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FREE GROUP FACTORS AND FACTORS WITH SOME DECOMPOSITIONS

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Abstract. In this paper, we show that if type II_1 von Neumann factors $\mathcal{M}$ have some decompositions introduced by Liming Ge and Sorin Popa, then these von Neumann factors are not isomorphic to free group factors $L(F_n)$ $(n \geq 2)$. Thus we have proved the number l_a defined by Ge and Popa bigger than 3 for all free group factors and we also extend some results of M. Stefan.

1. Introduction

In studying the structure of type II_1 factors, Connes and Jones [1] proved that von Neumann algebras with property T are not isomorphic to free group factors $L(F_n)$ $(n \geq 2)$. By introducing free entropy in [8] and [9], Voiculescu proved that free group factors $L(F_n)$ are not isomorphic to von Neumann algebras that have Cartan subalgbras. Also using Voiculescu's free entropy, Liming Ge [2], [3] proved that free group factors possess no simple masa and are *prime*. For the free group factors, Voiculescu [7] proved the following result.

Proposition 1. *Let $L(F_n)$ $(n > 1)$ be free group factors. Then*

$$(L(F_n))_{1/k} \cong L(F(nk^2 - k^2 + 1)),$$

where k is a positive integer.

In the study of von Neumann algebras, S. Popa [5] found a large class of non-hyperfinite type II_1 factors (called thin factors) having a decomposition of the form $M = \overline{sp} R_0 R_1$, with R_0, R_1 hyperfinite ($\overline{sp} X$ denotes the closed linear span of the set X in the Hilbert-space norm given by the trace of the ambient type II_1 factor). Liming Ge and Sorin Popa [4] proved that thin factors are not isomorphic to free group factors $L(F_n)$ $(n \geq 3)$. In the same paper [4], Liming Ge and Sorin Popa also proved that if a type II_1 factor $\mathcal{M}$ has an irreducible hyperfinite quasi-regular subfactor $\mathcal{R}_0 \subset \mathcal{M}$ (that is, so that $\mathcal{R}_0' \cap \langle \mathcal{M}, \mathcal{R}_0 \rangle$, where $\langle \mathcal{M}, \mathcal{R}_0 \rangle$ denotes the von Neumann algebra generated in $B(L^2(\mathcal{M}, \tau))$ by $\mathcal{M}$ and by the orthogonal projection $e_{\mathcal{R}_0}$ of $L^2(\mathcal{M}, \tau)$ onto $L^2(\mathcal{R}_0, \tau)$, is generated by projections of finite trace in $\langle \mathcal{M}, \mathcal{R}_0 \rangle$), then from [4] there exist an abelian subalgebra $\mathcal{A} \subset \mathcal{M}$ and an element $b \in \mathcal{M}$ such that $\mathcal{M} = \overline{\mathrm{sp}} \mathcal{R}_0 b \mathcal{A}$. These factors have vanishing second Hochschild cohomology groups, $H^2(\mathcal{M}, \mathcal{M}) = 0$. All symmetric enveloping factors

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mentioned above and all the cross-products of the hyperfinite factor by cocycle actions of a group do have this property. Hinted by Ge and Popa's work [4], Stefan [6] studied the indecomposabilty of free group factors over nonprime subfactors and abelian subalgebras and obtained the following result.

Proposition 2. *If $n \geq p+2f+2$, then the free group factor $L(F_n)$ does not admit a decomposition of the form*

$$\|\cdot\|_2 - \lim_{\omega\to 0} \sum_{\substack{1 \leq j_1, \dots, j_{t+1} \leq f \\ 1 \leq t \leq d}} \mathcal{M}_{j_1} Z^\omega \mathcal{M}_{j_2} Z^\omega \cdots \mathcal{M}_{j_t} Z^\omega \mathcal{M}_{j_{t+1}},$$

*where $\{Z^\omega\}_\omega$ are subsets of $L(F_n)$, each containing p self-adjoint elements, $\mathcal{M}_1, \dots, \mathcal{M}_f \subset L(F_n)$ are either abelian *-subalgebras or non-prime subfactors of $L(F_n)$, and $d \geq 1$ is an integer; and $\|\cdot\|_2$ denotes the L^2-norm (or trace-norm) of $L(F_n)$ with respect to the faithful normal tracial state τ (that is, $\|A\|_2^2 = \tau(A^*A)$ with $A \in L(F_n)$).*

In this paper, we use Proposition 1 and Proposition 2 to show that all free group factors $L(F_n)$ are not isomorphic to type II_1 factors $\mathcal{M}$ with composition $\overline{sp}\{\mathcal{A}_0 a_s \mathcal{A}_1 b_t \mathcal{A}_2 : s = 1, \dots, m, t = 1, \dots, q\}$ or $\overline{sp}\{\mathcal{A}_0 a_s \mathcal{R} b_t \mathcal{A}_1 : s = 1, \dots, m, t = 1, \dots, q\}$ or $\overline{sp}\mathcal{R}_0\mathcal{R}_1$, where $\mathcal{A}_0$, $\mathcal{A}_1$ and $\mathcal{A}_2$ are diffused abelian subalgebras of $\mathcal{M}$; $\mathcal{R}$, $\mathcal{R}_0$ and $\mathcal{R}_1$ are hyperfinite subfactors of $\mathcal{M}$ such that $\mathcal{R}_0$ and $\mathcal{R}_1$ satisfy some conditions; and $a_s, b_t, 1 \leq s \leq m, 1 \leq t \leq q$, are selfadjoint elements in $\mathcal{M}$. As a result, type II_1 factors with decomposition $\overline{sp}\mathcal{R}b\mathcal{A}$ are not isomorphic to all free group factors $L(F_n), n > 1$, on n generators. Our results also state that the abelian length defined in [4] of all free group factors is bigger than 3; thus we generalize some results in [6].

2. Main results and proof

Von Neumann algebras are algebras of bounded operators on a Hilbert space $\mathcal{H}$, containing the identity operator I and A^* when they contain A and closed in the strong-operator topology ($\{A_n\}$ converges to A in this topology when $\{A_n x\}$ converges to Ax for each $x \in \mathcal{H}$). For a von Neumann algebra $\mathcal{M}$, the commutant $\mathcal{M}'$ of $\mathcal{M}$ consists of those operators T that commute with each element of $\mathcal{M}$. *Factors* $\mathcal{M}$ are von Neumann algebras whose centers $\mathcal{M} \cap \mathcal{M}'$ consist of scalar multiples of I. A type II_1 factor $\mathcal{M}$ is a factor with a unique faithful tracial state τ such that $\tau(I) = 1$ and $\tau(\mathcal{P}(\mathcal{M})) = [0, 1]$, where $\mathcal{P}(\mathcal{M})$ is all projections in $\mathcal{M}$. If a von Neumann algebra has no minimal projections, we call such a von Neumann algebra diffused.

Let G be a discrete countable group with unit e. For each $g \in G$, let L_g denote the left translation of functions in the Hilbert space $l^2(G)$ by g^{-1}. Then $g \to L_g$ is a faithful unitary representation of G on $l^2(G)$. Let $L(G)$ be the von Neumann algebra generated by $\{L_g : g \in G\}$. In general, $L(G)$ is a finite von Neumann algebra. It is a factor (of type II_1) precisely when each conjugacy class in G (other than that of e) is infinite, that is to say, G is an i.c.c. group.

Specific examples of such II_1 factors result from choosing for G any of the free groups F_n on n generators ($n \geq 2$). We shall prove that none of the II_1 factors $L(F_n)$ are isomorphic to three classes of type II_1 factors with some decompositions.

Theorem 2.1. *Let $\mathcal{M}$ be a type II_1 factor with the decomposition*

$$\mathcal{M} = \overline{sp}\{\mathcal{A}_1 a_s \mathcal{A}_2 b_t \mathcal{A}_3 : s = 1, ..., m, t = 1, ..., q\},$$

where $\mathcal{A}_j, j = 1, 2, 3$, are diffused abelian subalgebra of $\mathcal{M}$ and a_s, b_t are non-zero elements in $\mathcal{M}$, then $\mathcal{M}$ is not isomorphic to any one of the free group factors $L(F_n)$ $(n > 1)$.

Proof. Since $\mathcal{A}_2$ is diffused, then for any k in $\mathbf{N}$, one can choose a projection $p \in \mathcal{A}_2$ such that $\tau(p) = \frac{1}{k}$. So there exist unitary elements u and v in $\mathcal{M}$ satisfying $upu^* \in \mathcal{A}_1$ and $vpv^* \in \mathcal{A}_3$. Also, we have

$$\begin{aligned}\mathcal{M} &= \overline{sp}\{u^*\mathcal{A}_1 uu^* a_s \mathcal{A}_2 bvv^* \mathcal{A}_3 v : 1 \le s \le m, 1 \le t \le q\}\\ &= \overline{sp}\{u^*\mathcal{A}_1 uu^* a_s \mathcal{A}_2 b_t vv^* \mathcal{A}_3 v : 1 \le s \le m, 1 \le t \le q\}\\ &= \overline{sp}\{\tilde{\mathcal{A}}_1 u^* a_s \mathcal{A}_2 b_t v \tilde{\mathcal{A}}_3 : 1 \le s \le m, 1 \le t \le q\},\end{aligned}$$

where $\tilde{\mathcal{A}}_1 = u^*\mathcal{A}_1 u$ and $\tilde{\mathcal{A}}_3 = v^*\mathcal{A}_3 v$. We have the fact that $\tilde{\mathcal{A}}_1$ and $\tilde{\mathcal{A}}_3$ are also diffused abelian, and $p \in \tilde{\mathcal{A}}_1 \cap \mathcal{A}_2 \cap \tilde{\mathcal{A}}_3$.

Since $\mathcal{A}_2$ is diffused, there exist projections $p_j \in \mathcal{A}_2$ and unitary elements $u_j \in \mathcal{M}$, $j = 1, ..., k$, such that $\sum_{j=1}^k p_j = I$ and $u_j^* p u_j = p_j$. Now

$$\begin{aligned}p\mathcal{M}p &= \overline{sp}\{p\tilde{\mathcal{A}}_1 pu^* a_s \textstyle\sum_{j=1}^k p_j \mathcal{A}_2 b_t vp\tilde{\mathcal{A}}_3 p : 1 \le s \le m, 1 \le t \le q\}\\ &= \overline{sp}\{p\tilde{\mathcal{A}}_1 pu^* a_s \textstyle\sum_{j=1}^k p_j \mathcal{A}_2 p_j b_t vp\tilde{\mathcal{A}}_3 p : 1 \le s \le m, 1 \le t \le q\}\\ &= \overline{sp}\{p\tilde{\mathcal{A}}_1 pu^* a_s u_j^* pu_j \mathcal{A}_2 u_j^* pu_j b_t vp\tilde{\mathcal{A}}_3 p : j = 1, ..., k; 1 \le s \le m, 1 \le t \le q\}.\end{aligned}$$

Because $p\tilde{\mathcal{A}}_1 p$, $pu_j\mathcal{A}_2 u_j^* p$ and $p\tilde{\mathcal{A}}_3 p$ are abelian subalgebras of $p\mathcal{M}p$ and by Proposition 2, $p\mathcal{M}p$ is not isomorphic to $L(F_n)$ $(n > 6 + 2(m + q + 1)k)$.

By Proposition 2 again, we know that $\mathcal{M}$ is not isomorphic to free group factors $L(F_n), n \ge 2(m + q) + 8$.

If $\mathcal{M}$ is isomorphic to some one of free group factors $L(F_n), n < 2(m + q) + 8$, and if we choose a projection $p \in \mathcal{M}(\tau(p) = \frac{1}{k})$, then $p\mathcal{M}p \cong L(F_{(n-1)k^2+1})$ by Proposition 1. On the other hand, when k is big enough, $(n-1)k^2 + 1 > 6+2(s+t+1)k$, from the previous paragraphs, one has that $p\mathcal{M}p$ is not isomorphic to $L(F_{(n-1)k^2+1})$. This is a contradiction. So for any one of free group factors, $L(F_n)$ is not isomorphic to the type II_1 factor $\mathcal{M}$ with decomposition $\overline{sp}\{\mathcal{A}_1 a_s \mathcal{A}_2 b_t \mathcal{A}_3 : 1 \ge s \le m, 1 \ge t \le q\}$. □

With the same method, one can prove the following theorem.

Theorem 2.2. *Any one of free group factors $L(F_n), n > 1$, is not isomorphic to the type II_1 factor $\mathcal{M}$ with the decomposition $\overline{sp}\{\mathcal{A}_0 a_s \mathcal{A}_1 : s = 1, ..., m\}$, where $\mathcal{A}_0$ and $\mathcal{A}_1$ are diffused von Neumann subalgebras and $a_s, s = 1, ..., m$, are non-zero elements in $\mathcal{M}$.*

Since all hyperfinite II_1 factors are all isomorphic, and a hyperfinite II_1 factor $\mathcal{R}$ has decomposition $\overline{sp}\mathcal{A}_0\mathcal{A}_1$ (by the irrational rotation algebra representation of $\mathcal{R}$), then as a corollary we have the following result.

Corollary 2.3. *None of free group factors $L(F_n), n > 1$, is isomorphic to the type II_1 factor $\mathcal{M}$ with the decomposition $\overline{sp}\{\mathcal{R} a_s \mathcal{A} : s = 1, ..., m\}$, where $\mathcal{R}$ is a hyperfinite subfactor of $\mathcal{M}$ and $\mathcal{A}$ are diffused von Neumann subalgebras and $a_s, s = 1, ..., m$, are non-zero elements in $\mathcal{M}$.*

Theorem 2.4. *None of free group factors $L(F_n), n > 1$, is isomorphic to a type II_1 factor $\mathcal{M}$ with the decomposition $\overline{sp}\{\mathcal{A}_0 a_s \mathcal{R} b_t \mathcal{A}_1 : s = 1, ..., m, t = 1, ..., q\}$, where $\mathcal{A}_0$ and $\mathcal{A}_1$ are diffused von Neumann subalgebras, $\mathcal{R}$ is a hyperfinite subfactor of $\mathcal{M}$ and $a_s, b_t, s = 1, ..., m; t = 1, 2, ..., q$, are non-zero elements in $\mathcal{M}$.*

Proof. By Proposition 2, we know that the factor $\mathcal{M}$ in Theorem 2.4 is not isomorphic to free group factors $L(F_n)$, $n \geq 2(m+q)+8$.

On the other hand, pick $p \in \mathcal{M}$ and $\tau(p) = \frac{1}{k}$. Then there exist unitary elements u_0, u_1 and u_R in $\mathcal{M}$ such that $u_0^* p u_0 \in \mathcal{A}_0, u_1^* p u_1 \in \mathcal{A}_1$ and $u_R^* p u_R \in \mathcal{R}$. We also know that

$$\begin{aligned}\mathcal{M} &= \overline{sp}\{u_0 \mathcal{A}_0 u_0^* u_0 a_s u_R^* u_R \mathcal{R} u_R^* u_R b_t u_1^* u_1 \mathcal{A}_1 u_1^* : s = 1, 2, ..., m; t = 1, 2, ..., q\} \\ &= \overline{sp}\{\tilde{\mathcal{A}}_0 \tilde{a}_s \tilde{\mathcal{R}} \tilde{b}_t \tilde{\mathcal{A}}_1 : 1 \leq s \leq m, 1 \leq t \leq q\},\end{aligned}$$

where $\tilde{\mathcal{A}}_i = u_i \mathcal{A}_i u_i^*$, $i = 0, 1$, and $\tilde{\mathcal{R}} = u_R \mathcal{R} u_R^*$. So $p \in \tilde{\mathcal{A}}_0 \cap \tilde{\mathcal{R}} \cap \tilde{\mathcal{A}}_1$. Then we assume that $\mathcal{M} = \overline{sp}\{\mathcal{A}_0 a_s \mathcal{R} b_t \mathcal{A}_1 : 1 \leq s \leq m, 1 \leq t \leq q\}$ and there is a projection $p \in \mathcal{A}_0 \cap \mathcal{R} \cap \mathcal{A}_1$ such that $\tau(p) = \frac{1}{k}$, where $a_s, 1 \leq s \leq m$, and $b_t, 1 \leq t \leq q$, are elements in $\mathcal{M}$ (not necessary self-adjoint elements).

Since $\mathcal{R}$ is a type II_1 factor, then there are orthogonal projections $q_j \in \mathcal{A}$ ($j = 1, ..., k$) and unitary elements $v_j \in \mathcal{R}$ ($j = 1, ..., k$) such that $\sum_{j=1}^{k} p_j = I, \tau(p_j) = \frac{1}{k}$ and $q_j = v_j^* p v_j$. Now

$$\begin{aligned}p\mathcal{M}p &= \overline{sp}\{p\mathcal{A}_0 p p a_s (\textstyle\sum q_j) \mathcal{R} (\textstyle\sum q_j) b_t p \mathcal{A}_1 p : 1 \leq s \leq m, 1 \leq t \leq q\} \\ &= \overline{sp}\{p\mathcal{A}_0 p p a_s v_j^* p v_j \mathcal{R} v_i^* p v_i b_t p \mathcal{A}_1 p : 1 \leq j, i \leq k, 1 \leq s \leq m, 1 \leq t \leq q\} \\ &= \overline{sp}\{p\mathcal{A}_0 p p a_s v_j^* p \mathcal{R} p v_i b_t p \mathcal{A}_1 p : 1 \leq j, i \leq k, 1 \leq s \leq m, 1 \leq t \leq q\},\end{aligned}$$

and $p\mathcal{A}_0 p$ and $p\mathcal{A}_1 p$ are also abelian subalgebras of $p\mathcal{M}p$, and $p\mathcal{R}p$ is a hyperfinite subfactor of $p\mathcal{M}p$. Then, by Proposition 2, $p\mathcal{M}p$ is not isomorphic to free group factors $L(F_n)$, $n \geq 2(m+q)k + 8$.

If $\mathcal{M}$ is isomorphic to one of free group factors $L(F_n)$, $n < 2(m+q)+8$, then $p\mathcal{M}p$ is isomorphic to $L(F_{(n-1)k^2+1})$ by Proposition 1, where p is as above. Since $\mathcal{A}_0$, $\mathcal{A}_1$ and $\mathcal{R}$ are diffused subalgebras of $\mathcal{M}$, then p can be chosen small enough. When k is big enough, $(n-1)k^2 + 1 > 2(m+q)k + 8$. From the second part of the above proof, $p\mathcal{M}p$ is not isomorphic to free group factors $L(F_{(n-1)k^2+1})$; this is contradiction. This completes the proof. □

In [4], Ge and Popa proved that free group factors $L(F_n), n > 2$, are not isomorphic to type II_1 factors $\mathcal{M}$ with decomposition $\overline{sp}\mathcal{R}_0\mathcal{R}_1$, where $\mathcal{R}_0$ and $\mathcal{R}_1$ are hyperfinite subfactors of $\mathcal{M}$. The following theorem states that if $\mathcal{R}_0$ and $\mathcal{R}_1$ satisfy some additional conditions, then all free group factors are not isomorphic to above type II_1 factors.

Theorem 2.5. *Let $\mathcal{M}$ be a type II_1 factor, and let $\mathcal{M} = \overline{sp}\mathcal{R}_0\mathcal{R}_1$. If the hyperfinite subfactor $\mathcal{R}_0 \cap \mathcal{R}_1$ of $\mathcal{R}_0$ and $\mathcal{R}_1$ has orthogonal projections $p_1, ..., p_s$ such that $p_1 + \cdots + p_s = I$ and $\max_{1 \leq j \leq s}\{\tau(p_j)\} < \frac{1}{2}$, then all free group factors $L(F_n)$ ($n > 1$) are not isomorphic to $\mathcal{M}$.*

Proof. Since $\mathcal{R}_0$ and $\mathcal{R}_1$ are type II_1 factors and $p_j \in \mathcal{R}_0 \cap \mathcal{R}_1$, then there are orthogonal projections

$$\begin{aligned}&e_1, ..., e_{m_1}; e_{m_1+1}, ..., e_{m_2}; ...; e_{m_{s-1}+1}, ..., e_{m_s} \in \mathcal{R}_0, \\ &f_1, ..., f_{m_1}; f_{m_1+1}, ..., f_{m_2}; ...; f_{m_{s-1}+1}, ..., f_{m_s} \in \mathcal{R}_1,\end{aligned}$$

such that $\tau(e_j) = \tau(f_j) = \frac{1}{k}$ $(k = m_s)$ and

$$\begin{array}{l} e_{m_{j-1}+1} + \cdots + e_{m_j} \leq p_j < e_{m_{j-1}+1} + \cdots + e_{m_j} + e_{m_j+1}, \\ f_{m_{j-1}+1} + \cdots + f_{m_j} \leq p_j < f_{m_{j-1}+1} + \cdots + f_{m_j} + f_{m_j+1} \end{array} \quad \forall 1 \leq j \leq s-1.$$

Since $\mathcal{R}_0$ and $\mathcal{R}_1$ are type II_1 factors, then there exist unitary elements $u_j \in \mathcal{R}_0, v_j \in \mathcal{R}_1$ and $w \in \mathcal{M}$ such that $u_j e_1 u_j^* = e_j, v_j f_1 v_j^* = f_j$ $(j = 1, ..., k)$ and $we_1 w^* = f_1$. So

$$\begin{aligned} M &= \overline{sp}\mathcal{R}_0\mathcal{R}_1 = \overline{sp}\mathcal{R}_0(\sum e_i)(\sum f_j)\mathcal{R}_1 w \\ &= \overline{sp}\{\mathcal{R}_0 e_i f_j \mathcal{R}_1 w; m_l \leq i, j \leq m_{l+1}, 0 \leq l \leq s-1\} \\ &= \overline{sp}\{\mathcal{R}_0 u_i e_1 u_i^* v_j f_j v_j^* \mathcal{R}_1 w; m_l \leq i, j \leq m_{l+1}, 0 \leq l \leq s-1\} \\ &= \overline{sp}\{\mathcal{R}_0 e_1 u_i^* v_j f_1 \mathcal{R}_1 w; m_l \leq i, j \leq m_{l+1}, 0 \leq l \leq s-1\}. \end{aligned}$$

Therefore,

$$\begin{aligned} e_1\mathcal{M}e_1 &= \overline{sp}\{e_1\mathcal{R}_0 e_1 u_i^* v_j w e_1 w^* \mathcal{R}_1 w e_1; m_l \leq i, j \leq m_{l+1}, 0 \leq l \leq s-1\} \\ &= \overline{sp}\{e_1\mathcal{R}_0 e_1 e_1 u_i^* v_j w e_1 e_1 w^* \mathcal{R}_1 w e_1; m_l \leq i, j \leq m_{l+1}, 0 \leq l \leq s-1\}. \end{aligned}$$

Also by Proposition 2, we know that $e_1\mathcal{M}e_1$ is not isomorphic to any one of free group factors $L(F_n)$, $n > 2\sum_{l=0}^{s-1}(m_{l+1} - m_l + 1)^2 + 6$. We also know that $\frac{m_{l+1}-m_l+1}{k} = \tau(e_{m_l} + \cdots + e_{m_{l+1}}) \leq \tau(p_{l+1}) + \frac{2}{k}$. Then

$$\begin{aligned} & 6 + 2\sum_{l=0}^{s-1}(m_{l+1} - m_l + 1)^2 \\ & \leq 6 + 2\sum_{l=0}^{s-1}(k\tau(p_{l+1}) + 2)^2 \\ & \leq 6 + 2\sum_l(k^2\tau(p_{l+1})^2 + 4k\tau(p_{l+1}) + 4) \\ & \leq 6 + 2ak^2 + 8k + 8s. \end{aligned}$$

Therefore, free group factors $L(F_n), n > 6 + 2ak^2 + 8k + 8s$, are not isomorphic to $e_1\mathcal{M}e_1$.

If free group factor $L(F_2)$ is isomorphic to $\mathcal{M}$, then $e_1\mathcal{M}e_1$ is isomorphic to free group factor $L(F_{k^2+1})$ by Proposition 1. On the other hand, when k is big enough, $k^2 + 1 > 6 + 2ak^2 + 8k + 8s$. From the above proof, the free group factor $L(F_{k^2+1})$ is not isomorphic to $e_1\mathcal{M}e_1$. This is contradiction. So all free group factors $L(F_n)$ $(n > 1)$ are not isomorphic to $\mathcal{M}$. □

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