,0,0,0,0,0,0], [30\zeta, -120(s+2\zeta^2+3\zeta+1), 120(s-\zeta^2-2z-2), -10(s+\zeta^2+\zeta), 60(s-\zeta^2-2\zeta-2), 10(s-1), -60(s+2\zeta^2+3\zeta+1), -40(s+\zeta^2+\zeta), 40(s-1), -20(5\zeta^2+9\zeta+5)], [-15(5\zeta^2+11\zeta+5), 120(2s+2\zeta^2+3\zeta), 120(-2s+\zeta+2), 10(2s+3\zeta^2+4\zeta+1), 60(-2s+\zeta+2), 10(-2s+\zeta^2+2\zeta+3), 60(2s+2\zeta^2+3\zeta), 40(2s+3\zeta^2+4\zeta+1), 40(-2s+\zeta^2+2\zeta+3), 20(10\zeta^2+17\zeta+10)], [-15(5\zeta^2+9\zeta+5), 120(s+3\zeta^2+4\zeta+2), 120(-s+2\zeta^2+3\zeta+3), 10(s-2\zeta^2-4\zeta-3), 60(-s+2\zeta^2+3\zeta+3), -10(s+3\zeta^2+5\zeta+2), 60(s+3\zeta^2+4\zeta+2), 40(s-2\zeta^2-4\zeta-3), -40(s+3\zeta^2+5\zeta+2), 20(5\zeta^2+8\zeta+5), -30(5\zeta^2+9\zeta+5), 120(4\zeta^2+5\zeta+3), 120(3\zeta^2+5\zeta+4), 10(3s-2\zeta^2-4\zeta-4), 60(3\zeta^2+5\zeta+4), -10(3s+4\zeta^2+7\zeta+2), 60(4\zeta^2+5\zeta+3), 40(3s-2\zeta^2-4\zeta-4), -40(3s+4\zeta^2+7\zeta+2), 40(5\zeta^2+8\zeta+5)], [-15(5\zeta^2+11\zeta+5), 120(s+\zeta^2+\zeta), 120(-s+1), 10(s-\zeta^2-2\zeta-2), 60(-s+1), -10(s+2\zeta^2+3\zeta+1), 60(s+\zeta^2+\zeta), 40(s-\zeta^2-2\zeta-2), -40(s+2\zeta^2+3\zeta+1), 20(10\zeta^2+17\zeta+10)], [-30\zeta, 240(s+\zeta^2+\zeta), 240(-s+1), 20(s-\zeta^2-2\zeta-2), 120(-s+1), -20(s+2\zeta^2+3\zeta+1), 120(s+\zeta^2+\zeta), 80(s-\zeta^2-3\zeta-3), -80(s+2\zeta^2+3\zeta+1), 20(5\zeta^2+9\zeta+5)]].$

Let $F_{ij} = f_j(F_i), 1 \leq i, j \leq 10$. Any invariant subspace of K isomorphic to V as an A_6-module then has a basis

$$G_i = \sum_j a_j F_{ij}, 1 \leq i \leq 10,$$

with $(a_1, \dots, a_{10}) \in \mathbb{P}_9(k)$, which behaves under the A_6-action as the basis in $U^{(3)}$.

The subspace $\langle G_1, \dots, G_{10} \rangle$ is a symmetric cube if the elements G_i can be written as $(u_1^3, u_2^3, u_3^3, u_1^2 u_2, u_1 u_2^2, u_1^2 u_3, u_1 u_3^2, u_2^2 u_3, u_2 u_3^2, u_1 u_2 u_3)$ for some parameters u_1, u_2, u_3. This is equivalent to the set of polynomials $\mathcal{P} = \{X_1X_2 - X_4X_5, X_1X_3 - X_6X_7, X_2X_3 - X_8X_9, X_1X_{10} - X_4X_6, X_2X_{10} - X_5X_8, X_1X_5 - X_4^2, X_5X_7 - X_{10}^2\}$ vanishing in $G_1, \dots, G_{10}$. Since the F_{ij} are known elements in the field K, the condition above provides degree 2 homogeneous equations defined over K on the coordinates $(a_1, \dots, a_{10})$ and so defines an algebraic variety Q in $\mathbb{P}_9(K)$. Now, we can add polynomials to the set $\mathcal{P}$ in order to obtain a family of equations with the property that, by the action of A_6, each of these equations is transformed into a linear combination of the equations in the family with coefficients in the field $\mathbb{Q}(\mu_{15})$, and so we obtain that the variety Q is defined over k.

The field $K(\sqrt[3]{G_1})$ is a cubic extension of K containing the elements $\sqrt[3]{G_1}$, $\sqrt[3]{G_2}$, $\sqrt[3]{G_3}$ which generate a k-subspace on which the action of $3A_6$ corresponds to the representation $\widetilde{\rho}$, and so we obtain the statements a) and b) in the theorem. □

Acknowledgment

We thank Professor Jean-Pierre Serre for his valuable comments on an earlier version of our work.

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