PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 134, Number 10, October 2006, Pages 2997–3002
S 0002-9939(06)08391-2
Article electronically published on May 5, 2006

ON CHAOTIC C_0-SEMIGROUPS AND INFINITELY REGULAR HYPERCYCLIC VECTORS

T. KALMES

(Communicated by Jonathan M. Borwein)

ABSTRACT. A C_0-semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ on a Banach space X is called hypercyclic if there exists an element $x \in X$ such that $\{T(t)x;\, t \geq 0\}$ is dense in X. $\mathcal{T}$ is called chaotic if $\mathcal{T}$ is hypercyclic and the set of its periodic vectors is dense in X as well. We show that a spectral condition introduced by Desch, Schappacher and Webb requiring many eigenvectors of the generator which depend analytically on the eigenvalues not only implies the chaoticity of the semigroup but the chaoticity of every $T(t)$, $t > 0$. Furthermore, we show that semigroups whose generators have compact resolvent are never chaotic. In a second part we prove the existence of hypercyclic vectors in $D(A^\infty)$ for a hypercyclic semigroup $\mathcal{T}$, where A is its generator.

1. Introduction

A continuous linear operator T on a separable Banach space $(X, ||\cdot||)$ is called *hypercyclic* if there is a hypercyclic vector $x \in X$ which means that $\{T^n x;\, n \in \mathbb{N}\}$ is dense in $(X, ||\cdot||)$. Using Baire's theorem, it can be shown that an operator T on a separable Banach space is hypercyclic if and only if it is *topologically transitive*, i.e. if and only if for every two open, non-empty subsets U, V of X there is a natural number n such that $U \cap T^n(V) \neq \emptyset$ (cf. [7, Theorem 1.2]). A hypercyclic operator T is called *chaotic* if the set of periodic points is dense in $(X, ||\cdot||)$. There are a number of articles dealing with hypercyclic operators; for a survey see, e.g., [8], [9].

Analogously, a C_0-semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ on a Banach space $(X, ||\cdot||)$ is called *hypercyclic* if there exists an element $x \in X$ such that $\{T(t)x;\, t \geq 0\}$ is dense in $(X, ||\cdot||)$ (by the strong continuity of the semigroup it follows that $\{T(t)x;\, t \geq 0, t \text{ rational}\}$ is a dense subset of $(X, ||\cdot||)$ for every hypercyclic vector x, so that a Banach space has to be separable in order to support a hypercyclic semigroup), and in this case x is again called a *hypercyclic vector* for the semigroup $\mathcal{T}$. The set of all hypercyclic vectors of $\mathcal{T}$ will be denoted by $\mathcal{HC}(\mathcal{T})$. If in addition $\{x \in X;\, \exists\, t_0 > 0 : T(t_0)x = x\}$ is dense in $(X, ||\cdot||)$, the semigroup is called *chaotic*. As for single operators, it can be shown (cf. [4, Theorem 2.2]) that a C_0-semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ on a separable Banach space is hypercyclic if and only if it is *topologically transitive*, i.e. if and only if for every two non-empty, open subsets U, V of X there is $t > 0$ such that $U \cap T(t)(V) \neq \emptyset$. Baire's theorem then implies that $\mathcal{HC}(\mathcal{T})$ is a dense G_δ-set in $(X, ||\cdot||)$ whenever it is not empty.

Received by the editors May 4, 2005.
2000 *Mathematics Subject Classification.* Primary 47A16, 47D03.

Obviously, a C_0-semigroup $\mathcal{T}$ is hypercyclic or chaotic if there is $t_0 > 0$ such that the operator $T(t_0)$ is hypercyclic or chaotic. Modifying arguments of [11, Theorem 6] slightly, one obtains that for a hypercyclic vector x of the C_0-semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ there is a dense G_δ-subset I of $[0,\infty)$ such that x is a hypercyclic vector for each of the operators $T(s)$, $s \in I$, so that a C_0-semigroup can only be hypercyclic if it contains "many" hypercyclic operators. It is not known whether all $T(t), t > 0$, then have to be hypercyclic or whether a chaotic C_0-semigroup must contain a chaotic operator.

2. Chaotic C_0-semigroups

A systematic investigation of hypercyclic and chaotic C_0-semigroups started with the article [4] of Desch, Schappacher and Webb. They gave the following sufficient condition on the spectrum of the generator $A : D(A) \to X$ for the semigroup to be chaotic:

(DSW) For some open subset U of the point spectrum $\sigma_p(A)$ of A intersecting the imaginary axis, there exist eigenvectors x_λ corresponding to $\lambda \in U$ such that for each $\phi \in X'\backslash\{0\}$ the mapping $F_\phi(\lambda) = \phi(x_\lambda)$ is holomorphic on U and does not vanish identically.

Theorem ([4, Theorem 3.1])**.** *A C_0-semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ on a separable Banach space is chaotic whenever its generator A satisfies* (DSW)*.*

We will show that condition (DSW) actually implies the chaoticity of each $T(t), t > 0$. The following theorem was inspired by deLaubenfels and Emamirad [3], and its proof uses an argument of [7].

Theorem 2.1. *Let $(X, ||\cdot||)$ be a separable Banach space and let A be the generator of the C_0-semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ on $(X, ||\cdot||)$. Suppose that condition* (DSW) *is fulfilled. Then for every $t > 0$ the operator $T(t)$ is chaotic.*

Proof. Let $t_0 > 0$. We define the sets $\Omega_1 := \{z \in U; \operatorname{Re} z > 0\}$ and $\Omega_2 := \{z \in U; \operatorname{Re} z < 0\}$ which are non-empty, open subsets of U by hypothesis and therefore contain accumulation points in U. Since U is open and intersects the imaginary axis, the set $\Omega_3 := \{z \in U; \operatorname{Re} z = 0,\ t_0 \operatorname{Im} z \in 2\pi\mathbb{Q}\}$ contains accumulation points in U, as well.

We set $V_j := \operatorname{span}\{x_\lambda;\ \lambda \in \Omega_j\}$, $j = 1, 2, 3$, and observe that V_j is a dense subspace of $(X, ||\cdot||)$: If $\phi \in X'$ is such that $0 = \phi(x_\lambda) = F_\phi(\lambda)$ for every $\lambda \in \Omega_j$, it follows that the holomorphic function F_ϕ vanishes identically on U, since Ω_j has accumulation points in U. By hypothesis this implies $\phi = 0$ so that from the Hahn-Banach theorem we obtain the density of V_j in $(X, ||\cdot||)$.

Using the spectral mapping theorem for the point spectrum of C_0-semigroups (cf. [6, Theorem IV 3.7]), i.e. $\sigma_p(T(t))\backslash\{0\} = e^{t\sigma_p(A)}, t \geq 0$, we get for $\sum_{k=1}^m \alpha_k x_{\lambda_k} \in V_2$ and $n \in \mathbb{N} : T(t_0)^n(\sum_{k=1}^m \alpha_k x_{\lambda_k}) = \sum_{k=1}^m \alpha_k e^{nt_0\lambda_k} x_{\lambda_k}$, which converges to zero as n tends to infinity since $|e^{t_0\lambda_k}| < 1$.

If we set $S : V_1 \to V_1, \sum_{k=1}^m \alpha_k x_{\lambda_k} \mapsto \sum_{k=1}^m \alpha_k e^{-t_0\lambda_k} x_{\lambda_k}$ (note that S is well defined because of the linear independence of $\{x_\lambda;\ \lambda \in U\}$) we obtain $T(t_0) \circ S = \operatorname{id}_{V_1}$ once again from the spectral mapping theorem. Because of $|e^{t_0\lambda_k}| > 1$ for $\lambda_k \in \Omega_2$ we see that $S^n x$ tends to zero as n tends to infinity for all $x \in V_1$. We have proved that $T(t_0)$ satisfies the so-called "Hypercyclicity Criterion" (cf. [8, Theorem 4] and the remark following it) which easily implies topological transitivity and therefore the hypercyclicity of $T(t_0)$.

It remains to show that the dense subspace V_3 consists of periodic points of $T(t_0)$. To this we take $p = \sum_{k=1}^{m} \alpha_k x_{\lambda_k} \in V_3$ with $\lambda_k \in \Omega_3$ and $e^{t_0\lambda_k} = e^{2\pi i \frac{j_k}{n_k}}$ (with j_k, n_k being integers). For $M := \prod_{k=1}^{m} n_k$ by applying the spectral mapping theorem again we then obtain $T(t_0)^M(p) = T(t_0)^M(\sum_{k=1}^{m} \alpha_k x_{\lambda_k}) = \sum_{k=1}^{m} \alpha_k x_{\lambda_k} = p$, i.e. the set of periodic vectors of $T(t_0)$ is dense in $(X, ||\cdot||)$. $\square$

Examples 2.2. a) In [4, Example 4.12] Desch et al. showed that the solution semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ on $L^2([0,\infty),\mathbb{C})$ of the partial differential equation

$$\begin{aligned} u_t(x,t) &= au_{xx}(x,t) + bu_x(x,t) + cu(x,t),\\ u(0,t) &= 0 \text{ for } t \geq 0,\\ u(x,0) &= f(x) \text{ for } x \geq 0 \text{ with some } f \in L^2([0,\infty),\mathbb{C}), \end{aligned}$$

satisfies condition (DSW) if $a, b, c > 0$ and $c < b^2/(2a) < 1$. So, by Theorem 2.1, each of the operators $T(t), t > 0$, is chaotic.

b) Let $\rho : [0,\infty) \to (0,\infty)$ be a Lebesgue-measurable function satisfying the growth condition $\sup_{s\geq 0} \frac{\rho(s)}{\rho(s+t)} < Me^{\omega t}$ for some $M > 1, \omega \in \mathbb{R}$ and all $t \geq 0$, and let $p \geq 1$. We consider the weighted Lebesgue space $L^p_\rho([0,\infty),\mathbb{C})$ of measurable complex-valued functions with the natural norm, i.e. $||u||^p := \int_{[0,\infty)} |u(t)|^p \rho(t)dt$. Then $(L^p_\rho([0,\infty),\mathbb{C}), ||\cdot||)$ is a Banach space on which we have defined a strongly continuous semigroup by $(T(t)u)(s) := u(t+s), t, s \geq 0$. It is a well-known fact that the domain of the generator of $\mathcal{T} = (T(t))_{t\geq 0}$ is given by $D(A) = \{u \in L^p_\rho([0,\infty),\mathbb{C}); u \text{ is absolutely continuous and } u' \in L^p_\rho([0,\infty),\mathbb{C})\}$ and that $Au = u'$.

For the special case $\rho(t) = e^{-\alpha t}$ we see that for $\lambda \in \mathbb{C}$ with $\mathrm{Re}\lambda < \alpha$ the function $x_\lambda(t) := e^{\lambda t}$ belongs to $L^p_\rho([0,\infty),\mathbb{C})$, so that λ belongs to the point spectrum of $(A, D(A))$. For every $g \in L^q_\rho([0,\infty),\mathbb{C})$, where $\frac{1}{p} + \frac{1}{q} = 1$, we have that $\{\lambda \in \mathbb{C}; \mathrm{Re}\lambda < \alpha\} \to \mathbb{C}, \lambda \mapsto \int_{[0,\infty)} e^{\lambda t} g(t)\rho(t)dt$ is holomorphic (the integrand is locally bounded in λ by an integrable function, so that we can apply [5, Theorem 13.8.6]). Since the considered function is a Laplace transform, the condition (DSW) is fulfilled whenever $\alpha > 0$.

It should be noted that in this particular example condition (DSW) is equivalent to the chaoticity of the translation semigroup $\mathcal{T}$ (cf. [3, Theorem 4.6]).

c) Let $C_{0,\rho} := \{f : \mathbb{R} \to \mathbb{C}; f \text{ continuous}, \lim_{x\to\infty} |f(x)|\rho(x) = \lim_{x\to-\infty} |f(x)|\rho(x) = 0\}$ equipped with $||f|| := \sup_{x\in\mathbb{R}} |f(x)|\rho(x)$, where $\rho : \mathbb{R} \to \mathbb{R}, x \mapsto \min\{1, 1/|x|\}$. According to [10, Theorem 4], the translation semigroup $\mathcal{T} = (T(t))_{t\geq 0}$ with $T(t)f := f(\cdot + t)$ is chaotic since $\lim_{t\to\infty} \rho(t) = \lim_{t\to-\infty} \rho(t) = 0$. It is easy to verify that the point spectrum of the generator A coincides with the imaginary axis, and that every $T(t), t > 0$, is chaotic, so that the "spectral part" of condition (DSW) is not necessary for the chaoticity of every $T(t)$ of a chaotic semigroup.

Next, we show that a C_0-semigroup $\mathcal{T}$ whose generator A has the property that $(D(A), ||\cdot||_A) \hookrightarrow (X, ||\cdot||)$ is compact, can never be chaotic. To do so, we need the following theorem.

Theorem 2.3. *Let $\mathcal{T} = (T(t))_{t\geq 0}$ be a C_0-semigroup on a Banach space $(X, ||\cdot||)$ with generator A such that $\sigma(A) \neq \emptyset$. Let $||\cdot||_A$ denote the graph norm on $D(A)$. If the imbedding $(D(A), ||\cdot||_A) \hookrightarrow (X, ||\cdot||)$ is compact, then $\mathcal{T}$ is not hypercyclic.*

Proof. Since the resolvent set $\rho(A)$ is not empty, we can choose $\lambda_0 \in \rho(A)$. By compactness of $(D(A), ||\cdot||_A) \hookrightarrow (X, ||\cdot||)$ it follows that the resolvent $(\lambda_0 - A)^{-1} : X \to X$ is compact. Because of $\sigma((\lambda_0 - A)^{-1})\backslash\{0\} = \frac{1}{\lambda_0 - \sigma(A)}$ and $\sigma(A) \neq \emptyset$ there is $\mu_0 \in \sigma((\lambda_0 - A)^{-1})\backslash\{0\}$, which has to be an eigenvalue by the compactness of $(\lambda_0 - A)^{-1}$. So μ_0 is an eigenvalue of $((\lambda_0 - A)^{-1})^* = (\lambda_0 - A^*)^{-1}$, too. Let ϕ be a corresponding eigenvector. Then $(\lambda_0 - A^*)^{-1}\phi = \mu_0\phi$, so that $\phi \in D(A^*)$ and $A^*\phi = (\lambda_0 - \frac{1}{\mu_o})\phi$, i.e. A^* has an eigenvalue. Using [4, Theorem 3.3] we see that $\mathcal{T}$ cannot be hypercyclic. $\square$

Corollary 2.4. *Let $\mathcal{T} = (T(t))_{t\geq 0}$ be a C_0-semigroup with generator A on a Banach space $(X, ||\cdot||)$. If the embedding $(D(A), ||\cdot||_A) \hookrightarrow (X, ||\cdot||)$ is compact, then $\mathcal{T}$ is not chaotic.*

Proof. We assume that $\mathcal{T}$ is chaotic. In particular, there is $t > 0$ and $x \in X\backslash\{0\}$ such that $T(t)x = x$. Using the spectral mapping theorem for the point spectrum, we see that $\sigma(A) \neq \emptyset$. So, by the above theorem, $\mathcal{T}$ cannot be hypercyclic, which is a contradiction to the assumed chaoticity of $\mathcal{T}$. $\square$

Remark. Let $\Omega \subset \mathbb{R}^d$ be an open and bounded region, $1 \leq p \leq \infty$ and let $L : D(L) \subset L^p(\Omega) \to L^p(\Omega)$ be a closable "linear differential operator" whose closure $(\bar{L}, D(\bar{L}))$ generates a C_0-semigroup on $L^p(\Omega)$ with $D(\bar{L})$ being contained in $W^{1,p}(\Omega)$, the first order Sobolev space of p-integrable functions.

If Ω has a "nice" boundary, it follows from Sobolev imbedding theorems (cf. [1, Theorem 6.2]) and the above corollary that the semigroup generated by $(\bar{L}, D(\bar{L}))$ is not chaotic.

3. Infinitely regular hypercyclic vectors

If the generator A of a hypercyclic C_0-semigroup $\mathcal{T}$ is an unbounded operator, it follows from the non-emptiness of its resolvent set and the Open-Mapping theorem that $D(A)$ is of first category in $(X, ||\cdot||)$. So, we cannot use the Baire argument to show the existence of a hypercyclic vector x in $D(A)$.

Nevertheless, we will now show that for a hypercyclic C_0-semigroup even $D(A^\infty) \cap \mathcal{HC}(\mathcal{T}) \neq \emptyset$. To this end we need the following simple modification of the so-called "Comparison Principle" for C_0-semigroups. We omit the simple proof.

Lemma 3.1. *Let $\mathcal{T} = (T(t))_{t\geq 0}$ and $\mathcal{S} = (S(t))_{t\geq 0}$ be C_0-semigroups on $(X, ||\cdot||)$ resp. $(Y, ||\cdot||)$ and let $\Phi : (X, ||\cdot||) \to (Y, ||\cdot||)$ be a continuous linear mapping with dense range such that $S(t) \circ \Phi = \Phi \circ T(t)$ for all $t \geq 0$. If $\mathcal{T}$ is hypercyclic, so is $\mathcal{S}$.*

We first show that $\mathcal{HC}(\mathcal{T}) \cap D(A^n) \neq \emptyset$ whenever $\mathcal{T}$ is hypercyclic.

Lemma 3.2. *Let $\mathcal{T}$ be a hypercyclic C_0-semigroup on $(X, ||\cdot||)$ with generator A. For $n \in \mathbb{N}$ we consider the graph norm $||x||_n := \sum_{j=0}^{n} ||A^j x||$ on $D(A^n)$, and we set*

$\mathcal{T}_n := (T_n(t))_{t\geq 0} := (T(t)_{|D(A^n)})_{t\geq 0}$. *Then* $\mathcal{T}_n$ *is a hypercyclic* C_0*-semigroup on* $(D(A^n), ||\cdot||_n)$ *with* $\mathcal{HC}(\mathcal{T}_n) \subseteq \mathcal{HC}(\mathcal{T})$*, and* $\mathcal{HC}(\mathcal{T}_n)$ *is dense in* $(X, ||\cdot||)$*. In particular* $\mathcal{HC}(\mathcal{T}) \cap D(A^n) \neq \emptyset$*.*

Proof. That $(D(A^n), ||\cdot||_n)$ is a Banach space and $\mathcal{T}_n$ is a C_0-semigroup on $(D(A^n), ||\cdot||_n)$ is a well-known result (cf. [6, Chapter II.5]).

Now let λ be in the resolvent set of A. Then $(\lambda - A)^{-n} : (X, ||\cdot||) \to (D(A^n), ||\cdot||_n)$ is a continuous isomorphism and obviously $T_n(t) \circ (\lambda - A)^{-n} = (\lambda - A)^{-n} \circ T(t)$ for every $t \geq 0$, so that $\mathcal{T}_n$ is a hypercyclic C_0-semigroup by Lemma 3.1 and $\mathcal{HC}(\mathcal{T}_n)$ is a dense G_δ-subset of $(D(A^n), ||\cdot||_n)$.

Since the continuous inclusion $\iota : (D(A^n), ||\cdot||_n) \hookrightarrow (X, ||\cdot||)$ has dense range and $T(t) \circ \iota = \iota \circ T_n(t), t \geq 0$, we see that $\mathcal{HC}(\mathcal{T}_n) \subseteq \mathcal{HC}(\mathcal{T})$ and that $\mathcal{HC}(\mathcal{T}_n)$ is dense in $(X, ||\cdot||)$. □

We equip $D(A^\infty) = \bigcap_{n\in\mathbb{N}} D(A^n)$ with the locally convex topology induced by the increasing family of seminorms $(||\cdot||_n)_{n\in\mathbb{N}_0}$, where again $||x||_n := \sum_{j=0}^{n} ||A^j x||$. Then $(D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0})$ is a Fréchet space and we obtain:

Theorem 3.3. *Let* $\mathcal{T}$ *be a hypercyclic* C_0*-semigroup on* $(X, ||\cdot||)$ *with generator* A*. Then* $\mathcal{T}_\infty := (T_\infty(t))_{t\geq 0} := (T(t)_{|D(A^\infty)})_{t\geq 0}$ *is a hypercyclic semigroup on* $(D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0})$ *(where hypercyclicity of a semigroup on a Fréchet space is defined in an obvious way) with* $\mathcal{HC}(\mathcal{T}_\infty) \subset \mathcal{HC}(\mathcal{T})$*, and* $\mathcal{HC}(\mathcal{T}_\infty)$ *is dense in* $(X, ||\cdot||)$*. In particular* $D(A^\infty) \cap \mathcal{HC}(\mathcal{T}) \neq \emptyset$*.*

Proof. Since $(D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0})$ is the projective limit of the countable family of Banach spaces $(D(A^n), ||\cdot||_n)$ and since each of the Banach spaces $(D(A^n), ||\cdot||_n)$ is separable (because $(D(A^n), ||\cdot||_n)$ is isomorphic to a closed subspace of $X^{(n+1)}$), $(D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0})$ is a separable Fréchet space, hence second countable as a topological space.

That $\mathcal{T}_\infty$ is a semigroup of continuous operators on $(D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0})$ is well known.

We will show that $\mathcal{T}_\infty$ is topologically transitive on $(D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0})$, hence hypercyclic. To do so, we choose $x, y \in D(A^\infty)$ and a neighbourhood U of zero in $(D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0})$. Then there are $\varepsilon > 0$ and $m_0 \in \mathbb{N}_0$ such that $\{z \in D(A^\infty);\, ||z||_{m_0} < \varepsilon\} \subseteq U$. Since $x, y \in D(A^\infty) \subset D(A^{m_0})$ and $\mathcal{T}_{m_0}$ is hypercyclic, hence topologically transitive, by Lemma 3.2 there is t_0 such that $W := (x + V) \cap T_{m_0}(t_0)^{-1}(y + V) \neq \emptyset$, where $V := \{z \in D(A^{m_0});\, ||z||_{m_0} < \varepsilon\}$. Since W is open in $(D(A^{m_0}), ||\cdot||_{m_0})$ and $D(A^\infty)$ is dense in $(D(A^{m_0}), ||\cdot||_{m_0})$, there is $z \in D(A^\infty)$ with $||x - z||_{m_0} < \varepsilon, ||y - T_{m_0}(t_0)(z)||_{m_0} < \varepsilon$, that is, $z - x \in U$, $T_\infty(t_0)(z) - y = T_{m_0}(t_0)(z) - y \in U$, which shows $(x + U) \cap T_\infty(t_0)^{-1}(y + U) \neq \emptyset$, i.e. the topological transitivity of $\mathcal{T}_\infty$.

Since the inclusion $\iota : (D(A^\infty), (||\cdot||_n)_{n\in\mathbb{N}_0}) \hookrightarrow (X, ||\cdot||)$ is continuous, has dense range, and $\iota \circ \mathcal{T}_\infty = \mathcal{T} \circ \iota$, we have $\mathcal{HC}(\mathcal{T}_\infty) \subseteq \mathcal{HC}(\mathcal{T})$, and $\mathcal{HC}(\mathcal{T}_\infty)$ is dense in $(X, ||\cdot||)$. □

Remark. a) In a more general setting, it can be shown that given a strongly reduced projective spectrum $\mathcal{X} = (X_\alpha)_{\alpha\in I}$ and a topological transitive semigroup $(T(t)_\alpha)_{t\geq 0}$ on each of the components, the induced semigroup on the projective limit of $\mathcal{X}$ is topologically transitive again; cf. [2, Proposition 2.1].

b) Returning to Examples 2.2 we get that there are C^∞-functions which are hypercyclic for the semigroups considered there.

Acknowledgement

The author gratefully thanks J. Wengenroth for useful discussions, and the referee for helpful comments. Moreover, financial support by the German National Academic Foundation is gratefully acknowledged.

References

[1] R. A. Adams, *Sobolev Spaces*, Academic Press, 1975. MR0450957 (56:9247)

[2] J. Bonet, L. Frerick, A. Peris, J. Wengenroth, *Transitive and hypercyclic operators on locally convex spaces*, Bull. London Math. Soc. **37** (2005), 254–264. MR2119025

[3] R. deLaubenfels, H. Emamirad, *Chaos for functions of discrete and continuous weighted shift operators*, Ergodic Theory Dynam. Systems **21** (2001), 1411-1427. MR1855839 (2002j:47030)

[4] W. Desch, W. Schappacher, G.F. Webb, *Hypercyclic and chaotic semigroups of linear operators*, Ergodic Theory Dynam. Systems **17** (1997), 793-819. MR1468101 (98j:47083)

[5] J. Dieudonné, *Treatise on analysis, Vol. II*, Academic Press, 1970. MR0258551 (41:3198)

[6] K.J. Engel, R. Nagel, *One-Parameter Semigroups for Linear Evolution Equations*, Springer, 2000. MR1721989 (2000i:47075)

[7] G. Godefroy, J.H. Shapiro, *Operators with dense, invariant, cyclic vector manifolds*, J. Funct. Anal. **98** (1991), 229-269. MR1111569 (92d:47029)

[8] K.G. Grosse-Erdmann, *Universal families and hypercyclic operators*, Bull. Amer. Math. Soc. **36** (1999), no. 3, 345-381. MR1685272 (2000c:47001)

[9] K.G. Grosse-Erdmann, *Recent developements in hypercyclicity*, RACSAM Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. **97** (2003), no. 2, 273-286. MR2068180 (2005c:47010)

[10] M. Matsui, M. Yamada, F. Takeo, *Supercyclic and chaotic translation semigroups*, Proc. Amer. Math. Soc. **131** (2003), no. 11, 3535-3546. MR1991766 (2004c:47018)

[11] J.C. Oxtoby, S.M. Ulam, *Measure-preserving homeomorphisms and metrical transitivity*, Ann. of Math. **42** (1941), no. 4, 874-920. MR0005803 (3:211b)

FB IV - Mathematik, Universität Trier, D - 54286 Trier, Germany
E-mail address: kalm4501@uni-trier.de