PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 136, Number 6, June 2008, Pages 1997–2006
S 0002-9939(08)09032-1
Article electronically published on February 19, 2008

SEMI-COMPACTNESS OF POSITIVE DUNFORD–PETTIS OPERATORS ON BANACH LATTICES

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(Communicated by Joseph A. Ball)

ABSTRACT. We investigate Banach lattices on which each positive Dunford–Pettis operator is semi-compact and the converse. As an interesting consequence, we obtain Theorem 2.7 of Aliprantis–Burkinshaw and an element of Theorem 1 of Wickstead.

1. INTRODUCTION AND NOTATION

An operator T from a Banach space E into a Banach lattice F is said to be semi-compact if for each $\varepsilon > 0$, there exists some $u \in F^+$ such that $T(B_E) \subset [-u, u] + \varepsilon B_F$ where B_H is the closed unit ball of $H = E$ or F and $F^+ = \{x \in F : 0 \leq x\}$. An operator T between two Banach spaces E and F is said to be Dunford–Pettis if the image of each weakly compact subset of E is a compact subset of F.

In contrast to compact operators, the class of semi-compact (resp. Dunford–Pettis) operators does not satisfy the analogue of Schauder's Theorem. However, the class of semi-compact operators satisfies the domination problem (Theorem 18.20 of [3]), but the class of Dunford–Pettis operators fails to satisfy this property, as was proved in [1], [7] and [9].

Finally, a semi-compact operator is not necessarily Dunford–Pettis and, conversely, a Dunford–Pettis operator is not necessarily semi-compact.

In [4] and [5] we studied the compactness of the class of positive Dunford–Pettis operators. And in [6] we characterized Banach lattices on which each positive Dunford–Pettis operator is weakly compact. Also, it follows from Aliprantis and Burkinshaw ([1], Theorem 3.4) that if E and F are two Banach lattices such that E has an order-continuous norm, then each regular Dunford–Pettis operator is AM-compact. Our objective in this paper is to continue the invistigation of Banach lattices on which each positive Dunford–Pettis operator is semi-compact and the converse. We will characterize Banach lattices for which each positive Dunford–Pettis operator is semi-compact, and we will give some interesting consequences. Next, we will prove a sufficient condition under which a positive semi-compact operator is weakly compact. More precisely, we will show that if the norm of a Banach lattice F is order continuous, then each positive semi-compact operator

Received by the editors February 24, 2006 and, in revised form, December 2, 2006.
2000 *Mathematics Subject Classification.* Primary 46A40, 46B40, 46B42.
Key words and phrases. Semi-compact operator, Dunford–Pettis operator, order continuous norm, discrete vector lattice.

from a Banach lattice E into F is weakly compact. As consequence, we shall obtain some conditions for which the class of Dunford–Pettis operators, the class of semi-compact operators, the class of weakly compact operators, and the class of compact operators coincide. Finally, whenever E is an order σ-complete Banach lattice, we will establish that if each positive semi-compact operator from E into E is weakly compact (resp. Dunford–Pettis), then the norm of E is order continuous.

To state our results, we need to fix some notation and recall some definitions. A vector lattice E is an ordered vector space in which $\sup(x, y)$ exists for every $x, y \in E$. A subspace F of a vector lattice E is said to be a sublattice if for every pair of elements a, b of F the supremum of a and b, taken in E, belongs to F. A subset B of a vector lattice E is said to be solid if it follows from $|y| \leq |x|$ with $x \in B$ and $y \in E$ that $y \in B$. An order ideal of E is a solid subspace. Let E be a vector lattice, for each $x, y \in E$ with $x \leq y$, the set $[x, y] = \{z \in E \,:\, x \leq z \leq y\}$ is called an order interval. A subset of E is said to be order bounded if it is included in some order interval. A Banach lattice is a Banach space $(E, \|.\|)$ such that E is a vector lattice and its norm satisfies the following property: for each $x, y \in E$ such that $|x| \leq |y|$, we have $\|x\| \leq \|y\|$. If E is a Banach lattice, its topological dual E', endowed with the dual norm, is also a Banach lattice. For more detail about Banach lattices, the reader is referred to the book of Zaanen [10].

2. Major results

We will use the term operator $T : E \longrightarrow F$ between two Banach lattices to mean a bounded linear mapping. It is positive if $T(x) \geq 0$ in F whenever $x \geq 0$ in E. An operator $T : E \longrightarrow F$ is regular if $T = T_1 - T_2$ where T_1 and T_2 are positive operators from E into F. It is well known that each positive linear mapping on a Banach lattice is continuous.

Also, a norm $\|.\|$ of a Banach lattice E is order continuous if for each net (x_α) such that $x_\alpha \downarrow 0$ in E, the sequence (x_α) converges to 0 for the norm $\|.\|$ where the notation $x_\alpha \downarrow 0$ means that (x_α) is decreasing, its infimum exists, and $\inf(x_\alpha) = 0$. For example, the norm of the Banach lattice l^1 is order continuous but the norm of the Banach lattice l^∞ is not.

To prove the next theorem, we need the following lemma:

Lemma 2.1. *Let E, F be Banach lattices and let T be a positive Dunford–Pettis operator from E into F. If the topological dual E' of E has an order continuous norm, then for each $\varepsilon > 0$, there exists some $y \in E^+$ such that*

$$T\left(B_E \cap E^+\right) \subset \varepsilon B_F + T\left([0, y]\right),$$

where B_H is the closed unit ball of $H = E, F$.

Proof. It follows from the proof of Theorem 2.7 and Theorem 2.8 of [1]. □

Recall that a nonzero element x of a vector lattice E is discrete if the order ideal generated by x equals the subspace generated by x. The vector lattice E is discrete if it admits a complete disjoint system of discrete elements. For example, the Banach lattice l^1 is discrete but $C\left([0, 1]\right)$ is not discrete.

A Dunford–Pettis operator is not necessarily semi-compact. In fact, the identity operator $\mathrm{Id}_c : c \longrightarrow c$ is semi-compact but it is not Dunford–Pettis where c is the Banach lattice of all convergent sequences. The following theorem gives a sufficient and necessary condition for which a regular Dunford–Pettis operator is semi-compact. In fact,

Theorem 2.2. *Let E be a Banach lattice. Then the following assertions are equivalent:*

1. E' has an order continuous norm.

2. Each positive Dunford–Pettis operator from E into F is semi-compact for each Banach lattice F.

3. Each positive Dunford–Pettis operator from E into E is semi-compact.

Proof. For the implication 1⟹2. Let F be a Banach lattice and let T be a positive Dunford–Pettis operator from E into F. It follows from Lemma 2.1 that for each $\varepsilon > 0$, there exists some $y \in E^+$ such that

$$T(B_E^+) \subset \varepsilon B_F^+ + T([0, y]),$$

where $B_H^+ = B_H \cap E^+$ for $H = E, F$.

Since

$$T([0, y]) \subset [0, T(y)]\,,$$

we obtain

$$T(B_E^+) \subset \varepsilon B_F^+ + [0, T(y)]\,.$$

This proves the result.

For the implication 3⟹1. If the norm of E' is not order continuous, then it follows from the proof of Theorem 1 of Wickstead [9], that E contains a sublattice that is isomorphic to l^1 and there exists a positive projection P from E into l^1.

Consider the operator product

$$i \circ P : E \longrightarrow l^1 \longrightarrow E,$$

where i is the inclusion operator of l^1 in E. It is clear that $i \circ P$ is a positive Dunford–Pettis operator. We have to prove that $i \circ P$ is not semi-compact. If not, its restriction to l^1 is a semi-compact operator, and then the restriction of the operator $P \circ (i \circ P)$ to l^1, which coincides with the operator identity Id_{l^1} of l^1, is semi-compact; i.e. for each $\varepsilon > 0$, there exists some $y \in \left(l^1\right)^+$ such that

$$B_{l^1} \subset \varepsilon B_{l^1} + [-y, y]\,.$$

Since the Banach lattice l^1 is discrete and has an order continuous norm, it follows from Theorem 3.22 of [2] that the order interval $[-y, y]$ is compact in l^1. Hence, the closed unit ball B_{l^1} is precompact. This presents a contradiction.

The implication 2⟹3 is trivial.

Remark 2.3. If we fix the Banach lattice F, the implication 2⟹1 of Theorem 2.2. is false. In fact, if we take F of finite dimension and $E = l^1$, it is clear that each positive Dunford–Pettis operator from l^1 into F is semi-compact, but it is well known that the norm of the topological dual $E' = l^\infty$ is not order continuous.

Recall from Zaanen [10] that a regular operator T from a vector lattice E into a Banach lattice F is said to be AM-compact if it carries each order-bounded subset of E onto a relatively compact subset of F.

As a consequence of Theorem 2.2 of [6] and the above theorem, we obtain

Corollary 2.4. *Let E be a Banach lattice. Then the following statements are equivalent:*

1. Each positive Dunford–Pettis operator from E into E is weakly compact.

2. Each positive Dunford–Pettis operator from E into E is semi-compact.

3. For each positive Dunford–Pettis operator T from E into E, the second power operator T^2 is compact.

4. For each pair of operators S and T from E into E such that $0 \leq S \leq T$ and T is Dunford–Pettis, the operator S is weakly compact.

5. The norm of the topological dual E' is order continuous.

The following result is a consequence of a theorem of Dodds and Fremlin ([10], Theorem 125.5) and Theorem 2.2:

Corollary 2.5. *Let E and F be two Banach lattices such that E' and F have order continuous norms. Let T be a positive operator from E into F. Then the following assertions are equivalent:*

1. T is compact.

2. T is Dunford–Pettis and AM-compact.

3. T is semi-compact and AM-compact.

Now by combining our Corollary 2.5 and Theorem 3.5 of Aliprantis and Burkinshaw [1], we obtain Theorem 2.7 of [1] as a consequence, which was the basic result of our papers [4] and [5].

Corollary 2.6. *Let E be a Banach lattice such that E and its topological dual E' have order continuous norms. Then each positive Dunford–Pettis operator from E into E is compact.*

Proof. Let T be a positive Dunford–Pettis operator from E into E. Since the norm of E is order continuous, it follows from Theorem 3.5 of [1] that T is AM-compact. Finally, Corollary 2.5 implies that T is compact.

Let T be a positive operator from a Banach lattice E into a Banach lattice F. If T' is the adjoint operator from F' into E' defined by $T'(f)(x) = f(T(x))$ for each $f \in F'$ and for each $x \in E$, it is clear that T' is positive.

Recall that a Banach space E has the Dunford–Pettis property if and only if each weakly compact operator on E, taking its value in another Banach space, is Dunford–Pettis.

Now, to give a sufficient condition under which each positive semi-compact operator is Dunford–Pettis, we study the compactness of a positive semi-compact operator.

Theorem 2.7. *Let E and F be Banach lattices. Then each positive semi-compact operator from E into F is compact if one of the following statements is valid:*

1. F is discrete and its norm is order continuous.

2. E' is a discrete order continuous norm, and F has an order continuous norm.

3. The norms of E, E' and F are order continuous, and E has the Dunford–Pettis property.

Proof. 1. Let T be a positive semi-compact operator from E into F. By Theorem 3.22 of [1], for each $y \in F^+$, the order interval $[0, y]$ is norm compact in F. On the other hand, for each $\varepsilon > 0$, there exists some $x \in F^+$ such that

$$T(B_E) \subset \varepsilon B_F + [-x, x].$$

This proves that $T(B_E)$ is norm relatively compact in F and hence T is compact.

2. Let T be a positive semi-compact operator from E into F. Since the norms of E' and F are order continuous, then the adjoint operator T' from F' into E' is semi-compact (Theorem 125.6 of [10]). As the Banach lattice E' is discrete and its norm is order continuous, then condition 1 implies that $T' : F' \longrightarrow E'$ is compact, and therefore T is compact.

3. Let T be a positive semi-compact operator from E into F. Since F has an order continuous norm, it follows from Theorem 2.2 that T is weakly compact. Now, the Dunford–Pettis property of E implies that T is Dunford–Pettis. On the other hand, E and E' have order continuous norms, so Theorem 2.7 of Aliprantis and Burkinshaw [1] implies that T is compact. □

As an immediate consequence of Theorem 2.6, we obtain a result of Wickstead ([8], Proposition 2.3 or [9], Theorem 1):

Corollary 2.8. *Let E and F be Banach lattices, and let S and T be positive operators from E into F such that $0 \leq S \leq T$ and T is compact. If F is discrete and its norm is order continuous, then S is compact.*

Recall that a semi-compact operator is not necessarily Dunford–Pettis. For example, the identity operator $\mathrm{Id}_{l^1} : l^1 \longrightarrow l^1$ is Dunford–Pettis but it is not semi-compact. Another consequence of Theorem 2.7 is the following:

Corollary 2.9. *Let E and F be Banach lattices. If F is discrete and its norm is order continuous, then each positive semi-compact operator from E into F is Dunford–Pettis.*

Recall that a semi-compact operator is not necessarily weakly compact, and conversely a weakly compact operator is not necessarily semi-compact. For example, the identity operator $\mathrm{Id}_{l^2} : l^2 \longrightarrow l^2$ is weakly compact but it is not semi-compact (if not, since l^2 is discrete and its norm is order continuous, it follows from Theorem 2.7 (1) that Id_{l^2} is compact) and conversely, the identity operator $\mathrm{Id}_c : c \longrightarrow c$ is semi-compact but it is not weakly compact.

The following proposition gives a sufficient condition under which a positive weakly compact operator is semi-compact:

Proposition 2.10. *Let E and F be two Banach lattices. If E has the Dunford–Pettis property and its topological dual E' has an order continuous norm, then each positive weakly compact operator from E into F is semi-compact.*

Proof. Let T be a positive weakly compact operator from E into F. Since E admits the Dunford–Pettis property, then T is Dunford–Pettis. Now the semi-compactness of T follows from the order continuousness of the norm of the topological dual E' (Theorem 2.2).

Conversely, we give a sufficient condition under which a positive semi-compact operator is weakly compact.

Theorem 2.11. *Let E and F be two Banach lattices. If the norm of F is order continuous, then each positive semi-compact operator from E into F is weakly compact.*

Proof. Let T be a positive semi-compact operator from E into F. Then for each $\varepsilon > 0$, there exists some $y \in F^+$ such that

$$T\left(B_E \cap E^+\right) \subset \varepsilon B_F + [0, y], \tag{1}$$

where B_H is the unit ball of $H = E, F$. Since F has an order continuous norm, then the order interval $[0, y]$ is weakly compact, and hence $T(B_E \cap E^+)$ is weakly precompact. To prove that $T(B_E \cap E^+)$ is weakly relatively compact, it is sufficient to show that the closure of $T(B_E \cap E^+)$, for the topology $\sigma(F, F')$, is weakly complete. To show this, we use the same proof as Theorem 2.2 of [6]. In fact, let $(T(x_i))_i$ be a Cauchy net for the topology $\sigma(F, F')$, where (x_i) is a net in $B_E \cap E^+$. Since $T(B_E \cap E^+)$ is relatively compact for $\sigma(F'', F')$ in the topological bidual F'', the sequence $(T(x_i))_i$ converges to some $\Psi \in F''$ for $\sigma(F'', F')$. Let $m \in \mathbb{N}^*$; it follows from (1), the existence of $y^m \in F^+$, $z_i^m \in B_F$ and $w_i^m \in [0, y^m]$ such that

$$T(x_i) = \frac{1}{m} z_i^m + w_i^m.$$

On the other hand, since $[0, y^m]$ is relatively weakly compact, there exists an accumulation point $a_m \in F$ of the net $(w_i^m)_i$.

If we fix $f \in F'$ with $\|f\| \leq 1$, then there exists some $i_0 \in I$ such that for each $i > i_0$, we have

$$|f \circ T(x_i) - \Psi(f)| < \frac{1}{m},$$

and hence

$$|f(w_i^m) - \Psi(f)| < \frac{2}{m}.$$

Since a_m is an accumulation point of the sequence $(w_i^m)_i$, there exists some $i > i_0$ such that

$$|f(w_i^m) - f(a_m)| < \frac{1}{m}.$$

This implies that

$$|\Psi(f) - f(a_m)| < \frac{3}{m},$$

and so

$$\|\Psi - a_m\|_{F''} \leq \frac{3}{m},$$

where $\|.\|_{F''}$ is the norm of F''. It follows that (a_m) converges in norm to Ψ, and hence $\Psi \in F$. This proves that the closure of $T(B_E \cap E^+)$ for $\sigma(F, F')$ in F coincides with its closure in $\hat{F}$ where $\hat{F}$ is the completion of F for $\sigma(F, F')$. Hence, $\overline{T(B_E \cap E^+)}^{\sigma(F,F')}$ is complete.

A Banach lattice E is said to be an AM-space if for each $x, y \in E$ such that $\inf(x, y) = 0$, we have $\|x + y\| = \max\{\|x\|, \|y\|\}$. It is an AL-space if its topological dual E' is an AM-space. For example, the Banach lattice l^1 is an AL-space and the Banach lattice l^∞ is an AM-space.

As consequence of Theorem 2.11, we have:

Corollary 2.12. *Let E be an AM-space with unit and F be a Banach lattice with an order continuous norm. Then each positive operator from E into F is weakly compact. In particular, each positive operator from E into l^1 is compact.*

Proof. Let $T : E \longrightarrow F$ be a positive operator. As E is an AM-space with unit, it follows that T is semi-compact. Now, since F has an order continuous norm, Theorem 2.11 implies that T is weakly compact.

Finally, let $S : E \longrightarrow l^1$ be a positive operator where E is an AM-space with unit. Since l^1 has an order continuous norm, the operator S is weakly compact. On the other hand, $S = \mathrm{Id}_{l^1} \circ S$, where $\mathrm{Id}_{l^1} : l^1 \longrightarrow l^1$ is the identity operator of l^1 which is Dunford–Pettis; hence, S is compact.

Remark 2.13. If E is a Banach lattice with a discrete topological dual E', then there exists a positive semi-compact operator which is not weakly compact. In fact, if we take $E = F = c$, then E' is discrete, and the identity operator Id_E of E is semi-compact but it is not weakly compact.

The following consequence of Theorem 2.11 gives another sufficient condition under which each positive semi-compact operator is Dunford–Pettis.

Corollary 2.14. *Let E and F be two Banach lattices. If E has the Dunford–Pettis property and the norm of F is order continuous, then each positive semi-compact operator is Dunford–Pettis.*

Proof. Let T be a positive semi-compact operator from E into F. Since the norm of F is order continuous, it follows from Theorem 2.11 that T is weakly compact. As E has the Dunford–Pettis property, the operator T is Dunford–Pettis.

Another consequence of Theorem 2.11 is the following:

Corollary 2.15. *Let E and F be two Banach lattices such that E' and F have order continuous norms and E has the Dunford–Pettis property. Let $T : E \longrightarrow F$ be a positive operator. Then the following assertions are equivalent:*

1. *T is Dunford–Pettis.*
2. *T is semi-compact.*
3. *T is weakly compact.*

In particular, if we take $E = F$, under the same conditions as Corollary 2.15, we have the following result:

Corollary 2.16. *Let E be a Banach lattice having the Dunford–Pettis property such that E and E' have order continuous norms. Let $T : E \longrightarrow E$ be a positive operator. Then the following assertions are equivalent:*

1. *T is Dunford–Pettis.*
2. *T is semi-compact.*
3. *T is weakly compact.*
4. *T is compact.*

Remark 2.17. Let E be a Banach lattice having the Dunford–Pettis property. If the topological dual E' is discrete with an order continuous norm, there exists a positive semi-compact operator which is not Dunford–Pettis. In fact, the identity operator Id_c of c is a positive semi-compact operator which is not Dunford–Pettis. However, the topological dual c' is discrete, c has the Dunford–Pettis property, and c' has an order continuous norm.

Whenever $E = F$ and E is order σ-complete (i.e. every majorized countable nonempty subset of E has a supremum), we obtain the following converse of Theorem 2.11:

Theorem 2.18. *Let E be an order σ-complete Banach lattice. If each positive semi-compact operator from E into E is weakly compact (resp. Dunford–Pettis), then the norm of E is order continuous.*

Proof. Assume that the norm of E is not order continuous. Since E is order σ-complete, it follows from the proof of Theorem 1 of Wickstead [9] that E contains a sublattice which is isomorphic to l^∞ and there exists a positive projection $P : E \longrightarrow l^\infty$.

If we take the operator

$$i \circ P : E \longrightarrow l^\infty \longrightarrow E,$$

it is clear that $i \circ P$ is a semi-compact operator which is not weakly compact (resp. Dunford–Pettis). If not, the restriction of the operator $P \circ (i \circ P)$ to l^∞, which coincides with the identity operator Id_{l^∞} of l^∞, would be weakly compact (resp. Dunford–Pettis). This presents a contradiction.

Recall that Wickstead characterized Banach lattices which satisfy the problem of domination for the class of positive weakly compact operators ([8], Theorem 17.10). We observe that by the same proof as Theorem 17.10 of Wickstead [8], we obtain the following result:

Theorem 2.19. *Let E and F be two Banach lattices. If every positive operator from E into F dominated by a compact operator is weakly compact, then either E' or F has an order continuous norm.*

Proof. If the norms of F and E' are not order continuous, Wickstead constructed in the proof of [8], Theorem 17.10 two positive operators $S, T : E \longrightarrow F$ such that $0 \leq S \leq T$ and T is compact but S is not weakly compact. This proves the result.

Whenever $E \neq F$, by combining Theorem 2.19 with Theorem 18.20 of [3] we obtain

Theorem 2.20. *Let E and F be two Banach lattices. If each positive semi-compact operator from E into F is weakly compact, then one of the following assertions is valid:*

1. F has an order continuous norm.
2. E' has an order continuous norm.

Proof. In fact, let $S, T : E \longrightarrow F$ be two operators such that $0 \leq S \leq T$ and T is compact. Since T is semi-compact, it follows that S is semi-compact (Theorem 18.20 of [3]). Hence S is weakly compact and the result comes from Theorem 2.19.

Also, recall that Wickstead ([9], Theorem 2) studied the converse of the domination problem of the class of positive Dunford–Pettis operators. We remark that by the same proof as Theorem 2 of [9], we can establish the following result:

Theorem 2.21. *For two Banach lattices E and F, the following assertions are equivalent:*

1. Every positive operator from E into F dominated by a compact operator is Dunford–Pettis.

2. The norm of F is order continuous or the lattice operations of E are weakly sequentially continuous.

Proof. If the norm of F is not order continuous and the lattice operations of E are not weakly sequentially continuous, Wickstead constructed in the proof of Theorem 2 of [9] two positive operators $S, T : E \longrightarrow F$ such that $0 \le S \le T$ and T is compact but S is not Dunford–Pettis. □

A combination of Theorem 2.21 and Theorem 18.20 of [3] gives

Theorem 2.22. *Let E and F be Banach lattices. If each positive semi-compact operator from E into F is Dunford–Pettis, then one of the following assertions is valid:*

1. *The norm of F is order continuous.*
2. *The lattice operations of E are weakly sequentially continuous.*

Proof. Let S and T be two positive operators from E into F such that $0 \le S \le T$ and T is compact. Since T is semi-compact, S is semi-compact ([3], Theorem 18.20) and then Dunford–Pettis. Now, the result follows from Theorem 2.21. □

Remark 2.23. Let E and F be two Banach lattices such that one of the following properties is valid:

1. The topological dual E' of E is discrete.
2. F is discrete.
3. The norm of E' is order continuous.

Then we can always find a positive semi-compact operator from E into F which is not Dunford–Pettis. In fact, the identity operator $\mathrm{Id}_c : c \longrightarrow c$ is semi-compact which is not Dunford–Pettis but the above three conditions are satisfied by the Banach lattice c.

Remark 2.24. There exist Banach lattices E and F and there exists a positive Dunford–Pettis operator from E into F which is semi-compact but not necessarily weakly compact. In fact, we take the Banach lattice $E = F = l^1 \oplus l^\infty$. Since the norms of E and E' are not order continuous, it follows, from Theorem 1 of Wickstead [9], the existence of two operators S and T from E into E, such that $0 \le S \le T$ and T is compact but S is not compact. On the other hand, the lattice operations of E are weakly sequentially continuous (i.e. the sequence $(|x_n|)$ converges to 0 for the weak topology $\sigma(E, E')$ whenever the sequence (x_n) converges to 0 for $\sigma(E, E')$), then an application of Theorem 2 of Wickstead [9] implies that S is Dunford–Pettis. Now, as the norms of E and E' are not order continuous, the operator S is not necessarily weakly compact ([8], Theorem 2.2).

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