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ON POSITIVE PERIODIC SOLUTIONS OF LOTKA-VOLTERRA COMPETITION SYSTEMS WITH DEVIATING ARGUMENTS

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ABSTRACT. By using Krasnoselskii's fixed point theorem, we prove that the following periodic $n-$species Lotka-Volterra competition system with multiple deviating arguments

$$(*)\qquad \dot{x}_i(t) = x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t-\tau_{ij}(t))\right], \quad i = 1, 2, \ldots, n,$$

has at least one positive $\omega-$periodic solution provided that the corresponding system of linear equations

$$(**)\qquad \sum_{j=1}^{n} \bar{a}_{ij}\, x_j = \bar{r}_i, \quad i = 1, 2, \ldots, n,$$

has a positive solution, where $r_i, a_{ij} \in C(\mathbf{R}, [0, \infty))$ and $\tau_{ij} \in C(\mathbf{R}, \mathbf{R})$ are $\omega-$periodic functions with

$$\bar{r}_i = \frac{1}{\omega}\int_0^{\omega} r_i(s)ds > 0; \quad \bar{a}_{ij} = \frac{1}{\omega}\int_0^{\omega} a_{ij}(s)ds \geq 0, \quad i, j = 1, 2, \ldots, n.$$

Furthermore, when $a_{ij}(t) \equiv a_{ij}$ and $\tau_{ij}(t) \equiv \tau_{ij}$, $i, j = 1, \ldots, n$, are constants but $r_i(t)$, $i = 1, \ldots, n$, remain ω-periodic, we show that the condition on $(**)$ is also necessary for $(*)$ to have at least one positive $\omega-$periodic solution.

1. INTRODUCTION

In recent years, various delay differential equation models have been proposed in the study of ecological systems, population dynamics and infectious diseases. One of the most celebrated models for dynamics of population is the Lotka-Volterra system. Due to its theoretical and practical significance, the Lotka-Volterra system have been studied extensively [2]–[12], [14]–[19], [21]–[25]. In particular, [4]–[7], [14], [16]–[19], [21]–[23] investigated the existence of periodic solutions of some special cases of the following *periodic* $n-$species Lotka-Volterra competition system with

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several deviating arguments:

$$\dot{x}_i(t) = x_i(t)\left[r_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t-\tau_{ij}(t))\right], \quad i = 1, 2, \ldots, n, \tag{1.1}$$

where $r_i, a_{ij} \in C(\mathbf{R}, [0, \infty))$ and $\tau_{ij} \in C(\mathbf{R}, \mathbf{R})$ are $\omega-$periodic functions ($\omega > 0$) with

$$\bar{r}_i = \frac{1}{\omega}\int_0^{\omega} r_i(s)ds > 0; \quad \bar{a}_{ij} = \frac{1}{\omega}\int_0^{\omega} a_{ij}(s)ds \geq 0, \quad i, j = 1, 2, \ldots, n. \tag{1.2}$$

For example, Shibata and Saito [21] studied a two-species delay Lotka-Volterra competition system and showed that the delays in the system can lead to chaotic behavior. When $n = 1$, (1.1) reduces to the following delayed periodic logistic equation:

$$\dot{x}(t) = x(t)\left[r(t) - a(t)x(t-\tau(t))\right]. \tag{1.3}$$

It was shown in Li [17] that Eq. (1.3) always has a positive $\omega-$periodic solution if $r, a, \tau \in C(\mathbf{R}, [0, \infty))$ are $\omega-$periodic functions with $\int_0^{\omega} r(s)ds > 0$ and $\int_0^{\omega} a(s)ds > 0$.

Recently, by using the method of coincidence degree, Fan et al. [7] and Li [18] investigated the existence of periodic solutions of Eq. (1.1) and established the following two results respectively.

Theorem 1.1 ([7]). *Assume that $\bar{a}_{ii} > 0$ and*

$$\bar{r}_i > \sum_{j \neq i} \frac{\bar{a}_{ij}\bar{r}_j}{\bar{a}_{jj}}\, e^{2\bar{r}_j\omega}, \quad i = 1, 2, \ldots, n. \tag{1.4}$$

Then Eq. (1.1) *has at least one positive periodic solution of periodic ω.*

Theorem 1.2 ([18]). *Assume that $\tau_{ii}(t) = 0$, $i = 1, 2, \ldots, n$, and that*

(C): *the linear system*

$$\sum_{j=1}^{n} \bar{a}_{ij}\; x_j = \bar{r}_i, \quad i = 1, 2, \ldots, n, \tag{1.5}$$

has a positive solution.

In addition, suppose that

$$\bar{r}_i > \sum_{j \neq i} \bar{a}_{ij} \max_{0 \in [0,\omega]} \left|\frac{r_j(t)}{a_{jj}(t)}\right|, \quad i = 1, 2, \ldots, n. \tag{1.6}$$

Then Eq. (1.1) *has at least one positive $\omega-$periodic solution.*

In the the proof of Theorem 1.2, the author took advantage of the fact that there is no deviating argument in the negative feedback terms $a_{ii}(t)x_i(t)$, $i = 1, 2, \ldots, n$. Thus, Theorem 1.2 may fail for Eq. (1.1) when $\tau_{ii}(t) \not\equiv 0$. Furthermore, by Lemma 4.1 in [11], it is not difficult to see that condition (1.4) implies (C). But conditions (1.4) and (1.6) are independent in the sense that neither of them implies the other, and therefore, Theorems 1.1 and 1.2 are complementary.

In both Theorems 1.1 and 1.2, (C) is an essential condition. Obviously, when $a_{ij}(t) \equiv a_{ij}$, $r_i(t) \equiv r_i$, $i, j = 1, 2, \ldots, n$, are all constants, (C) is also a sufficient and necessary condition for Eq. (1.1) to have a trivial positive periodic solution (i.e. positive equilibrium). Also, note that as a special case of (1.1), Eq. (1.3) always has a positive ω-periodic solution provided that

(C_0): $\bar{r} = \int_0^\omega r(s)ds > 0$ and $\bar{a} = \int_0^\omega a(s)ds > 0$,

which is implied by (C) under (1.2) in this case (since Eq. (1.5) becomes $\bar{a}x = \bar{r}$). Motivated by these two observations, we conjecture that (1.4) and (1.6) may not be necessary, and condition (C) may only be enough to guarantee that (1.1) has at least one positive ω-periodic solution.

The purpose of this paper is to give a positive answer to the above conjecture. More precisely, in Section 2, we prove that if (C) holds, then Eq. (1.1) has at least one positive $\omega-$periodic solution. Furthermore, when $a_{ij}(t) \equiv a_{ij}$ and $\tau_{ij}(t) \equiv \tau_{ij}, \ \dots, n$ are constants but $r_i(t), \ i = 1, \dots, n$, remain ω-periodic, we show that (C) is even a sufficient and necessary condition for Eq. (1.1) to have at least one positive $\omega-$periodic solution.

Throughout of this paper, we say a vector $x = (x_1, x_2, \dots, x_n)^T$ is positive if $x_i > 0, \ i = 1, 2, \dots, n$.

2. Main results

For convenience, we introduce the definition of cone and the well-known Krasnoselskii's fixed point theorem.

Definition 2.1. Let X be a Banach space and let P be a closed, nonempty subset of X. P is a cone if

(i) $\alpha x + \beta y \in P$ for all $x, y \in P$ and all $\alpha, \beta \geq 0$;
(ii) $x, -x \in P$ imply $x = 0$.

Lemma 2.2 (Krasnoselskii, [13]). *Let X be a Banach space, and let $P \subset X$ be a cone in X. Assume that Ω_1, Ω_2 are open bounded subsets of X with $0 \in \Omega_1 \subset \bar{\Omega}_1 \subset \Omega_2$, and let*

$$\varphi : P \cap \ (\bar{\Omega}_2 \backslash \Omega_1) \to P$$

be a completely continuous operator such that either

(i) $||\varphi x|| \leq ||x||, \ \ \forall \ x \in P \cap \partial\Omega_1$ *and* $||\varphi x|| \geq ||x||, \ \ \forall \ x \in P \cap \partial\Omega_2$;

or

(ii) $||\varphi x|| \geq ||x||, \ \ \forall \ x \in P \cap \partial\Omega_1$ *and* $||\varphi x|| \leq ||x||, \ \ \forall \ x \in P \cap \partial\Omega_2$. *Then φ has a fixed point in $P \cap \ (\bar{\Omega}_2 \backslash \Omega_1)$.*

Let

$$X = \left\{ x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in C(\mathbf{R}, \mathbf{R}^n) : \ x(t + \omega) = x(t) \right\}, \tag{2.1}$$

$$||x|| = \sum_{j=1}^n |x_j|_0, \quad |x_j|_0 = \max_{t \in [0,\omega]} |x_j(t)|, \quad i = 1, 2, \dots, n. \tag{2.2}$$

Then X is Banach space endowed with the above norm $|| \cdot ||$. If $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T \in X$ is a solution of Eq. (1.1), then

$$\begin{aligned} &\left[x_i(t) \exp\left(-\int_0^t r_i(s)ds \right) \right]' \\ &= -\exp\left(-\int_0^t r_i(s)ds \right) x_i(t) \sum_{j=1}^n a_{ij}(t) x_j(t - \tau_{ij}(t)), \quad i = 1, 2, \dots, n. \end{aligned} \tag{2.3}$$

Integrating both sides of (2.3) over $[t, t+\omega]$, we obtain

$$x_i(t) = \int_t^{t+\omega} G_i(t,s)x_i(s)\sum_{j=1}^{n} a_{ij}(s)x_j(s-\tau_{ij}(s))ds, \quad i=1,2,\dots,n, \tag{2.4}$$

where

$$G_i(t,s) = \frac{1}{1-e^{-\bar{r}_i\omega}} \exp\left(-\int_t^s r_i(\xi)d\xi\right), \quad i=1,2,\dots,n. \tag{2.5}$$

Let $\sigma = \min\{e^{-\bar{r}_i\omega} : \ i=1,2,\dots,n\}$. Now, choose the cone defined by

$$P = \left\{x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T \in X : x_i(t) \geq \sigma|x_i|_0, \ i=1,2,\dots,n\right\}, \tag{2.6}$$

and define an operator $\Phi : X \to X$ by

$$(\Phi x)(t) = ((\Phi x)_1(t), (\Phi x)_2(t), \dots, (\Phi x)_n(t))^T, \tag{2.7}$$

where

$$(\Phi x)_i(t) = \int_t^{t+\omega} G_i(t,s)x_i(s)\sum_{j=1}^{n} a_{ij}(s)x_j(s-\tau_{ij}(s))ds, \quad i=1,2,\dots,n. \tag{2.8}$$

By (2.4), it is easy to verify that $x = x(t) \in X$ is a $\omega-$periodic solution of Eq. (1.1) provided x is a fixed point of Φ.

Lemma 2.3. *The mapping Φ maps P into P, i.e. $\Phi P \subset P$.*

Proof. It is easy to see that for $t \leq s \leq t+\omega$,

$$A_i := \frac{e^{-\bar{r}_i\omega}}{1-e^{-\bar{r}_i\omega}} \leq G_i(t,s) \leq \frac{1}{1-e^{-\bar{r}_i\omega}} := B_i, \quad i=1,2,\dots,n. \tag{2.9}$$

From (2.8) and (2.9), we have for $x \in P$

$$|(\Phi x)_i|_0 \leq B_i \int_0^{\omega} x_i(s)\sum_{j=1}^{n} a_{ij}(s)x_j(s-\tau_{ij}(s))ds$$

and

$$(\Phi x)_i(t) \geq A_i \int_0^{\omega} x_i(s)\sum_{j=1}^{n} a_{ij}(s)x_j(s-\tau_{ij}(s))ds \geq \frac{A_i}{B_i}|(\Phi x)_i|_0 \geq \sigma|(\Phi x)_i|_0.$$

Hence, $\Phi P \subset P$. The proof is completed. □

Lemma 2.4. *$\Phi : P \to P$ is completely continuous.*

Proof. Set

$$f_i(t, x_t) = x_i(t)\sum_{j=1}^{n} a_{ij}(t)x_j(t-\tau_{ij}(t)), \quad i=1,2,\dots,n.$$

We first show that Φ is continuous. For any $L > 0$ and $\varepsilon > 0$, there exists a $\delta > 0$ such that for $\phi, \psi \in X, ||\phi|| \leq L, ||\psi|| \leq L$, and $||\phi - \psi|| < \delta$ imply

$$\max_{s\in[0,\omega]} |f_i(s,\phi_s) - f_i(s,\phi_s)| < \frac{\varepsilon}{nB\omega}, \quad i=1,2,\dots,n, \tag{2.10}$$

where $B = \max_{1\le i\le n} B_i$. If $x, y \in X$ with $||x|| \le L, ||y|| \le L$, and $||x-y|| \le \delta$, then from (2.8), (2.9) and (2.10), we have

$$\begin{aligned} |(\Phi x)_i - (\Phi y)_i|_0 &\le \int_t^{t+\omega} |G_i(t,s)|\ |f_i(s,x_s) - f_i(s,y_s)|ds \\ &\le B\int_0^{\omega} |f_i(s,x_s) - f_i(s,y_s)|ds \\ &< \frac{\varepsilon}{n}, \quad i = 1, 2, \dots, n. \end{aligned}$$

This yields

$$||\Phi x - \Phi y|| = \sum_{i=1}^{n} |(\Phi x)_i - (\Phi y)_i|_0 < \varepsilon.$$

Thus, Φ is continuous.

Next, we show that Φ is compact. Set $a = \max_{1\le i\le n} \sum_{j=1}^{n} \bar{a}_{ij}$. Let $M > 0$ be any constant and let $S = \{x \in X : ||x|| \le M\}$ be a bounded set. For any $x \in S$, it follows from (2.8) and (2.9) that

$$|(\Phi x)_i|_0 \le B_i \int_0^{\omega} |x_i(s)| \sum_{j=1}^{n} a_{ij}(s)|x_j(s - \tau_{ij}(s))|ds \le \omega BM^2 \sum_{j=1}^{n} \bar{a}_{ij} \le a\omega BM^2,$$

and so

$$||\Phi x|| = \sum_{i=1}^{n} |(\Phi x)_i|_0 \le na\omega BM^2, \quad \forall\ x \in S.$$

Again, from (2.8), we have

$$[(\Phi x)_i(t)]' = r_i(t)(\Phi x)_i(t) - x_i(t)\sum_{j=1}^{n} a_{ij}(t)x_j(t - \tau_{ij}(t)), \quad i = 1, 2, \dots, n.$$

Then for $x \in S$,

$$\begin{aligned} |[(\Phi x)_i(t)]'| &\le r_i(t)|(\Phi x)_i(t)| + |x_i(t)|\sum_{j=1}^{n} a_{ij}(t)|x_j(t - \tau_{ij}(t))| \\ &\le r_i^u a\omega BM^2 + M^2 \sum_{j=1}^{n} a_{ij}^u \\ &\le KM^2, \quad i = 1, 2, \dots, n, \end{aligned}$$

where $K = \max_{1\le i\le n}(r_i^u a\omega B + \sum_{j=1}^{n} a_{ij}^u)$ and

$$r_i^u = \max_{t\in[0,\omega]} r_i(t), \quad a_{ij}^u = \max_{t\in[0,\omega]} a_{ij}(t), \quad i, j = 1, 2, \dots, n.$$

Hence, $\Phi S \subset X$ is a family of uniformly bounded and equi-continuous functions. By the Ascoli-Arzela Theorem (see, e.g., [20, p. 169]), the operator Φ is compact, and so it is completely continuous. The proof is completed. □

We are now in a position to state and prove our main results of this paper.

Theorem 2.5. *Assume that* (C) *holds. Then Eq.* (1.1) *has at least one positive* $\omega-$*periodic solution.*

Proof. Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ with $x_i^* > 0,\ i = 1, 2, \ldots, n$, be a positive solution of (1.5). Set

$$A = \min\{\bar{r}_i A_i : i = 1, 2, \ldots, n\}, \quad B = \max\{\bar{r}_i B_i : i = 1, 2, \ldots, n\}. \tag{2.11}$$

Then $0 < A < B < \infty$. Define

$$\Omega_1 = \left\{ x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in X : |x_i|_0 < \frac{x_i^*}{B\omega},\ i = 1, 2, \ldots, n \right\}. \tag{2.12}$$

If $x = x(t) \in P \cap \partial\Omega_1$, then $\sigma|x_i|_0 \le x_i(t) \le |x_i|_0 = (B\omega)^{-1}x_i^*,\ i = 1, 2, \ldots, n$, and

$$\begin{aligned}
|(\Phi x)_i|_0 &\le B_i \int_0^\omega x_i(s) \sum_{j=1}^n a_{ij}(s) x_j(s - \tau_{ij}(s)) ds \\
&\le B_i \omega |x_i|_0 \sum_{j=1}^n \bar{a}_{ij} |x_j|_0 \\
&= B_i \omega (B\omega)^{-1} |x_i|_0 \sum_{j=1}^n \bar{a}_{ij} x_j^* \\
&= B_i \bar{r}_i \omega (B\omega)^{-1} |x_i|_0 \\
&\le |x_i|_0, \quad i = 1, 2, \ldots, n,
\end{aligned}$$

and so

$$||\Phi x|| = \sum_{i=1}^n |(\Phi x)_i|_0 \le \sum_{i=1}^n |x_i|_0 = ||x||, \quad \forall\ x = x(t) \in P \cap \partial\Omega_1. \tag{2.13}$$

Next, we define

$$\Omega_2 = \left\{ x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in X : |x_i| < \frac{x_i^*}{\sigma^2 A\omega},\ i = 1, 2, \ldots, n \right\}. \tag{2.14}$$

If $x = x(t) \in P \cap \partial\Omega_2$, then $\sigma|x_i|_0 \le x_i(t) \le |x_i|_0 = (\sigma^2 A\omega)^{-1}x_i^*,\ i = 1, 2, \ldots, n$, and

$$\begin{aligned}
(\Phi x)_i(t) &\ge A_i \int_0^\omega x_i(s) \sum_{j=1}^n a_{ij}(s) x_j(s - \tau_{ij}(s)) ds \\
&\ge \sigma^2 A_i \omega |x_i|_0 \sum_{j=1}^n \bar{a}_{ij} |x_j|_0 \\
&= A_i \omega (A\omega)^{-1} |x_i|_0 \sum_{j=1}^n \bar{a}_{ij} x_j^* \\
&= A_i \bar{r}_i \omega (A\omega)^{-1} |x_i|_0 \\
&\ge |x_i|_0, \quad i = 1, 2, \ldots, n,
\end{aligned}$$

and so

$$||\Phi x|| = \sum_{i=1}^n |(\Phi x)_i|_0 \ge \sum_{i=1}^n |x_i|_0 = ||x||, \quad \forall\ x = x(t) \in P \cap \partial\Omega_2. \tag{2.15}$$

Obviously, Ω_1 and Ω_2 are open bounded subsets of X with $0 \in \Omega_1 \subset \bar{\Omega}_1 \subset \Omega_2$. Hence, $\Phi : P \cap\ (\bar{\Omega}_2 \backslash \Omega_1) \to P$ is a completely continuous operator and satisfies condition (i) in Lemma 2.2. By Lemma 2.2, there exists a point $x = x(t) \in P \cap (\bar{\Omega}_2 \backslash \Omega_1)$ such that $x(t) = (\Phi x)(t)$, i.e., $x(t)$ is a positive $\omega-$periodic solution of Eq. (1.1). The proof is completed. □

Theorem 2.6. *Assume that* $a_{ii}(t) \equiv a_{ij} \geq 0, \tau_{ij}(t) \equiv \tau_{ij},\ i, j = 1, 2, \ldots, n$. *Then Eq.* (1.1) *has at least one positive* $\omega-$*periodic solution if and only if the system of linear equations*

$$\sum_{j=1}^{n} a_{ij}\ x_j = \bar{r}_i, \quad i = 1, 2, \ldots, n, \tag{2.16}$$

has a positive solution.

Proof. If (2.16) has a positive solution, then by Theorem 2.5, Eq. (1.1) has at least one positive $\omega-$periodic solution. On the other hand, if Eq. (1.1) has at least one positive $\omega-$periodic solution, say $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$. Then from (1.1), we have

$$\int_0^\omega \left[r_i(t) - \sum_{j=1}^{n} a_{ij}\ x_j(t - \tau_{ij}) \right] dt = 0, \quad i = 1, 2, \ldots, n.$$

It follows that

$$\sum_{j=1}^{n} a_{ij} \left(\frac{1}{\omega} \int_0^\omega x_j(t) dt \right) = \bar{r}_i, \quad i = 1, 2, \ldots, n.$$

This shows that the system (2.16) of linear equations has a positive solution $x_j = \frac{1}{\omega} \int_0^\omega x_j(t) dt,\ \ j = 1, 2, \ldots, n$. The proof is completed. □

Remark 2.7. The method in this paper may be used to more general Lotka-Volterra competition systems than Eq. (1.1).

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