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ERRATUM TO "EXOTIC SMOOTH STRUCTURES ON $3\mathbf{CP}^2\#n\overline{\mathbf{CP}}^2$"

B. DOUG PARK

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The proofs of Lemmas 2.2 and 4.2(ii) in [3] are incorrect as they assume that the numerical Godeaux surface in [1] is simply-connected as was claimed in that paper. From [2], we have learned that this surface may not be simply-connected after all.

However, we can still construct an infinite family of pairwise non-diffeomorphic irreducible 4-manifolds that are homeomorphic to $3\mathbf{CP}^2\#10\overline{\mathbf{CP}}^2$. One such construction can be given by slightly modifying the construction of an exotic $3\mathbf{CP}^2\#9\overline{\mathbf{CP}}^2$ in [4]. In the notation of [4], we can find an embedded sphere of self-intersection -16 in the 4-manifold $Z_{K_1,K_2,K_3}\#4\overline{\mathbf{CP}}^2$. This sphere is obtained by resolving the intersection of a pseudo-section of Z_{K_1,K_2,K_3} and a single fishtail fiber (instead of two fishtail fibers as in [4]) and then blowing up at the three positive double points of the pseudo-section and at the double point of the fishtail fiber. Using twelve (-2)-spheres in the I_{16} fiber, we can then extend this (-16)-sphere to the configuration $C_{14,1}$. Let M_{K_1,K_2,K_3} denote the result of the rational blowdown of this configuration of spheres. The proof of Proposition 3.2 in [4] applies almost verbatim to show that M_{K_1,K_2,K_3} is homeomorphic to $3\mathbf{CP}^2\#10\overline{\mathbf{CP}}^2$. By setting $K_1 = K_2 = K_3 = T_n$, the n-twist knot and mimicking the proof of Theorem 3.3 in [4], we can show that $M_n = M_{T_n,T_n,T_n}$ $(n = 1, 2, 3, \dots)$ are pairwise non-diffeomorphic irreducible 4-manifolds with inequivalent non-zero Seiberg-Witten invariants.

It remains an intriguing open problem whether the 4-manifolds X and P in [3] are simply-connected or not. Note that the 4-manifold M in Section 4 of [3] is unaffected and still is an exotic $3\mathbf{CP}^2\#12\overline{\mathbf{CP}}^2$.

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Department of Pure Mathematics, University of Waterloo, Waterloo, ON, N2L 3G1, Canada

E-mail address: bdpark@math.uwaterloo.ca

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