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ERRATA TO "HECKE ALGEBRAS OF SEMIDIRECT PRODUCTS"

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There is a mistake in Lemma 1.3 of our paper [3] that invalidates the proofs of [3, Lemma 1.10] and [3, Theorem 1.9]. We are grateful to Iain Raeburn, Jacqui Ramagge and Udo Baumgartner for communicating this to us and for suggesting that Lemma 1 below should be used in place of [3, Lemma 1.3] to fix the results. It is also necessary to normalize the products mentioned in part (i) of [3, Theorem 1.9]. Indeed, the correct linear basis for the Hecke algebra there is the set

$$\left\{ \frac{1}{R(n)} \mu_t^*[n]\mu_s : s,t \in \Sigma \text{ and } n \in N \right\}.$$

Lemma 1 (cf. [2, Corollary I. 4.5])**.** *If* $\Gamma_0 x \Gamma_0 y \Gamma_0 = \Gamma_0 xy \Gamma_0$, *then* $[x][y] = \frac{R(x)R(y)}{R(xy)}[xy]$.

The following lemma should replace [3, Lemma 1.10].

Lemma 2. *For* $s, t, \tau, \sigma \in \Sigma$ *and* $n, m \in N$,

$$[t^{-1}ns] = [\tau^{-1}m\sigma] \quad \iff \quad \mu_t^* \frac{[n]}{R(n)} \mu_s = \mu_\tau^* \frac{[m]}{R(m)} \mu_\sigma, \text{ in } H(\Gamma, \Gamma_0).$$

Proof. First notice that the partial products in $\mu_\tau^* \frac{[m]}{R(m)} \mu_\sigma$ are supported on single double cosets. To compute them we use Lemma 1 to get

$$[\tau^{-1}][m] = \frac{R(\tau^{-1})R(m)}{R(\tau^{-1}m)}[\tau^{-1}m] \quad \text{and} \quad [\tau^{-1}m][\sigma] = \frac{R(\tau^{-1}m)R(\sigma)}{R(\tau^{-1}m\sigma)}[\tau^{-1}m\sigma],$$

and then combine the results to obtain the triple product

$$\begin{aligned} \mu_\tau^* \frac{[m]}{R(m)} \mu_\sigma &= \frac{[\tau^{-1}][m][\sigma]}{R(\tau)^{1/2} R(m) R(\sigma)^{1/2}} \\ &= \frac{1}{R(\tau)^{1/2}\ R(m) R(\sigma)^{1/2}}\ \frac{R(\tau^{-1})R(m)}{R(\tau^{-1}m)}\ \frac{R(\tau^{-1}m)R(\sigma)}{R(\tau^{-1}m\sigma)}\ [\tau^{-1}m\sigma]. \end{aligned}$$

Since the triple product is supported on a single double coset, the implication ($\Leftarrow$) follows. Next we simplify the coefficient, using $R(\tau^{-1}) = L(\tau) = 1$, to obtain

$$(\dagger) \qquad \mu_\tau^* \frac{[m]}{R(m)} \mu_\sigma = \left(\frac{R(\sigma)}{R(\tau)} \right)^{1/2} \frac{[\tau^{-1}m\sigma]}{R(\tau^{-1}m\sigma)}.$$

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Suppose now that $[\tau^{-1}m\sigma] = [t^{-1}ns]$. Taking quotients modulo N we see that $\tau^{-1}\sigma = t^{-1}s$ in $G = \Gamma/N$ so there exist elements γ and r in $S = \Sigma/N$ such that $\gamma\tau = rt$ and $\gamma\sigma = rs$ because S is directed. Since R is a homomorphism on Σ,

$$\frac{R(\sigma)}{R(\tau)} = \frac{R(\gamma\sigma)}{R(\gamma\tau)} = \frac{R(rs)}{R(rt)} = \frac{R(s)}{R(t)},$$

and the implication ($\Rightarrow$) now follows from (†). □

To correct the proof of [3, Theorem 1.9] we need a direct proof of the (rescaled) key identity

$$(\star) \qquad \mu_x\mu_x^*\mu_r\frac{[n]}{R(n)}\mu_r^*\mu_y\mu_y^* = \mu_x\mu_x^*\mu_\gamma\frac{[m]}{R(m)}\mu_\gamma^*\mu_y\mu_y^*.$$

Suppose $\mu_t^*\frac{[n]}{R(n)}\mu_s = \mu_\tau^*\frac{[m]}{R(m)}\mu_\sigma$ in $H(\Gamma,\Gamma_0)$, and let γ and r be as in the proof of Lemma 2 above. Then

$$\mu_{rt}^*\mu_r\frac{[n]}{R(n)}\mu_r^*\mu_{rs} = \mu_{\gamma\tau}^*\mu_\gamma\frac{[m]}{R(m)}\mu_\gamma^*\mu_{\gamma\sigma}.$$

Letting $x = rt = \gamma\tau$ and $y = rs = \gamma\sigma$ and multiplying on the left by μ_x and on the right by μ_y^* shows that equation ($\star$) holds in $H(N,\Gamma_0)$ because of the relation ($\mathfrak{h}_3$). Thus ($\star$) also holds for the universal (tilded) generators. Multiplying this now on the left by $\tilde\mu_x^*$ and on the right by $\tilde\mu_y$, and simplifying, yields $\tilde\mu_t^*\frac{\tilde e(n)}{R(n)}\tilde\mu_s = \tilde\mu_\tau^*\frac{\tilde e(m)}{R(m)}\tilde\mu_\sigma$, as desired. It follows that the canonical homomorphism $H(N,\Gamma_0)\rtimes_\alpha S \to H(\Gamma,\Gamma_0)$ maps the spanning set $\{\frac{1}{R(m)}\tilde\mu_\tau^*\tilde e(m)\tilde\mu_\sigma\}$ of the universal algebra of the relations one-to-one and onto the linear basis $\{\frac{1}{R(m)}\mu_\tau^* e(m)\mu_\sigma\}$ of the Hecke algebra, hence is an isomorphism.

Note: A very interesting generalization of the results of [3] to group extensions has been obtained by Baumgartner et al. in [1] which implicitly provides, in the particular case of split extensions, a correction to the error they found in [3].

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