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UNIQUENESS AND STABILITY OF STEADY STATES FOR A PREDATOR-PREY MODEL IN HETEROGENEOUS ENVIRONMENT

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ABSTRACT. In this paper, we deal with a predator-prey model with diffusion in a heterogeneous environment, and we study the uniqueness and stability of positive steady states as the diffusion coefficient of the predator is small enough.

1. Introduction

In [1], after some simple scaling changes, Du and Hsu considered the following prey-predator model with diffusion:

$$
\begin{cases}
u_t - \Delta u = \lambda u - \alpha(x)u^2 - \beta uv & \text{in } \Omega \times (0,\infty),\\
v_t - \Delta v = \mu v(1 - \dfrac{v}{u}) & \text{in } \Omega \times (0,\infty),\\
\partial_\nu u = \partial_\nu v = 0 & \text{on } \partial\Omega \times (0,\infty),\\
u(x,0) = u_0(x) > 0 & \text{in } \Omega,\\
v(x,0) = v_0(x) \geq 0, \not\equiv 0 & \text{in } \Omega,
\end{cases} \tag{1.1}
$$

where $\Omega \subset \mathbf{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$ and ν is the outward unit normal vector over $\partial\Omega$. In biological terms, u and v represent the densities of prey and predator, respectively, and λ stand for the intrinsic growth rate of the prey, β is the predation ability of the predator and μ accounts for the diffusion coefficient of the predator. Throughout the paper, λ, β, μ are assumed to be positive constants. Also, $\alpha(x)$ is a nonnegative continuous function involving the spatial variable x, which implies the spatial environment is heterogeneous. In addition, the homogeneous Neumann boundary on $\partial\Omega$ for u and v means there is no-flux population across $\partial\Omega$ and so the prey and predator live in a closed environment, and the admissible initial data $u_0(x)$ and $v_0(x)$ are continuous functions on $\bar{\Omega}$.

First, in [1], when the spatial environment is homogeneous, that is, $\alpha(x)$ is a positive constant, by constructing a technical Lyapunov functional, Du and Hsu proved

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that the unique positive constant steady state $(u, v) = (\lambda/(\alpha+\beta), \lambda/(\alpha+\beta))$ is globally stable if $\alpha/\beta > s_0 \in (1/5, 1/4)$, where s_0 is the unique positive root of the equation $32s^3 + 16s^2 - s - 1 = 0$.

Then, Du and Hsu investigated the case that $\alpha(x)$ is a general nonnegative function on $\bar{\Omega}$ and studied the steady states of (1.1), which satisfy

$$\begin{cases} -\Delta u = \lambda u - \alpha(x)u^2 - \beta uv & \text{in } \Omega, \\ -\Delta v = \mu v(1 - \dfrac{v}{u}) & \text{in } \Omega, \\ \partial_\nu u = \partial_\nu v = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.2}$$

In particular, if $\alpha(x) > 0$ on $\bar{\Omega}$, they used the topological degree argument to prove that (1.2) always has a positive solution; see Theorem 3.1 of [1] for the details. On the other hand, [1] paid more attention to the so-called degenerate environment case, i.e., $\alpha(x)$ can vanish in some proper subdomains of Ω. The authors studied the effect of such a spatial degeneracy on the steady state solution of (1.2), and they observed some very interesting behaviours of solutions of (1.2).

In the following, for the brief description of some of the results in [1] and also for our later purposes, we first introduce some notation and assumptions.

When we say that $\alpha(x)$ is *degenerate* or has the *degeneracy*, we always mean the assumption: Ω_0 *is a smooth domain with* $\bar{\Omega}_0 \subset \Omega$, *and* $\alpha(x) = 0$ *on* $\bar{\Omega}_0$ *and* $\alpha(x) > 0$ *in* $\bar{\Omega} \setminus \bar{\Omega}_0$.

Now, assume that O is a bounded domain with smooth boundary, and let $f(x)$ be a continuous function. We denote by $\lambda_1^D(f, O)$ and $\lambda_1^N(f, O)$ the first eigenvalue of the operator $-\Delta + f$ over O, with Dirichlet or Neumann boundary condition, respectively. If the potential function $f(x)$ is omitted, then we understand that $f = 0$. It is well known that the following properties hold:

(1) $\lambda_1^D(f, O) > \lambda_1^N(f, O)$;
(2) $\lambda_1^B(f_1, O) > \lambda_1^B(f_2, O)$ if $f_1 \geq f_2$ and $f_1 \not\equiv f_2$, for $B = D$ or $B = N$;
(3) $\lambda_1^D(f, O_1) \geq \lambda_1^D(f, O_2)$ if $O_1 \subset O_2$.

It was proved in [1] that if $\lambda > \lambda_1^D(\Omega_0) > \mu$, (1.2) has no positive solution for all small $\beta > 0$; if $0 < \lambda < \lambda_1^D(\Omega_0)$ or $\lambda_1^D(\Omega_0) \leq \lambda < \mu$ holds, (1.2) has at least a positive solution for any $\beta > 0$. The authors also showed that the small perturbation of (1.2) with respect to $\alpha(x)$ (namely, (1.2) with $\alpha(x)$ replaced by $\alpha(x) + \varepsilon$ and letting $\varepsilon \to 0$) can develop sharp spatial patterns if $\lambda_1^D(\Omega_0) \leq \lambda < \mu$. For the related details, please refer to the results of subsection 3.2 in [1]. Moreover, in the case that the space dimension is one, i.e., Ω is a finite interval, applying Theorem 3.1 in [4], the authors proved that (1.2) has at most one positive solution, which is also nondegenerate if it exists (namely, zero is not an eigenvalue of the linearized problem of (1.2) at such a positive solution). See Theorem 3.2 of [1].

In a very recent paper [3], to further understand the effect of the degeneracy of $\alpha(x)$ on the steady-state solution, Du and Wang studied the asymptotic behaviour of positive solutions to (1.2) as the parameter μ or β is large or β is small. Their results demonstrate that the effect of the degeneracy can be clearly observed only when β is large enough, which implies the weak predation ability of the predator in the ecological explanation.

Now, in this paper, we shall deduce the uniqueness and stability for the positive steady state of (1.1) if μ is large enough. As mentioned earlier, for any fixed λ, $\beta > 0$

and nonnegative function $\alpha(x)$, (1.2) always has a positive solution if μ is large. Precisely speaking, the main result of this paper can be stated below.

Theorem 1.1. *There exists μ^* depending only on λ, β, $\alpha(x)$, Ω and Ω_0 such that if $\mu > \mu^*$, (1.2) has a unique positive solution (u_μ, v_μ), which is also linearly stable in the sense that $\operatorname{Re}\eta > 0$ if η is an eigenvalue of the linearized eigenvalue problem of (1.1) at (u_μ, v_μ).*

2. Proof of Theorem 1.1

To prove Theorem 1.1, we first need some preliminary results. From now on, we always denote by (u_μ, v_μ) the positive solution of (1.2).

From [2], when $\alpha(x)$ is degenerate, it is known that the problem

$$-\Delta U = \lambda U - \alpha(x)U^2 \quad \text{in } \Omega, \qquad \partial_\nu U = 0 \ \text{ on } \partial\Omega \tag{2.1}$$

has a unique positive solution if and only if $0 < \lambda < \lambda_1^D(\Omega_0)$. If $\alpha(x) > 0$ on $\bar{\Omega}$, it is also well known that problem (2.1) always has a unique positive solution for any $\lambda > 0$. In any case, if the positive solution of (2.1) exists, we denote it by U_λ.

Lemma 2.1. *Let m, M and μ_0 be arbitrary positive constants. For any fixed $\lambda \in [m, M]$ and $\mu \in [\mu_0, \infty)$, there exist positive constants $\underline{C}$ and $\bar{C}$ independent of λ and μ such that any positive solution (u_μ, v_μ) of (1.2) satisfies*

$$\underline{C} < u_\mu < \bar{C} \quad \textit{and} \quad \underline{C} < v_\mu < \bar{C} \quad \textit{for} \ \ \forall x \in \bar{\Omega}.$$

Proof. From the proof of Step 1 and Step 2 of Theorem 3.1 in [3], it is not hard to see that Lemma 2.1 holds when $\alpha(x)$ is degenerate.

If $\alpha(x)$ is positive on $\bar{\Omega}$, a simple comparison analysis shows $u_\mu < U_\lambda \le U_M$. Since v_μ is a $C^2(\bar{\Omega})$ function by the classical regularity theory for elliptic equations, it follows from the second equation of (1.2) that $v_\mu < \|U_M\|_\infty$ by the Maximum Principle of [5]. Therefore, similarly to Step 2 of Theorem 3.1 in [3], it is also easy to verify the existence of the desired $\underline{C}$. □

Lemma 2.2. *As $\mu \to \infty$, $(u_\mu,\, v_\mu) \to (w,\, w)$ uniformly on $\bar{\Omega}$, where w^* is the unique positive solution of*

$$-\Delta w = \lambda w - (\alpha(x) + \beta)w^2 \quad \text{in } \Omega, \qquad \partial_\nu w = 0 \ \text{ on } \partial\Omega. \tag{2.2}$$

Proof. Using Lemma 2.1, by the argument similar to Step 3 of Theorem 3.1 and Remark 4.1 in [3], we easily see that Lemma 2.2 is true. □

Now, based on the results indicated above, we can claim the linear stability of $(u_\mu,\, v_\mu)$ as μ is large enough. In fact, we have

Theorem 2.1. *There exists a large μ^* depending only on λ, $\alpha(x)$, Ω and Ω_0 such that if $\mu > \mu^*$, every positive solution (u_μ, v_μ) of (1.2) is linearly stable.*

Proof. We argue in an indirect manner. Suppose that Theorem 2.1 is not true. Then, we can find a sequence μ_n and the corresponding positive solution (u_n, v_n) of (1.2) with $\mu = \mu_n$ such that the eigenvalue problem

$$\begin{cases} -\Delta\phi_n = (\lambda - 2\alpha(x) - \beta v_n)\phi_n - \beta u_n \psi_n + \eta_n \phi_n & \text{in } \Omega, \\ -\Delta\psi_n = \mu_n(1 - \dfrac{2v_n}{u_n})\psi_n + \mu_n \dfrac{v_n^2}{u_n^2}\phi_n + \eta_n \psi_n & \text{in } \Omega, \\ \partial_\nu \phi_n = \partial_\nu \psi_n = 0 & \text{on } \partial\Omega \end{cases} \tag{2.3}$$

has an eigenvalue solution pair (ϕ_n, ψ_n, η_n) which satisfies $\mathrm{Re}\,\eta_n \leq 0$ for $n \geq 1$ and $(\phi_n,\, \psi_n) \in W^{2,p}(\Omega) \times W^{2,p}(\Omega)$ for $p > N$ with

$$\|\phi_n\|_{L^2(\Omega)} + \|\psi_n\|_{L^2(\Omega)} = 1 \quad \text{for all } n \geq 1. \tag{2.4}$$

We note that ψ_n, ϕ_n may be complex functions.

To obtain a contradiction, we first prove that η_n is bounded. To this end, it is sufficient to show that both $\mathrm{Re}\,\eta_n$ and $\mathrm{Im}\,\eta_n$ are bounded. Here, $\mathrm{Re}\,\eta_n$ and $\mathrm{Im}\,\eta_n$, respectively, represent the real part and imaginary part of η_n.

We first claim the boundedness of $\mathrm{Re}\,\eta_n$. According to our assumption, it remains to prove that $\mathrm{Re}\,\eta_n$ is bounded below. If this is false, we may assume that $\mathrm{Re}\,\eta_n \to -\infty$.

It is clear that φ_n, $\phi_n \not\equiv 0$. By Kato's inequality, we have from the equation for ϕ_n in (2.3) that

$$-\Delta(|\phi_n|) \leq \mathrm{Re}\left(\frac{\bar{\phi}_n}{|\phi_n|}\Delta\phi_n\right) \leq (\lambda - 2\alpha(x) - \beta v_n)|\phi_n| + \beta u_n|\psi_n| + \mathrm{Re}\,\eta_n|\phi_n|. \tag{2.5}$$

Similarly, by the equation for ψ_n in (2.3), it follows that

$$-\Delta(|\psi_n|) \leq \mu_n(1 - \frac{2v_n}{u_n})|\psi_n| + \mu_n\frac{v_n^2}{u_n^2}|\phi_n| + \mathrm{Re}\,\eta_n|\psi_n|. \tag{2.6}$$

Multiplying (2.5) by $|\phi_n|$ and integrating by parts over Ω, we obtain that

$$0 \leq \int_\Omega |\nabla(|\phi_n|)|^2 \leq \int_\Omega (\lambda - 2\alpha(x) - \beta v_n)|\phi_n|^2 + \beta\int_\Omega u_n|\phi_n||\psi_n| + \mathrm{Re}\,\eta_n\int_\Omega |\phi_n|^2. \tag{2.7}$$

Using Lemma 2.1, we observe that $\lambda - 2\alpha(x) - \beta v_n$ and βu_n are bounded. Thus, by (2.7), there exists a positive constant C independent of n such that

$$(-\mathrm{Re}\,\eta_n - C)\int_\Omega |\phi_n|^2 \leq C\int_\Omega |\phi_n||\psi_n|, \tag{2.8}$$

from which, together with Hölder's inequality, we deduce

$$\|\phi_n\|_{L^2(\Omega)} \leq \frac{C}{-\mathrm{Re}\,\eta_n - C}\|\psi_n\|_{L^2(\Omega)}. \tag{2.9}$$

On the other hand, due to (2.4), $\|\psi_n\|_{L^2(\Omega)}$ is bounded. Hence, by (2.4) and (2.9), the previous assumption $\mathrm{Re}\,\eta_n \to -\infty$ implies

$$\lim_{n\to\infty}\|\phi_n\|_{L^2(\Omega)} \to 0 \quad \text{and} \quad \lim_{n\to\infty}\|\psi_n\|_{L^2(\Omega)} \to 1. \tag{2.10}$$

Now, we need to use (2.6). Similarly, multiplying (2.6) by $|\psi_n|$ and integrating by parts over Ω, it follows that

$$0 \leq \int_\Omega |\nabla(|\psi_n|)|^2 \leq \mu_n\int_\Omega \Big[(1 - \frac{2v_n}{u_n})|\psi_n|^2 + \frac{v_n^2}{u_n^2}|\phi_n||\psi_n|\Big] + \mathrm{Re}\,\eta_n\int_\Omega |\psi_n|^2. \tag{2.11}$$

In addition, from Lemma 2.2 we see that

$$1 - \frac{2v_n}{u_n} \to -1 \quad \text{and} \quad \frac{v_n^2}{u_n^2} \to 1 \ \text{ uniformly on } \bar{\Omega} \ \text{ as } \ n \to \infty. \tag{2.12}$$

As a result, using (2.11) and (2.12), for all large n, we can yield the following inequality:

$$0 \leq -\frac{1}{2}\mu_n\int_\Omega |\psi_n|^2 + 2\mu_n\Big(\int_\Omega |\phi_n|^2\Big)^{1/2}\Big(\int_\Omega |\psi_n|^2\Big)^{1/2} + \mathrm{Re}\,\eta_n\int_\Omega |\psi_n|^2. \tag{2.13}$$

Recalling our assumption $\mathrm{Re}\,\eta_n \to -\infty$, together with (2.10), it is easily seen that the right-side of (2.13) is negative, which is a contradiction. This confirms our previous claim: $\mathrm{Re}\,\eta_n$ is bounded.

Next, we claim that $\mathrm{Im}\,\eta_n$ is also bounded. Once again, we adopt a contradiction argument. Suppose that $|\mathrm{Im}\,\eta_n| \to \infty$ as $n \to \infty$. Then, from the equation for ϕ_n in (2.3), we find that

$$\int_\Omega |\nabla\phi_n|^2 = \int_\Omega (\lambda - 2\alpha(x) - \beta v_n)|\phi_n|^2 - \beta \int_\Omega u_n \bar{\phi}_n \psi_n + \eta_n \int_\Omega |\phi_n|^2. \tag{2.14}$$

Hence, (2.14) implies that the following holds:

$$\mathrm{Im}\,\eta_n \int_\Omega |\phi_n|^2 = \beta\, \mathrm{Im} \int_\Omega u_n \bar{\phi}_n \psi_n,$$

which yields

$$|\mathrm{Im}\,\eta_n| \int_\Omega |\phi_n|^2 \le \beta \int_\Omega u_n |\phi_n||\psi_n|. \tag{2.15}$$

Consequently, by Lemma 2.1 and Hölder's inequality, (2.15) guarantees that there exists a positive constant C independent of n such that

$$|\mathrm{Im}\,\eta_n|^2 \int_\Omega |\phi_n|^2 \le C \int_\Omega |\psi_n|^2.$$

Thus, our assumption $|\mathrm{Im}\,\eta_n| \to \infty$ implies that (2.10) holds. On the other hand, we easily observe that (2.12)-(2.13) are still true. Since $\mu_n \to \infty$ and $\mathrm{Re}\,\eta_n$ is bounded, due to (2.13), a contradiction occurs, which verifies our previous claim.

Until now, it has been proved that η_n is bounded. Hence, we may assume that $\eta_n \to \eta$ with $\mathrm{Re}\,\eta \le 0$. Since $\|\psi_n\|_{L^2(\Omega)}$ is bounded, by the first equation of (2.3) and Lemma 2.1, the standard L^p theory for elliptic equations guarantees that $\|\phi_n\|_{W^{2,2}(\Omega)}$ is bounded. Therefore, passing to a subsequence if necessary, assume that $\phi_n \to \phi$ weakly in $H^1(\Omega)$ and strongly in $L^2(\Omega)$. In addition, by virtue of the boundedness of $\|\psi_n\|_{L^2(\Omega)}$, we may also let $\psi_n \to \psi$ weakly in $L^2(\Omega)$.

Now, choose φ to be an arbitrary $C_0^\infty(\Omega)$ function. Multiplying the equation for ψ_n in (2.3) by φ/μ_n, integrating by parts over Ω, and then letting $n \to \infty$, it is easily seen that $\int_\Omega (\phi - \psi)\varphi = 0$, and this implies $\phi = \psi$.

Using the equation for ϕ_n in (2.3) and Lemma 2.2 again, we find that ϕ satisfies weakly

$$-\Delta\phi + (-\lambda + 2\alpha(x)w^* + 2\beta w^*)\phi = \eta\phi \quad \text{in } \Omega, \quad \partial_\nu \phi = 0 \quad \text{on } \partial\Omega, \tag{2.16}$$

where w^* is defined in Lemma 2.2. Furthermore, the standard regularity theory for elliptic equations implies that ϕ is a $W^{2,p}(\Omega)$ solution of (2.16) for $p > N$.

We note that $\phi \not\equiv 0$. On the contrary, if $\phi \equiv 0$, then $\phi_n \to 0$ in $L^2(\Omega)$, from which it follows that (2.10) holds. Moreover, in this case, (2.11)-(2.13) are also true. As $\mu_n \to \infty$ and η_n is bounded, (2.13) leads to an obvious contradiction. Hence, ϕ is a nontrivial solution of (2.16), which also shows that η is a real eigenvalue of (2.16) and $\eta \le 0$. Thus, by (2.16), we get

$$0 \ge \eta \ge \lambda_1^N(-\lambda + 2\alpha(x)w^* + 2\beta w^*, \Omega) > \lambda_1^N(\Omega, -\lambda + \alpha(x)w^* + \beta w^*, \Omega). \tag{2.17}$$

However, by the definition of w^* and the property of the first eigenvalue, it follows that $\lambda_1^N(-\lambda + \alpha(x)w^* + \beta w^*, \Omega) = 0$, contradicting (2.17). This finishes the proof of Theorem 2.1. $\square$

In the following, we prove the uniqueness of positive solutions of (1.1) as $\mu \to \infty$, which will verify the first part of Theorem 1.1. Our main tool to be employed is topological degree theory.

Theorem 2.2. *There exists a large μ^* depending only on λ, $\alpha(x)$, Ω and Ω_0 such that* (1.2) *has a unique positive solution (u_μ, v_μ) if $\mu > \mu^*$.*

Proof. Let μ^* be as in Theorem 2.1, and fix λ, μ satisfying $\lambda \in [m, M]$ and $\mu > \mu^*$. Denote

$$\Theta = \{(u, v) \in C(\bar{\Omega}) \times C(\bar{\Omega}) |\, \underline{C} < u,\ \ v < \bar{C}\},$$

where $\underline{C}$ and $\bar{C}$ are given in Lemma 2.1. Thus, for such λ and μ, (1.2) has no positive solution $(u, v) \in \partial\Theta$.

Let us define

$$A(\lambda, u, v) = (-\Delta + \mathrm{I})^{-1}((1 + \lambda)u - \alpha(x)u^2 - \beta uv,\, (1 + \mu)v - \mu\frac{v^2}{u}),$$

where $(-\Delta+\mathrm{I})^{-1}$ stands for the inverse operator of $-\Delta+\mathrm{I}$ subject to the Neumann boundary condition over $\partial\Omega$. It is well-known that A is a compact operator from $[m, M] \times \Theta$ to $C(\bar{\Omega}) \times C(\bar{\Omega})$, and $(u, v) \in \Theta$ solves (1.2) if and only if (u, v) satisfies $(u, v) = A(\lambda, u, v)$. In addition,

$$(u, v) \neq A(\lambda, u, v), \quad \forall \lambda \in [m, M] \text{ and } \forall (u, v) \in \partial\Theta.$$

As a result, the topological degree $\deg(\mathrm{I} - A(\lambda, \cdot), \Theta, 0)$ is well-defined, which is also independent of $\lambda \in [m, M]$.

Note that, by the argument as in the proof of Theorem 3.1 of [1], if $\lambda = m \in (0, \lambda_1^D(\Omega_0))$, we have

$$\deg(\mathrm{I} - A(m, \cdot), \Theta, 0) = 1.$$

Therefore, it follows that

$$\deg(\mathrm{I} - A(\lambda, \cdot), \Theta, 0) = 1, \quad \forall \lambda \in [m, M].$$

Now, by Theorem 2.1, every positive solution (u_μ, v_μ) of (1.2) is nondegenerate and linearly stable. Hence, the fixed point index of $A(\lambda, (u_\mu, v_\mu))$ is well-defined and is 1. Furthermore, by the compactness of $A(\lambda, \cdot)$, it is easy to show that there are at most finitely many such fixed points in Θ, denoted by $\{(u_i, v_i)\}_1^l$. Then, from the additivity property of the fixed point index, it follows that

$$1 = \deg(\mathrm{I} - A(\lambda, \cdot), \Theta, 0) = \Sigma_1^l \operatorname{index}(\mathrm{I} - A(\lambda, \cdot), (u_i, v_i)) = l,$$

from which we deduce the uniqueness of the positive solution of (1.2) for $\mu > \mu^*$. Our proof is complete. □

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References

[1] Y. H. Du and S. B. Hsu, *A diffusive predator-prey model in heterogeneous environment*, J. Diff. Equns., 203(2004), 331-364. MR2073690 (2005e:35051)

[2] Y. H. Du and Q. G. Huang, *Blow-up solutions for a class of semilinear elliptic and parabolic equations*, SIAM J. Math. Anal., 31(1999), 1-18. MR1720128 (2000g:35059)

[3] Y. H. Du and M. X. Wang, *Asymptotic behavior of positive steady-states to a predator-prey model*, Proc. Roy. Soc. Edinburgh Sect. A, 136(2006), 759–778. MR2250444

[4] J. López-Gómez and R. M. Pardo, *Invertiblility of linear noncooperative elliptic systems*, Nonlinear Anal., TMA, **31**(1998), 687-699. MR1487855 (99b:34046)

[5] Y. Lou and W. M. Ni, *Diffusion vs cross-diffusion: an elliptic approach*, J. Diff. Equns., 154 (1999), 157-190. MR1685622 (2000c:35042)

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