

**CLASSIFICATION OF TIGHT CONTACT STRUCTURES
 ON SMALL SEIFERT 3-MANIFOLDS WITH $e_0 \geq 0$**

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ABSTRACT. We classify positive, tight contact structures on closed Seifert fibered 3-manifolds with base S^2 , three singular fibers and $e_0 \geq 0$.

1. INTRODUCTION

The classification of positive, tight contact structures on lens spaces is due to K. Honda and E. Giroux [8, 5]. Let M be a closed, small Seifert fibered 3-manifold which is not a lens space. Then, M has base S^2 and exactly three singular fibers. Equivalently, M is orientation-preserving diffeomorphic to $M(r_1, r_2, r_3)$ for some $r_1, r_2, r_3 \in \mathbb{Q} \setminus \mathbb{Z}$, where $M(r_1, r_2, r_3)$ denotes the oriented 3-manifold given by the surgery diagram of Figure 1.

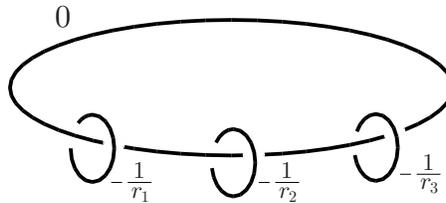


FIGURE 1. Surgery diagram for the Seifert fibered 3-manifold $M(r_1, r_2, r_3)$

Applying Rolfsen twists to the diagram of Figure 1 it is easy to show that

$$(1.1) \quad M(r_1, r_2, r_3) = M(r_1 - h - k, r_2 + h, r_3 + k), \quad h, k \in \mathbb{Z}.$$

The integer

$$e_0(M(r_1, r_2, r_3)) := \sum_{i=1}^3 [r_i]$$

is an invariant of the Seifert fibered 3-manifold $M(r_1, r_2, r_3)$.

Recently H. Wu obtained the classification up to isotopy of positive tight contact structures on $M(r_1, r_2, r_3)$ (and therefore on small Seifert 3-manifolds) assuming $e_0 \neq -2, -1, 0$ [13]. He used convex surface theory to derive an upper bound for

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the number of isotopy classes of tight contact structures, and produced Legendrian surgery diagrams which show that the upper bound found in the first step is sharp.

In this note we extend Wu’s results to the case $e_0 = 0$. More precisely, we classify positive tight contact structures on $M(r_1, r_2, r_3)$ assuming $e_0 \geq 0$, by using a set of Legendrian surgery diagrams which is slightly different from the one used in [13]. We will assume the reader’s familiarity with the standard techniques of contact topology [2, 3, 8].

Observe that if $e_0(M(s_1, s_2, s_3)) \geq 0$, then by (1.1) we have

$$M(s_1, s_2, s_3) = M(r_1, r_2, r_3) \quad \text{for some } r_1 > 0 \quad \text{and} \quad 1 > r_2, r_3 > 0.$$

For each of the three rational numbers r_1, r_2, r_3 , we can write

$$-\frac{1}{r_i} = [a_0^i, a_1^i, \dots, a_{n_i}^i] := a_0^i - \frac{1}{a_1^i - \frac{1}{\dots - \frac{1}{a_{n_i}^i}}}, \quad i = 1, 2, 3,$$

for some uniquely determined integer coefficients

$$a_0^1 \leq -1 \quad \text{and} \quad a_0^2, a_0^3, a_1^i, \dots, a_{n_i}^i \leq -2, \quad i = 1, 2, 3.$$

We define

$$T(r_1, r_2, r_3) := \left| \left(\prod_{i=1}^3 (a_0^i + 1) \right) - \prod_{i=1}^3 a_0^i \prod_{i=1}^3 \prod_{k=1}^{n_i} (a_k^i + 1) \right|.$$

The following is our main result.

Theorem 1.1. *Suppose $r_1 > 0$ and $1 > r_2, r_3 > 0$. Then, $M = M(r_1, r_2, r_3)$ carries exactly $T(r_1, r_2, r_3)$ positive tight contact structures up to isotopy. Moreover, each tight contact structure on M has a Stein filling whose underlying 4-manifold has the handlebody decomposition given by Figure 2.*

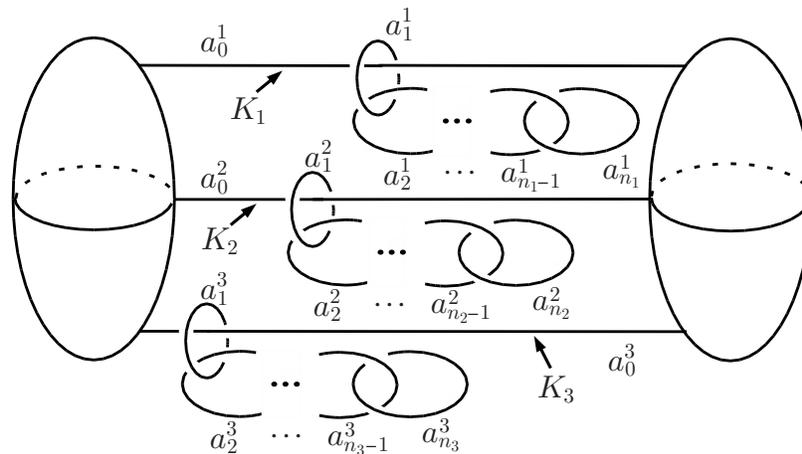


FIGURE 2. Handlebody decomposition of Stein fillings of $M(r_1, r_2, r_3)$

2. UPPER BOUNDS

In this section we establish an upper bound on the number of isotopy classes of tight contact structures on the Seifert fibered 3-manifold $M = M(r_1, r_2, r_3)$. Let ξ be a tight contact structure on M . Then, a Legendrian knot in M smoothly isotopic to a regular fiber admits two framings: one coming from the fibration and another one coming from the contact structure ξ . The difference between the contact framing and the fibration framing is the *twisting number* of the Legendrian curve. We say that ξ has *maximal twisting equal to zero* if there is a Legendrian knot L isotopic to a regular fiber such that L has twisting number zero.

Proposition 2.1 ([12], Theorem 1.3). *If $r_1, r_2, r_3 > 0$, then any tight contact structure on $M(r_1, r_2, r_3)$ has maximal twisting equal to zero.* \square

We can give an explicit construction of the Seifert manifold $M(r_1, r_2, r_3)$ as follows. Let Σ be an oriented pair of pants, and identify each connected component of

$$-\partial(\Sigma \times S^1) = T_1 \cup T_2 \cup T_3$$

with $\mathbb{R}^2/\mathbb{Z}^2$, so that $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ gives the direction of $-\partial(\Sigma \times \{1\})$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ gives the direction of the S^1 factor. Then glue a solid torus $D^2 \times S^1$ to each T_i using the map $\varphi_{A_i} : \partial(D^2 \times S^1) \rightarrow T_i$ defined by the matrix

$$A_i = \begin{pmatrix} \alpha_i & \alpha'_i \\ -\beta_i & -\beta'_i \end{pmatrix},$$

where

$$\frac{\beta_i}{\alpha_i} = r_i, \quad \alpha'_i \beta_i - \alpha_i \beta'_i = 1, \quad \text{and} \quad 0 < \alpha'_i < \alpha_i.$$

Since $r_i > 0$, it follows that $\beta_i > 0$. The vertical S^1 -fibration on $\Sigma \times S^1$ can be extended in a natural way to the Seifert fibration on $M(r_1, r_2, r_3)$. The singular fibers of the Seifert fibration will be denoted by F_i , $i = 1, 2, 3$.

Lemma 2.2. *Let $r_1, r_2, r_3 > 0$, and let ξ be a tight contact structure on $M = M(r_1, r_2, r_3)$. Then, the singular fibers F_i can be made Legendrian with twisting number -1 . Moreover, there exist convex neighbourhoods U_i of F_i such that $-\partial(M \setminus U_i)$ has infinite slope.*

Proof. Using Proposition 2.1, one can isotope ξ until there is a Legendrian regular fiber L with twisting number 0. Then, one can make the singular fibers F_i Legendrian with very low twisting numbers $n_i < 0$. Let V_i , $i = 1, 2, 3$, be disjoint, standard convex neighbourhoods of the F_i 's. Without loss of generality we may assume $L \cap V_i = \emptyset$ for $i = 1, 2, 3$. Let A_i be a convex vertical annulus between L and a ruling of $\partial(M \setminus V_i)$. By the Imbalance Principle [8, Proposition 3.17], A_i produces a bypass attached to ∂V_i along a Legendrian curve with slope

$$-\frac{\alpha_i}{\alpha'_i} < -1.$$

Using the Twisting Number Lemma [8, Lemma 4.4] we can increase the twisting number of F_i up to -1 . Then, we can thicken V_i further in order to obtain a convex solid torus U_i such that $-\partial(M \setminus U_i)$ has infinite slope. \square

Let Σ be a pair of pants. Following [12, 13], we say that a tight contact structure ξ on $\Sigma \times S^1$ is *appropriate* if there is no contact embedding $(T^2 \times I, \xi_\pi) \hookrightarrow (\Sigma \times S^1, \xi)$,

with $T^2 \times \{0\}$ isotopic to a boundary component, where ξ_π is a tight contact structure with convex boundary and twisting π (see [8, § 2.2.1] for the definition of twisting).

Lemma 2.3. *Let Σ be a pair of pants and let ξ be an appropriate contact structure on $\Sigma \times S^1$ with convex boundary $-\partial(\Sigma \times S^1) = T_1 \cup T_2 \cup T_3$, boundary slopes $s(T_1) = s(T_2) = -1$, $s(T_3) = -n \leq -1$, and $\#\Gamma_{T_i} = 2$, $i = 1, 2, 3$. Then:*

- (1) *There exist a pair of pants Σ' contained in Σ and a factorisation $\Sigma \times S^1 = (\Sigma' \times S^1) \cup B_1 \cup B_2 \cup B_3$ such that $\Sigma' \times S^1$ has convex boundary with infinite boundary slopes and B_i is a basic slice for $i = 1, 2, 3$.*
- (2) *The isotopy class of ξ is determined by the signs of B_1, B_2 and B_3 .*
- (3) *If $-n < -1$ or the signs of B_1, B_2 and B_3 are not all the same, different sign configurations correspond to distinct tight contact structures on $\Sigma \times S^1$. If $-n = -1$, the two configurations for which the signs of B_1, B_2 and B_3 are all the same correspond to isotopic tight contact structures.*

Proof. We are going to refer to the statement and proof of [9, Lemma 5.1]. The reader should be aware that our conventions differ from those of [9, Lemma 5.1] in the sense that our slopes are computed with respect to $-\partial(\Sigma \times S^1)$, as opposed to $\partial(\Sigma \times S^1)$.

Part (1) is an immediate consequence of [9, Lemma 5.1(3a)]. If the signs of the basic slices are not all the same, the proof of [9, Lemma 5.1] shows that ξ can be uniquely extended to a tight contact structure with infinite boundary slopes on $\Sigma'' \times S^1$, where Σ'' is a pair of pants containing Σ . The same proof also shows that ξ is universally tight and is completely determined by its extension $\tilde{\xi}$ to $\Sigma'' \times S^1$. Since the contact structure ξ is appropriate, we can apply [13, Lemma 4.1], which says that $\tilde{\xi}$ is completely determined by the signs of B_1, B_2 , and B_3 (strictly speaking, the statement of [13, Lemma 4.1] requires $n > 1$, but it is easy to check that the proof works for $n = 1$ as well).

If the signs are all the same, the proof of [9, Lemma 5.1] shows that ξ is virtually overtwisted. If $-n < -1$, by [9, Lemma 5.1(3c)] there are two distinct virtually overtwisted contact structures corresponding to the two possible sign configurations. If $-n = -1$, then according to [9, Lemma 5.1(3c)] there is only one virtually overtwisted contact structure up to isotopy, so the two configurations of signs give isotopic contact structures. This proves (2) and (3). □

Theorem 2.4. *Suppose $r_1 > 0$ and $1 > r_2, r_3 > 0$. Then, $M(r_1, r_2, r_3)$ carries at most*

$$T(r_1, r_2, r_3) := \left| \left(\prod_{i=1}^3 (a_0^i + 1) - \prod_{i=1}^3 a_0^i \prod_{i=1}^3 \prod_{k=1}^{n_i} (a_k^i + 1) \right) \right|$$

distinct tight contact structures up to isotopy.

Proof. Let ξ be a tight contact structure on $M = M(r_1, r_2, r_3)$. Fix a decomposition of M as in Lemma 2.2:

$$M = M \setminus (U_1 \cup U_2 \cup U_3) \bigcup_{i=1}^3 U_i.$$

Let N_i be the outermost continued fraction block of $(U_i, \xi|_{U_i})$. Applying [11, Lemma A4], we get

$$(2.1) \quad -\frac{\alpha_i}{\alpha'_i} = [a_{n_i}^i, \dots, a_0^i], \quad i = 1, 2, 3.$$

By [8, § 4.4.4], this implies that, in the basis of ∂U_i , N_i has boundary slopes

$$[a_{n_i}^i, \dots, a_1^i + 1] \quad \text{and} \quad [a_{n_i}^i, \dots, a_0^i] \quad \text{for} \quad i = 2, 3,$$

while N_1 has boundary slopes

$$[a_{n_1}^1, \dots, a_1^1 + 1] \quad \text{and} \quad [a_{n_1}^1, \dots, a_0^1] \quad \text{if} \quad a_0^1 < -1,$$

or

$$[a_{n_1}^1, \dots, a_{k+1}^1 + 1] \quad \text{and} \quad [a_{n_1}^1, \dots, a_k^1 + 1] = [a_{n_1}^1, \dots, a_0^1] \quad \text{if} \quad a_0^1 = -1,$$

where $k \in \{1, \dots, n_1\}$ is the first index such that $a_k^1 < -2$.

If we let $V'_i := U_i \setminus N_i$, applying [8, Theorem 2.3] we see that the number of distinct isotopy classes of tight contact structures on V'_i is

$$(2.2) \quad \left| \prod_{h=k+1}^{n_1} (a_h^1 + 1) \right| \quad \text{for} \quad i = 1 \quad \text{if} \quad a_0^1 = -1,$$

$$(2.3) \quad \left| \prod_{h=1}^{n_1} (a_h^1 + 1) \right| \quad \text{for} \quad i = 1 \quad \text{if} \quad a_0^1 < -1,$$

and

$$(2.4) \quad \left| \prod_{h=1}^{n_i} (a_h^i + 1) \right| \quad \text{for} \quad i = 2, 3.$$

Since

$$A_i \begin{pmatrix} -\alpha'_i \\ \alpha_i \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{and} \quad A_i \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha_i \\ -\beta_i \end{pmatrix},$$

by [1, Lemma 3.16], for any $s_i \in [-\infty, -r_i)$ the solid torus U_i contains a convex torus parallel to ∂U_i with slope s_i when computed in the basis of $-\partial(M \setminus U_i)$. Since $r_1 > 0$ and $1 > r_2, r_3 > 0$, the outermost basic slice B_1 of N_1 has boundary slopes $-n$ (for some integer $n \geq 1$) and ∞ , while the outermost basic slices B_2 and B_3 of N_2 and N_3 have boundary slopes -1 and ∞ when computed in the basis of $-\partial(M \setminus U_i)$. Moreover, by [12, Lemma 3.2] the restriction of a tight contact structure ξ to $(M \setminus \bigcup_i U_i) \bigcup_i B_i$ is appropriate. Therefore, by Lemma 2.3(2) such restriction is determined by $\xi|_{\bigcup_i B_i}$. It follows that the restriction of ξ to $M \setminus \bigcup_i V'_i$ is determined by $\xi|_{\bigcup_i N_i}$.

Now define

$$p_i := \#\{\text{positive basic slices in } N_i\}.$$

Then, we have

$$0 \leq p_1 \leq \begin{cases} |a_0^1 + 1| & \text{if } a_0^1 < -1, \\ |a_k^1 + 2| & \text{if } a_0^1 = -1 \end{cases}$$

and

$$0 \leq p_i \leq |a_0^i + 1| \quad \text{for} \quad i = 2, 3,$$

If $a_0^1 = -1$, there are $|(a_k^1 + 1)a_0^2 a_0^3|$ possible configurations of signs (p_1, p_2, p_3) in $\bigcup_i N_i$. When multiplied by the quantities (2.2) and (2.4), this number gives

exactly $T(r_1, r_2, r_3)$. This proves the statement when $a_0^1 = -1$ (which is equivalent to $e_0(M) > 0$).

If $a_0^1 < -1$, in $\bigcup_i N_i$ there are $|a_0^1 a_0^2 a_0^3|$ possible configurations of signs (p_1, p_2, p_3) . If N_1, N_2 and N_3 contain basic slices B_1, B_2 and B_3 with the same sign, by [8, Lemma 4.14] we can arrange the basic slice decomposition of each N_i so that B_i is the outermost basic slice. Applying Lemma 2.3(3), we are allowed to change the sign of all three basic slices simultaneously without changing the isotopy type of the contact structure. This shows that the configuration (p_1, p_2, p_3) is equivalent to $(p_1 \pm 1, p_2 \pm 1, p_3 \pm 1)$ (with the same signs chosen in each slot) whenever the sums are defined.

Now we can easily estimate the number of possible tight contact structures on $M \setminus \bigcup_i V'_i$: by the above argument we can always arrange that one of the p_i 's is maximal, i.e. equal to $|a_0^i + 1|$. For the other two we have $|a_0^j| \cdot |a_0^k|$ many choices (where $\{i, j, k\} = \{1, 2, 3\}$). A simple computation shows that the total number of possibilities is equal to

$$|a_0^1| \cdot |a_0^2| + |a_0^2| \cdot |a_0^3| + |a_0^3| \cdot |a_0^1| - |a_0^1| - |a_0^2| - |a_0^3| + 1,$$

and this expression is equal to

$$|(\prod_{i=1}^3 (a_0^i + 1) - \prod_{i=1}^3 a_0^i)|.$$

When multiplied by the quantities given by (2.3) and (2.4), this number gives $T(r_1, r_2, r_3)$. This proves the statement when $a_0^1 < -1$, and concludes the proof. \square

3. LOWER BOUNDS

In this section we construct $T(r_1, r_2, r_3)$ distinct isotopy classes of Stein fillable, hence tight, contact structures on $M = M(r_1, r_2, r_3)$ assuming $r_1, r_2, r_3 > 0$.

Note that the diagram of Figure 2 gives the handlebody decomposition of a 4-manifold X with boundary diffeomorphic to M . The decomposition involves a single 1-handle and some 2-handles.

Since $a_0^i \leq -1$ and $a_k^i \leq -2$ for $k > 0$, following [6] it is easy to describe a Stein structure on X by putting the knots into Legendrian position and stabilizing them until the prescribed framing coefficient becomes -1 with respect to their contact framing. This way we get

$$|a_0^1 a_0^2 a_0^3 \prod_{i=1}^3 \prod_{k=1}^{n_i} (a_k^i + 1)|$$

different Legendrian diagrams giving Stein structures on X and therefore tight contact structures on $\partial X = M$. Let us denote by ξ_J the contact structure corresponding to a Stein structure J on X . According to [10], if $c_1(J_1) \neq c_1(J_2)$, then the induced contact structures ξ_{J_1} and ξ_{J_2} are nonisotopic. Our aim is to count the number of distinct first Chern classes obtainable in this way.

In order to do this, we start by fixing a basis of $H_2(X; \mathbb{Z})$. We will present the second homology group using cellular homology. It is well known [7] that the framed knots in the diagram correspond to 2-cells. Hence, a choice of orientation for the knots gives rise to a basis for the group $C_2(X)$ of 2-chains for X .

Let $C_1(X)$ denote the group generated by the 1-cells, i.e. by the 1-handles in the handle decomposition. Since there are no 3-handles present in the handle

decomposition, $H_2(X; \mathbb{Z})$ can be computed as the kernel of the map $\varphi: C_2(X) \rightarrow C_1(X)$, given on a basis element $K \in C_2(X)$ corresponding to the knot K as

$$\varphi(K) = \sum a_i L_i,$$

where L_i runs through all 1–handles and $a_i \in \mathbb{Z}$ is the algebraic number of times K passes through the 1–handle L_i . In our case we have $C_1(X) \cong \mathbb{Z}$ and, for a suitable choice of orientations,

$$\varphi(K_i) = 1 \quad \text{for } i = 1, 2, 3,$$

where the knots K_i are indicated in Figure 2, and φ is zero on each basis element $K \neq K_1, K_2, K_3$. Consequently, a basis of $H_2(X; \mathbb{Z})$ can be given by the homology classes corresponding to the unknots of Figure 2 together with the classes of

$$K_1 - K_2, K_1 - K_3 \in C_2(X).$$

It follows from the results of [6] that if $K \neq K_1, K_2, K_3$, then

$$\langle c_1(J), [K] \rangle = \text{rot}(K),$$

while if $\{i, j\} = \{1, 2\}$ or $\{i, j\} = \{1, 3\}$,

$$\langle c_1(J), [K_i - K_j] \rangle = \text{rot}(K_i) - \text{rot}(K_j).$$

Theorem 1.1 follows immediately from Theorem 2.4 together with the following

Proposition 3.1. *Suppose $r_1, r_2, r_3 > 0$. Then, $M(r_1, r_2, r_3)$ carries at least*

$$T(r_1, r_2, r_3) := \left| \left(\prod_{i=1}^3 (a_0^i + 1) - \prod_{i=1}^3 a_0^i \prod_{i=1}^3 \prod_{k=1}^{n_i} (a_k^i + 1) \right) \right|$$

distinct Stein fillable contact structures up to isotopy.

Proof. Let J_1 and J_2 be Stein structures on X resulting from oriented Legendrian surgery diagrams as above. Denote by $r_i^k(1)$ and $r_i^k(2)$ ($i = 1, 2, 3$, $k = 0, \dots, n_i$) the rotation numbers of the Legendrian knots appearing in the two diagrams. It follows from the above discussion that if either

$$r_1^0(1) - r_2^0(1) \neq r_1^0(2) - r_2^0(2), \quad r_1^0(1) - r_3^0(1) \neq r_1^0(2) - r_3^0(2),$$

or

$$r_i^k(1) \neq r_i^k(2) \quad \text{for some } k > 0,$$

then $c_1(J_1) \neq c_1(J_2)$, and therefore by [10, Theorem 1.2] the induced contact structures ξ_1 and ξ_2 on $\partial X = M$ are not isotopic. The conclusion follows from a computation similar to the one given in the proof of Theorem 2.4. \square

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