INTERPOLATING SEQUENCES
FOR BOUNDED ANALYTIC FUNCTIONS

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Abstract. We prove that any sequence in the open ball of a complex Banach space \( E \), even in that of \( E^{**} \), whose norms are an interpolating sequence for \( H^\infty \), is interpolating for the space of all bounded analytic functions on \( B_E \). The construction made yields that the interpolating functions depend linearly on the interpolated values.

1. Introduction and preliminaries

Let \( A \) be a space of functions defined on \( X \). A sequence \( \{x_n\} \) in \( X \) is called interpolating for \( A \) if for any sequence \( (a_n) \in \ell_\infty \), there exists \( f \in A \) such that \( f(x_n) = a_n \) for all \( n \in \mathbb{N} \). It is a classical result in function theory that a sequence \( (z_n) \) in the open unit disc \( D \subset \mathbb{C} \) is interpolating for \( H^\infty \), if, and only if, Carleson’s condition holds, i.e.:

\[
\prod_{k \neq j} \left| \frac{z_k - z_j}{1 - z_kz_j} \right| \geq \delta \quad \forall j \in \mathbb{N},
\]

Some years later B. Berndtsson [B] showed that a sequence \( (x_n) \) in the open unit ball \( B_n \) of \( \mathbb{C}^n \) is interpolating for \( A = H^\infty(B_n) \) if the following condition holds:

\[
\prod_{k \neq j} \rho_A(x_j, x_k) \geq \delta > 0 \quad \forall j \in \mathbb{N},
\]

where \( \rho_A \) denotes the pseudohyperbolic distance in \( B_n \). Shortly after that, B. Berndtsson, S. Chang and K. Lin [BCL] proved that such a type of condition is also sufficient to guarantee that if \( (x_n) \) lies in the polidisc, \( D^n \), then it is interpolating for \( A = H^\infty(D^n) \).

Very recently, T. Gamelin, M. Lindström and the first author [GGL1] have extended the above type of results to the ball of a complex Hilbert space. However, when turning to complex Banach spaces \( E \) there were, to the best of our knowledge, no known general sufficient conditions for a sequence to be interpolating. The aim of this paper is to provide a sufficient condition for a sequence \( (x_n) \) in the open...
unit ball, $B_E$, of $E$ to be interpolating for $H^\infty(B_E)$. The resulting condition is:

(2) There is $\delta > 0$ such that

$$\prod_{k \neq j} \left| \frac{\|x_j\| - \|x_k\|}{1 - \|x_j\|\|x_k\|} \right| \geq \delta \quad \forall j \in \mathbb{N}.$$ 

Actually, we construct a sequence $(F_j) \subset H^\infty(B_E)$ such that $F_j(x_n) = \delta_{j,n}$ and $\sum_j |F_j(x)| \leq M$ for all $x \in B_E$, where $M$ is a constant depending only on $\delta$. Therefore the mapping $T : \ell_\infty \to H^\infty(B_E)$ defined by $T(\alpha_n) = \sum_n \alpha_n F_n$ is a well-defined operator such that $\|T\| \leq M$ and $T(\alpha_n)(x_k) = \alpha_k \quad \forall k \in \mathbb{N}$. When such a linear interpolation operator exists, we say that $\{x_n\}$ is a linear interpolating sequence. Its existence has shown to be very useful in the study of composition operators; see [GLR] or [GGL].

A function $f : B_E \to \mathbb{C}$ is said to be analytic if it is Fréchet differentiable. Denote by $H^\infty(B_E)$ the space $\{f : B_E \to \mathbb{C} : f$ is analytic and bounded $\}$. It is a uniform Banach algebra when endowed with the sup-norm $\|f\| = \sup\{|f(x)| : x \in B_E\}$ and it is, obviously, the analogue of the space $H^\infty$. Davie and Gamelin [DG] proved that each $f \in H^\infty(B_E)$ extends, by means of the Aron-Berner extension, to an element $\tilde{f} \in H^\infty(B_{E^*})$ and that the extension mapping is a multiplicative linear isometry. Therefore, the question whether a sequence in $B_{E^*}$ is interpolating for $H^\infty(B_E)$ is meaningful. Our techniques allow us to assert that (2) is also a sufficient condition whenever the sequence $(x_n)$ lies in $B_{E^*}$.

Recall that the pseudohyperbolic distance between two given points $z, w \in \mathbb{D}$ is

$$\rho(z, w) = \frac{|z - w|}{|1 - zw|}.$$ 

2. The result

**Theorem 1.** Let $\{x_n\}$ be a sequence in $B_{E^*}$ satisfying the following condition:

(2) There is $\delta > 0$ such that

$$\prod_{k \neq j} \left| \frac{\|x_j\| - \|x_k\|}{1 - \|x_j\|\|x_k\|} \right| \geq \delta \quad \forall j \in \mathbb{N}.$$ 

Then there are functions $\{F_j\} \subset H^\infty(B_E)$ and a constant $M$, depending only on $\delta$, such that $\tilde{F}_j(x_n) = \delta_{j,n}$ and $\sum_j |\tilde{F}_j(x)| \leq M$ for all $x \in B_{E^*}$.

The construction of the Beurling functions $\{F_j\}$ is based on the one given in [BCL] for the case of $H^\infty(\mathbb{D}^2)$. A number of lemmas is required, and we proceed to give all of them. The first one is just a set of well-known calculations.

**Lemma 2.** The following holds:

(3) $\rho(a, c) \geq \rho(a, b)$ for real numbers $0 \leq a \leq b \leq c < 1$.

(4) $\rho(|z|, |w|) \leq \rho(z, w)$ for $z, w \in \mathbb{D}$.

(5) $1 - \frac{|z - w|^2}{|1 - \overline{w}z|^2} = \frac{(1 - |z|^2)(1 - |w|^2)}{|1 - \overline{w}z|^2}$ for $z, w \in \mathbb{D}$.

(6) $1 - x \leq -\log x$ for $0 < x \leq 1$.

(7) $\Re \left[ \frac{1 + \alpha z}{1 - \alpha z} \right] = \frac{1 - \alpha^2|z|^2}{|1 - \alpha z|^2}$ for $0 \leq \alpha \leq 1, z \in \mathbb{D}$.

**Lemma 3.** Let $\{x_n\}$ be a sequence in $B_{E^*}$ such that $\{|x_n|\}$ does not accumulate at 0. Then for $k, j \in \mathbb{N}$ and $\alpha_{k,j} > 0$, there exists $T_{k,j} \in B_{E^*}$ such that

(8) $\rho(T_{k,j}(x_k), T_{k,j}(x_j)) \geq \rho(|x_k|, |x_j|) - \alpha_{k,j}.$
Moreover, there is \( m > 0 \), which depends only on the sequence, such that
\[
\tag{9} m \leq |T_{k,j}(x_k)| \text{ for } k, j \in \mathbb{N} \text{ such that } \|x_k\| > \|x_j\|.
\]
If, further, \( \lim \|x_k\| = 1 \), then for any \( \alpha_k > 0 \), there is \( L_k \in \overline{B_{E^*}} \) such that
\[
\tag{10} \rho(L_k(x_k), L_k(x_j)) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_k \text{ if } \|x_k\| \geq \|x_j\|, \text{ and}
\]
\[
\tag{11} L_k(x_k) \geq \|x_j\| \text{ if } \|x_k\| > \|x_j\|.
\]
\[\text{Proof.} \] Since 0 is not an accumulation point, there is \( m > 0 \) so that
\[
m < \inf_{n \in \mathbb{N}} \{\|x_n\| : \|x_n\| \neq 0\}.
\]
For every \( k \in \mathbb{N}, \) there is a sequence \( \{T_n^k\}_n \) in \( \overline{B_{E^*}} \) such that \( T_n^k(x_k) \geq 0 \) and
\[
\tag{12} \lim_{n \to \infty} T_n^k(x_k) = \|x_k\|.
\]
If \( x_k \neq 0 \), we may assume as well that \( T_n^k(x_k) \geq m \) for any \( n \in \mathbb{N}. \)
Let \( j \in \mathbb{N}. \) There is nothing to prove if \( \|x_k\| = \|x_j\|. \) So, suppose \( \|x_k\| > \|x_j\|. \)
When considering the pseudohyperbolic distance between \( \|x_k\| \) and \( \|x_j\| \), there are two possibilities:
\[\text{i) Either, there is } n_1 \in \mathbb{N} \text{ so that } \rho(T_n^k(x_k), T_n^k(x_j)) \geq \rho(\|x_k\|, \|x_j\|). \]
In this case, we choose \( T_{k,j} = T_n^k \) and then \( 8 \) holds for arbitrary \( \alpha_{k,j} \geq 0. \)
\[\text{ii) Or such } n_1 \text{ does not exist, that is, } \rho(T_n^k(x_k), T_n^k(x_j)) < \rho(\|x_k\|, \|x_j\|) \text{ \forall n.} \]
Since \( \|x_j\| < \|x_k\| \), there is \( n_2 \in \mathbb{N} \) such that \( \|x_j\| \leq T_n^k(x_k) \) for \( n \geq n_2. \) Thus, after using \( 9 \), we have \( \rho(T_n^k(x_k), T_n^k(x_j)) \geq \rho(T_n^k(x_k), \|x_j\|). \) Consequently,
\[
\rho(T_n^k(x_k), \|x_j\|) \leq \rho(T_n^k(x_k), T_n^k(x_j)) \leq \rho(\|x_k\|, \|x_j\|) \ \forall n \geq n_2.
\]
Hence by the choice in \( 12 \), \( \lim_{n \to \infty} \rho(T_n^k(x_k), T_n^k(x_j)) = \rho(\|x_k\|, \|x_j\|). \) Then, for \( \alpha_{k,j} > 0 \) there is \( n_3 > n_2 \) such that
\[
\rho(T_n^k(x_k), T_n^k(x_j)) \geq \rho(T_n^k(x_k), T_n^k(x_j)) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_{k,j} \text{ for } n \geq n_3.
\]
So we choose \( T_{k,j} = T_n^k \), and \( 8 \) holds as well in this case.
Assume now that \( \lim \|x_k\| = 1. \) Fix \( k \in \mathbb{N}. \) Observe that there are only a finite number of terms \( x_j \) in the sequence with \( \|x_j\| < \|x_k\|. \) For such \( j \in \mathbb{N} \) we consider
\[
B_j = \{ n : \rho(T_n^k(x_k), T_n^k(x_j)) < \rho(\|x_k\|, \|x_j\|) \}.
\]
In case \( B_j \) is a finite set, we may find \( n_j \in \mathbb{N} \) so that
\[
\tag{13} \rho(T_n^k(x_k), T_n^k(x_j)) \geq \rho(\|x_k\|, \|x_j\|) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_k \ \forall n \geq n_j.
\]
Whenever \( B_j \) is not finite, we argue as in ii) above to show that
\[
\lim_{n \in B_j} \rho(T_n^k(x_k), T_n^k(x_j)) = \rho(\|x_k\|, \|x_j\|).
\]
Therefore, an \( n_j \in \mathbb{N} \) can be found so that
\[
\rho(T_n^k(x_k), T_n^k(x_j)) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_k \ \forall n \in B_j, n \geq n_j.
\]
Note that this inequality also holds for \( n \notin B_j. \) Consequently, \( 13 \) also holds in this case.
Since \( \max\{n_j\} \) is finite, \( \lim_{n \to \infty} T_n^k(x_k) = \|x_k\| \text{ and } \max\{\|x_j\| : \|x_j\| < \|x_k\|\} < \|x_k\|, \) we may take \( n_0 \) big enough so that \( T_n^k(x_k) > \|x_j\| \text{ if } \|x_j\| < \|x_k\|. \) By setting \( L_k = T_n^k \) and using \( 14, \) we are done. \( \square \)
Let \( \{x_n\} \subset B_{E^*} \) and \( \delta > 0 \) satisfying condition (2). Then there are functionals \( \{T_{k,j}\} \subset B_{E^*} \) satisfying (4) such that

\[
\prod_{k \neq j} \rho(T_{k,j}(x_k), T_{k,j}(x_j)) \geq \frac{\delta}{2}
\]

and

\[
\sum_{k=1}^{\infty} (1 - |T_{k,j}(x_k)|^2) \leq (1 + 2 \log \frac{2}{\delta}) \frac{1 + \|x_j\|}{1 - \|x_j\|} \quad \forall j \in \mathbb{N}.
\]

**Proof.** Choose a sequence \( (\beta_k) \) such that \( \prod (1 - \beta_k) \geq \frac{1}{2} \) and then put \( \alpha_{k,j} = \beta_k \rho(\|x_k\|, \|x_j\|). \) Now, apply Lemma 3 to find the functionals \( \{T_{k,j}\} \) such that

\[
\rho(T_{k,j}(x_k), T_{k,j}(x_j)) \geq \rho(\|x_k\|, \|x_j\|) - \alpha_{k,j} = (1 - \beta_k) \rho(\|x_k\|, \|x_j\|).
\]

Hence (14) follows. From here we get

\[
- \sum_{k \neq j} \log \left| \frac{T_{k,j}(x_j) - T_{k,j}(x_k)}{1 - T_{k,j}(x_k)T_{k,j}(x_j)} \right|^2 \leq -2 \log \frac{\delta}{2}.
\]

Further, by (15) and then bearing in mind (4), we obtain

\[
\sum_{k \neq j} \frac{(1 - |T_{k,j}(x_k)|^2)(1 - |T_{k,j}(x_j)|^2)}{|1 - T_{k,j}(x_k)T_{k,j}(x_j)|^2} \leq -2 \log \frac{\delta}{2}.
\]

Finally,

\[
\sum_{k=1}^{\infty} (1 - |T_{k,j}(x_k)|^2) = \sum_{k=1}^{\infty} \frac{(1 - |T_{k,j}(x_k)|^2)(1 - |T_{k,j}(x_j)|^2)}{|1 - T_{k,j}(x_k)T_{k,j}(x_j)|^2} \leq (1 - 2 \log \frac{\delta}{2}) \frac{1 + \|x_j\|^2}{1 - \|x_j\|^2} = (1 + 2 \log \frac{2}{\delta}) \frac{1 + \|x_j\|}{1 - \|x_j\|}.
\]

**Lemma 5.** Let \( \{x_n\} \subset B_{E^*} \) and \( \delta > 0 \) satisfying condition (2). Then there are functionals \( \{L_k\} \subset B_{E^*} \) such that

\[
\prod_{\{k : \|x_k\| > \|x_j\|\}} \rho(L_k(x_k), L_k(x_j)) \geq \frac{\delta}{2}
\]

and

\[
\sum_{\{k : \|x_k\| > \|x_j\|\}} (1 - |L_k(x_k)|^2) \leq (1 + 2 \log \frac{2}{\delta}) \frac{1 + \|x_j\|}{1 - \|x_j\|} \quad \forall j \in \mathbb{N}.
\]

**Proof.** The proof follows the same pattern as Lemma 4. Simply choose \( \alpha_k = \min_{\{j : \|x_j\| < \|x_k\|\}} \{\beta_k \rho(\|x_k\|, \|x_j\|)\} \) and pick \( L_k \) from Lemma 3. □

**Lemma 6 (BCL).** Let \( \{S_j\} \) be a sequence of subsets of \( \mathbb{N} \) such that if \( k \notin S_j \), then \( j \in S_k \). Let \( \{c_j\} \) be a sequence of nonnegative numbers. Then

\[
\sum_{j=1}^{\infty} c_j e^{-\sum_{k \in S_j} e^\alpha} \leq 2e.
\]

Now we proceed to prove our main result.

**Proof of Theorem 1.** Let \( \{T_{k,j}\} \) be the set of functionals furnished by Lemma 4. Define

\[
g_{k,j}(x) = \alpha_{k,j} \frac{T_{k,j}(x_k) - T_{k,j}(x)}{1 - T_{k,j}(x_k)T_{k,j}(x)} \quad \forall k, j \in \mathbb{N}, \ k \neq j,
\]
where \( a_{k,j} \) is given by

\[
a_{k,j} = \begin{cases} \frac{r_{k,j}(x_k)}{|T_{k,j}(x_k)|} & \text{if } T_{k,j}(x_k) \neq 0, \\ -1 & \text{if } T_{k,j}(x_k) = 0. \end{cases}
\]

For each \( j \in \mathbb{N} \) we define \( B_j(x) = \prod_{k \neq j} g_{k,j}(x) \). First we check that this infinite product converges uniformly on \( rB_{E^{**}} \) for fixed \( 0 < r < 1 \). We have

\[
1 - g_{k,j}(x) = \frac{1}{|T_{k,j}(x_k)|} \left[ 1 - \frac{|T_{k,j}(x_k)|^2 - T_{k,j}(x_k)T_{k,j}(x)}{1 - T_{k,j}(x_k)T_{k,j}(x)} \right] + 1 - \frac{1}{|T_{k,j}(x_k)|}.
\]

Moreover, \( |T_{k,j}(x)| \leq \|x\| \leq r < 1 \), so we get

\[
|1 - g_{k,j}(x)| \leq \frac{1}{|T_{k,j}(x_k)|} \left[ \frac{2}{1 - r} + 1 \right].
\]

Since \( \lim_k \|x_k\| = 1 \), it follows that \( \|x_k\| > \|x_j\| \), for \( k = k_j, k_j + 1, \ldots \) and recalling (14),

\[
|1 - g_{k,j}(x)| \leq \frac{1}{m} \left[ \frac{2}{1 - r} + 1 \right] (1 - |T_{k,j}(x_k)|) \quad \forall k \geq k_j.
\]

Now we use inequality (17) to show that the series \( \sum_{k=1}^{\infty} |1 - g_{k,j}(x)| \) is uniformly convergent on \( \{ x : \|x\| \leq r \} \), as we wanted. In particular, \( \prod_{k \neq j} g_{k,j}(x) \) converges uniformly on compact sets, so \( B_j \in H^{\infty}(B_{E^{**}}) \).

Note that \( \|B_j\|_{\infty} \leq 1 \), \( B_j(x_k) = 0 \) for \( k \neq j \) and that, according to (14), \( |B_j(x_k)| \geq \frac{\delta}{2} \).

Next we take the sequence \( \{L_k\} \) found in Lemma 5 and we define

\[
A_j(x) = \sum_{k: \|x_k\| \geq \|x_j\|} \left[ \frac{1 + L_k(x_k)L_k(x)}{1 - L_k(x_k)L_k(x)} \right] (1 - L_k(x_k)^2).
\]

Since \( \frac{1 + L_k(x_k)L_k(x)}{1 - L_k(x_k)L_k(x)} \leq \frac{1 + \|x\|}{\|x_k\|} \), it follows from (17) that the convergence of this series is uniform on \( rB_{E^{**}} \). Therefore \( G_j := A_j - A_j(x_j) \in H(B_{E^{**}}) \).

Let \( F_j \in H^{\infty}(B_{E^{**}}) \) be given by

\[
F_j(x) = \frac{B_j(x)}{B_j(x_j)} q_j(x) \exp \left\{ -\frac{1}{2(1 + 2 \log \frac{\delta}{2})} G_j(x) \right\}
\]

where \( q_j(x) = \left[ \frac{1 - |L_j(x_j)|^2}{1 - L_j(x_j)L_j(x)} \right]^2 \).

Recall that the Aron-Berner extension is also continuous whenever both \( H^{\infty}(B_E) \) and \( H^{\infty}(B_{E^{**}}) \) are endowed with the topology of the uniform convergence on balls of radii less than 1 (see ACC (10.1) p. 86). Consequently, each of the functions \( g_{k,j}, B_j, q_j \) and \( F_j \) is the Aron-Berner extension of its restriction to \( B_E \).

We claim that

\[
F_j(x_k) = \delta_{j,k} \quad \text{for } j, k \in \mathbb{N} \text{ and that}
\]

\[
\text{there exists } M > 0 \text{ such that } \sum |F_j(x)| \leq M.
\]
Condition (13) is trivially verified. To prove (19), we begin by recalling that 
\[ |B_j(x)| \geq \delta/2. \]
Then
\[
\sum_{j=1}^{\infty} |F_j(x)| \leq \frac{2\delta}{\delta} \sum_{j=1}^{\infty} |q_j(x)| \exp \left\{ -\frac{1}{2(1+2 \log \frac{2}{\delta})} \Re G_j(x) \right\}.
\]

Applying (7) we have
\[
(20) \quad \Re A_j(x) = \sum_{\{k: \|x_k\| \geq \|x_j\|\}} \frac{(1 - L_k(x_k)^2|L_k(x)|^2)(1 - L_k(x_k)^2)}{1 - L_k(x_k)\bar{L}_k(x)|^2}.
\]
If \( \|x_k\| > \|x_j\| \), then \( |L_k(x_j)| \leq \|x_j\| \leq L_k(x_k) \) by (11). Now, applying the inequality \( 1 - \alpha \beta \leq 2(1 - \beta) \) for \( \alpha \geq \beta \), it turns out that \( 1 - L_k(x_k)^2|L_k(x_j)|^2 \leq 2(1 - L_k(x_j)^2) \). Thus
\[
\Re A_j(x) \leq 2 \sum_{\{k: \|x_k\| > \|x_j\|\}} \frac{(1 - L_k(x_j)^2)(1 - L_k(x_k)^2)}{1 - L_k(x_k)\bar{L}_k(x)|^2} = \sum_{\{k: \|x_k\| > \|x_j\|\}} 2 \left[ 1 - \rho(L_k(x_j), L_k(x_k)^2 \right] + 2
\]
after bearing in mind (5). Next, using inequalities (6) and (10) we obtain
\[
\Re A_j(x) \leq -2 \log \prod_{\{k: \|x_k\| > \|x_j\|\}} \rho(L_k(x_j), L_k(x_k)^2) + 2 \leq 2(1 + 2 \log \frac{2}{\delta})
\]
and then
\[
\sum_{j=1}^{\infty} |F_j(x)| \leq \frac{2\delta}{\delta} \sum_{j=1}^{\infty} |q_j(x)| \exp \left\{ -\frac{1}{2(1+2 \log \frac{2}{\delta})} \Re A_j(x) \right\}.
\]

Therefore, bearing in mind (20), we have that \( \sum_{j=1}^{\infty} |F_j(x)| \) is bounded by
\[
\frac{2\delta}{\delta} \sum_{j=1}^{\infty} \left| 1 - L_j(x_j)^2 \frac{2}{1 - L_j(x_j)\bar{L}_j(x)|^2} \right| \exp \left\{ -\frac{1}{2(1+2 \log \frac{2}{\delta})} \sum_{\{k: \|x_k\| \geq \|x_j\|\}} \frac{(1 - L_k(x_k)^2)^2}{1 - L_k(x_k)\bar{L}_k(x)|^2} \right\}.
\]
To apply Lemma [6] to the above series we put \( c_j = \frac{1}{2(1+2 \log \frac{2}{\delta})} \left| 1 - L_j(x_j)^2 \right| \). Then we obtain
\[
\sum_{j=1}^{\infty} |F_j(x)| \leq \frac{4\epsilon(1 + 2 \log \frac{2}{\delta})}{\delta} 2e = \frac{8\epsilon^2}{\delta}(1 + 2 \log \frac{2}{\delta}) := M.
\]

The converse of Theorem 1 is false for any Banach space \( E \). It is enough to consider a unit vector \( x \in E \) and an interpolating sequence \( (\lambda_n) \) in \([0,1]\) such that \( \lambda_1 = 1/2 \). Let \( \phi \neq 0 \) be a linear mapping whose norm is 1 and \( \phi(x) = 1 \). Since the sequence \( \{-\lambda_1, \lambda_1, \ldots, \lambda_n, \ldots\} \) is interpolating for \( H^\infty \), the sequence \( \{-\lambda_1 x, \lambda_1 x, \ldots, \lambda_n x, \ldots\} \), is interpolating for \( H^\infty(B_E) \). However \( \| -\lambda_1 x \| = \| \lambda_1 x \| = 1/2 \), so condition (2) clearly fails.

**Corollary 7.** Let \( \{x_n\} \) be a sequence in \( B_E \). If \( \{\|x_n\|\} \) is an interpolating sequence for \( H^\infty \), then \( \{x_n\} \) is a linear interpolating sequence for \( H^\infty(B_E) \).
Proof. Carleson’s condition for the disc \( \text{(1)} \) states that \( \{\|x_n\|\} \) is an interpolating sequence for \( H^\infty \) if, and only if, condition \( \text{(2)} \) is satisfied. Then Theorem 1 shows that \( \{x_n\} \) is a linear interpolating sequence for \( H^\infty(B_E) \). \( \square \)

**Corollary 8.** Let \( \{x_n\} \) be a sequence in \( B_{E^*} \) and \( 0 < c < 1 \) such that
\[
\frac{1 - \|x_{k+1}\|}{1 - \|x_k\|} < c.
\]
Then \( \{x_n\} \) is a linear interpolating sequence for \( H^\infty(B_E) \).

Proof. It is well-known (see [H]) that, if Hayman-Newman’s condition \( \text{(21)} \) is satisfied by a sequence in \( D \), then it also fulfills Carleson’s condition \( \text{(1)} \). So it suffices to apply the above corollary. \( \square \)

Note that from Corollary 8 we derive the following interpolation result.

**Theorem 9 ([ACG]).** Let \( \{x_n\} \) be a sequence in \( B_{E^*} \) satisfying \( \lim_{n \to \infty} \|x_n\| = 1 \). Then there is a linear interpolating subsequence for \( H^\infty(B_E) \).

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