PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 136, Number 7, July 2008, Pages 2417–2423
S 0002-9939(08)08820-5
Article electronically published on March 4, 2008

EXTENSIONS BY SPACES OF CONTINUOUS FUNCTIONS

JESÚS M. F. CASTILLO AND YOLANDA MORENO

(Communicated by Jonathan M. Borwein)

ABSTRACT. We present two complementary results on the splitting of exact sequences having the form $0 \to C(K) \to E \to X \to 0$. The first one characterizes the Banach spaces X such that $\mathrm{Ext}(X, C(K)) = 0$ for every compact space K. The second is a nonlinear generalization of Zippin's criterion for the extension of $C(K)$-valued operators.

1. Introduction

See the background section below for all unexplained terminology or notation. The paper deals with the splitting of exact sequences $0 \to C(K) \to E \to X \to 0$,

The first problem we consider is when every exact sequence

$$0 \longrightarrow C(K) \longrightarrow E \longrightarrow X \longrightarrow 0$$

splits, which will be denoted by $\mathrm{Ext}(X, C(K)) = 0$. Previous results in this direction were obtained by Kalton in [7], where he shows that if $\mathrm{Ext}(X, C(K)) = 0$, then X must also have the strong-Schur property (see also [9]). On the other hand, Johnson and Zippin show in [5] that $\mathrm{Ext}(X, C(K)) = 0$ for every dual of a subspace of c_0, and Kalton obtains in [7] the converse implication under the assumption that X has an unconditional FDD. Since there is a well-established equivalence between exact sequences and z-linear maps, the problem can be reformulated as when $C(K)$-valued z-linear maps defined on X are trivial; this means that every z-linear map F from X into $C(K)$ can be decomposed as $F = B(F) + L(F)$ with $B(F)$ a bounded map and $L(F)$ a linear map. Our characterization given in Theorem 1 asserts that $\mathrm{Ext}(X, C(K)) = 0$ if and only if the correspondence $F \longrightarrow L(F)$ is pointwise continuous.

The second result we present as Theorem 2 is a nonlinear generalization of Zippin's criterion [10] for the extension of $C(K)$-valued operators. Where Zippin shows, ignoring for the moment quantitative estimates, $C(K)$-valued operators on a subspace Y of a Banach space X can be extended to X if and only if there exists a certain weak*-continuous selection for functionals; we will show that $C(K)$-valued z-linear maps on Y can be extended to X if and only if there exists a certain pointwise continuous selection for scalar z-linear maps.

Received by the editors May 5, 2004.
2000 *Mathematics Subject Classification.* Primary 46M18, 46B20.
This work was supported in part by MTM2004-02635.

2. Background on z-linear maps

An exact sequence of Banach spaces is a diagram $0 \to Y \xrightarrow{j} E \xrightarrow{q} X \to 0$ formed with Banach spaces and operators in such a way that the kernel of each arrow coincides with the image of the preceding. It is sometimes called an extension of X by Y. The open mapping theorem makes Y a subspace of E through the embedding j and X the corresponding quotient space through q. An exact sequence is said to split if $j(Y)$ is complemented in X. We write $\mathrm{Ext}(X, Y) = 0$ to mean that every exact sequence with Y as subspace and X as quotient splits.

Following the theory developed by Kalton [6] and Kalton and Peck [8] in which extensions $0 \to Y \to E \to X \to 0$ of quasi-Banach spaces were identified with quasi-linear maps $F : X \to Y$, extensions of Banach spaces were represented in [4, 1] with a particular type of quasi-linear maps called z-linear maps. These are homogeneous maps such that for some constant C and every finite set of points $\{z_1, \ldots, z_n\} \subset X$ one has

$$\|F(\sum_{i=1}^{n} z_i) - \sum_{i=1}^{n} F(z_i)\| \le C \sum_{i=1}^{n} \|z_i\|.$$

The infimum of the constants C above is denoted $Z(F)$.

A z-linear map $F : X \to Y$ determines a quasi-norm on the product space $Y \times X$ given by $\|(y, x)\|_F = \|y - Fx\| + \|x\|$. Let us call this quasi-Banach space $Y \oplus_F X$. If $co(Y \oplus_F X)$ denotes its Banach envelope (i.e., the Banach space having as unit ball the closed convex hull of the unit ball of $\|\cdot\|_F$), it is easy to see that $co(Y \oplus_F X)$ is $Z(F)$-isomorphic to $Y \oplus_F X$. In this way each z-linear map induces an exact sequence $0 \to Y \xrightarrow{j_F} co(Y \oplus_F X) \xrightarrow{q_F} X \to 0$ of Banach spaces with embedding $j_F(y) = (y, 0)$ and quotient map $q_F(y, x) = x$. To obtain a z-linear map associated to a given exact sequence $0 \to Y \to E \to X \to 0$ of Banach spaces one can proceed as follows: take a homogeneous and bounded selection $b : X \to E$ for the quotient map, then take a linear selection $l : X \to E$ for the quotient map, and finally make the difference $F = b - l$, which is a z-linear map $X \to Y$. Two z-linear maps F and G are said to be *equivalent*, and written $F \equiv G$, if there is a linear map $L : X \to Y$ such that $\|F - G - L\| = \sup\{\|(F - G - L)(x)\| : \|x\| \le 1\} < +\infty$. We will sometimes say that G is a version of F. The map F is said to be trivial if $F \equiv 0$, which means that there is a linear map $L : X \to Y$ such that $\|F - L\| < +\infty$. Of course $F \equiv G$ if and only if the associated exact sequences are equivalent in the classical sense (see [4]), and F is trivial if and only if the associated sequences split.

3. Linearization and factorization of z-linear maps

It will be necessary for us to deal with the space $Z(X, Y)$ of z-linear maps $F : X \curvearrowright Y$ considered as mere functions (i.e., without equivalence relation). The space $Z(X, \mathbb{R})$ admits a semi-normed (not necessarily Hausdorff) topology induced by the seminorm $Z(\cdot)$.

In order to get the first characterization we need a refinement of Theorem 2.1 in [2]. There it was shown that for each separable space X there exists a universal exact sequence $0 \to C[0, 1] \to E \to X \to 0 \equiv U$ with the property that for each sequence $0 \to C[0, 1] \to V \to X \to 0 \equiv F$ there exists an operator $\phi : C[0, 1] \to C[0, 1]$ such that $F \equiv \phi U$. Our aim is to set an equality instead of the mere equivalence. With that purpose in mind we introduce the linearization and factorization processes, and recall Zippin's extension method for operators.

Linearization process. Given a z-linear map $F : X \curvearrowright Y$ and a Hamel basis (e_γ) for X, we define a linear map $\ell_F : X \to Y$ by setting $\ell_F(e_\gamma) = F(e_\gamma)$. The process $F \to \ell_F$ is linear. The *linearized form* of F (with respect to a given Hamel basis) is its version $F - \ell_F$. We shall call the correspondence $L : Z(X,Y) \to Z(X,Y)$ given by $L(F) = F - \ell_F$ the *linearization process* (with respect to a given Hamel basis). We shall omit from now on the coda "with respect to a given Hamel basis".

Factorization process. Consider a minimal (in the purely algebraic sense) set S^+ such that the unit sphere S of X coincides with $\bigcup_{z\in S^+}\{z,-z\}$. Naming $(e_z)_{z\in S^+}$ the canonical basis of $l_1(S^+)$, we construct the quotient map $q : l_1(S^+) \to Z$ defined by $q(e_z) = z$. A homogeneous bounded section for q comes defined as: If $p \in S^+$, then $b(p) = e_p$; the map b can be extended by homogeneity to S and then to the whole X. The z-linear map $P_X = b - \ell_b$ is associated with the exact sequence $0 \to K(X) \to l_1(S^+) \stackrel{q}{\to} X \to 0$, which we call the projective presentation of X. Given a z-linear map $F : X \curvearrowright Y$ we construct the associated sequence $0 \to Y \stackrel{j_F}{\to} co(Y \oplus_F X) \stackrel{q_F}{\to} X \to 0 \equiv F$. A homogeneous bounded selection for q_F is $B(x) = (Fx, x)$; it has norm 1 since $\|Bx\| = \|x\|_F \le \|x\|$. A linear selection for q_F is $\Upsilon(x) = (0, x)$, and one has $F = B - \Upsilon$. We define a norm one operator $\phi(F) : l_1(S^+) \to co(Y \oplus_F X)$ as

$$\phi(F)(\sum_{z\in S^+}\lambda_z e_z) = \sum_{z\in S^+}\lambda_z Bqe_z.$$

The restriction of this operator to $K(X)$ shall be called ϕ_F, and it takes values in Y. Since the norm of Y is $Z(F)$-equivalent to that of $co(Y \oplus_F X)$, we get $\|\phi_F : K(X) \to Y\| \le Z(F)$. We call the correspondence $Z(X,Y) \to \mathcal{L}(K(X),Y)$ given by $F \to \phi_F$ the *factorization process*. Observe now that $B = \phi(F)b$ because if $p \in S^+$, then $\phi(F)b(p) = \phi(F)(e_p) = Bqe_p = Bp$. Hence

$$\phi_F P_X = \phi(F)b - \phi(F)\ell_b = B - \phi(F)\ell_b = B - \Upsilon + \Upsilon - \phi(F)\ell_b = F + \Upsilon - \phi(F)\ell_b.$$

Since P_X is a linearized form so must $\phi(F)P_X$ be and therefore $\Upsilon - \phi(F)\ell_b = -\ell_F$. This means that when the factorization process is applied to a z-linear map in linearized form $G = F - \ell_F$, then one obtains $\phi_G P_X = G$.

Zippin's extension method. Let $\delta_X : X \to C(B_{X^*}, w^*)$ be the canonical embedding. As it was observed by Zippin in [10], the w^*-continuous map $\omega : B_{X^*} \to B_{C(B_{X^*})^*}$ defined by $\omega(x^*)(f) = f(x^*)$ provides an extension for every norm-one operator $T : X \to C(K)$ to $C(B_{X^*})$ through δ_X in the following way: $T^\omega(f)(k) = \omega(T^*(\delta_k))(f)$. We will say that T^ω is the Zippin extension of T.

Putting the three processes together we get:

Lemma 1. *For every Banach space X there exists a compact space $\Xi[X]$ and a z-linear map $\Delta_X : X \to C(\Xi[X])$ such that for every z-linear map $F : X \curvearrowright C(K)$ in linearized form there exists an operator $\Phi_F : C(\Xi[X]) \curvearrowright C(K)$ with norm $\|\Phi_F\| \le Z(F)$ such that $\Phi_F\Delta_X = F$.*

Proof. Set $\Xi[X] = B_{K(X)^*}$ and $\Delta_X = \delta_{K(X)}P_X$. Given $F : X \curvearrowright C(K)$ in linearized form, consider the operator ϕ_F given by the factorization process as described above. Let Φ_F be Zippin's norm one extension of $Z(F)^{-1}\phi_F$. One has

$$Z(F)\Phi_F\Delta_X = Z(F)\Phi_F\delta_{K(X)}P_X = Z(F)Z(F)^{-1}\phi_F P_X = F.$$

$\square$

4. Characterization of the spaces X such that $\mathrm{Ext}(X, C(K)) = 0$

Besides the semi-normed topology, we will consider on the space $Z(X, \mathbb{R})$ the topology w^* of pointwise convergence: we shall say that $F = w^* - \lim F_\alpha$ if for every $x \in X$ one has $F(x) = \lim F_\alpha(x)$. A map $Z(X, \mathbb{R}) \to Z(X, \mathbb{R})$ will be called w^*-continuous if it is w^*-continuous on the unit ball; namely, it transforms w^*-convergent nets on the unit ball into w^*-convergent nets. If X' denotes the algebraic dual of X, then $X' \subset Z(X, \mathbb{R})$ and the restriction of the w^*-topology to X' is the weak $w(X', X)$-topology.

The so-called "nonlinear Hahn-Banach theorem" shown in [1] asserts that given a z-linear map $F : X \to \mathbb{R}$ there exists a linear map $L \in X'$ such that $\|F - L\| \leq Z(F)$. This makes nonvacuous the following definition.

Definitions. A map $m : Z(X, \mathbb{R}) \to X'$ will be called a λ-metric projection if

$$\|F - m(F)\| \leq \lambda Z(F).$$

If we do not need to emphasize λ we just speak of metric projection. We shall say that a Banach space X admits a metric projection with a given property $\mathcal{P}$ if there is a metric projection $m : Z(X, \mathbb{R}) \to X'$ with property $\mathcal{P}$. For instance, every Banach space admits a 1-metric projection. In [3] it was shown that a Banach space X is an $\mathcal{L}_1$-space if and only if it admits a *linear* metric projection. This yields that X admits a *linear* metric projection if and only if $\mathrm{Ext}(X, Y^*) = 0$ for every dual space. A characterization of Banach spaces such that all their extensions by any $C(K)$-space are trivial can also be obtained in terms of properties of metric projections.

Theorem 1. *A Banach space X admits a w^*-continuous metric projection if and only if* $\mathrm{Ext}(X, C(K)) = 0$ *for every compact Hausdorff space K.*

Proof of the necessity. Let $F : X \curvearrowright C(K)$ be a z-linear map and let $m : Z(X, \mathbb{R}) \to X'$ be a w^*-continuous λ-metric projection. We define a linear map $M : X \to C(K)$ by

$$M(x)(k) = m(\delta_k F)(x),$$

where δ_k is the evaluation at k. The map M is well defined since $M(x)$ is a continuous function: whenever $k = \lim k_\alpha$ on K then $M(x)(k) = m(\delta_k F)(x) = m(w^* - \lim \delta_{k_\alpha} F)(x) = \lim m\left(\delta_{k_\alpha} F\right)(x) = \lim M(x)(k_\alpha)$. Moreover,

$$|F(x)(k) - M(x)(k)| = |\delta_k F(x) - m(\delta_k F)(x)| \leq \lambda Z(\delta_k F)\|x\|,$$

which implies $\|F - M\| \leq \lambda Z(F)$. $\square$

Proof of the sufficiency. If $\mathrm{Ext}(X, C(K)) = 0$ for every $C(K)$-space, then

$$\mathrm{Ext}(X, C(\Xi[X])) = 0.$$

In particular, $\Delta_X \equiv 0$ and there exists a linear map $\Lambda : X \to C(\Xi[X])$ such that $\|\Delta_X - \Lambda\| < +\infty$. Let $F : X \curvearrowright \mathbb{R}$ be a z-linear map with $Z(F) \leq 1$. Consider the projective presentation $0 \to K(X) \to l_1(S^+) \to X \to 0 \equiv P_X$ and the operator $\phi_F : K(X) \to \mathbb{R}$ such that $\phi_F P_X = F - \ell_F$, given by the factorization process. Let

$\phi_F^\omega : C(\Xi[X]) \to \mathbb{R}$ be its Zippin's extension. We define a map $m : Z(X, \mathbb{R}) \to X'$ as

$$m(F) = \phi_F^\omega \Lambda.$$

To prove that $m(\cdot)$ is w^*-continuous we decompose it in three applications:

(1) The linearization process $L(F) = F - \ell_F$, which is w^*-continuous.

(2) The factorization process. We show now it is w^*-continuous. Observe that if we restrict ourselves to work with the subspace $\varphi_0(S^+)$ of $l_1(S^+)$ of all finitely supported sequences, then $K_0 = K(X) \cap \varphi_0(S^+)$ is dense in $K(X)$. On this dense subspace the operator ϕ_F takes the form

$$\phi_F(\sum \lambda_z e_z) = \sum \lambda_z F q e_z.$$

Thus, if $\{G_\alpha\}$ is a net in the unit ball such that $G = w^* - \lim G_\alpha$, then $\phi_G(u) = \lim \phi_{G_\alpha(u)}$ for all $u \in K_0$. Since $\|\phi_{G_\alpha}\| \leq 1$, the sequence of uniformly bounded operators is w^*-convergent on the whole $K(X)$.

(3) Zippin's extension method is a w^*-continuous process $B_{K(X)^*} \to B_{C(\Xi[X])^*}$, as we have already remarked.

The process

$$F \to m(F) + \ell_F$$

is obviously w^*-continuous, and it remains to show that it is a metric projection:

$$\|F - m(F) - \ell_F\| = \|\Phi_F \Delta_X - \Phi_F \Lambda\| \leq \|\phi_F\| \|\Delta_X - \Lambda\| \leq Z(F) \|\Delta_X - \Lambda\|.$$

□

Remark about the role of the $C(K)$-space. It is not difficult to see that for separable spaces X the condition "$\mathrm{Ext}(X, C(K)) = 0$ for all compact spaces K" is equivalent to "$\mathrm{Ext}(X, C[0,1]) = 0$". It has been shown in [2] that $\mathrm{Ext}(X, C(\omega^\omega)) = 0$ is a strictly weaker condition; precisely, if T denotes the dual of the original Tsirelson space, then $\mathrm{Ext}(T, C[0,1]) \neq 0$, while $\mathrm{Ext}(T, C(\omega^\omega)) = 0$. It seems a touchy question how metric projections would reflect the change of the $C(K)$ target space. For instance, it is clear that, without separability assumptions, if X admits a metric projection which is w^*-continuous at 0, then $\mathrm{Ext}(X, c_0) = 0$.

Question. Is it true that $\mathrm{Ext}(X, c_0) = 0$ if and only if X admits a metric projection w^*-continuous at 0? In particular: Does every separable Banach space admit a metric projection w^*-continuous at 0?

sectionOn the extension of z-linear maps

Let $j : Y \to X$ be an embedding. We observe that the induced map $j^* : Z(X, \mathbb{R}) \to Z(Y, \mathbb{R})$ is surjective since every scalar-valued z-linear map can be extended to any superspace: since z-linear maps $Y \to \mathbb{R}$ are trivial, $F = B + L$, just taking $m : X \to Y$ a homogeneous bounded projection, $l : X \to Y$ a linear projection and then putting $\widehat{F} = Bm + Ll$. Given $\lambda > 0$, we shall say that a map $\omega : B_{Z(Y,\mathbb{R})} \to \lambda B_{Z(X,\mathbb{R})}$ is a w^*-*selector* for j^* if ω is w^*-continuous and verifies $j^* \omega = id$.

We now consider an extension $0 \to Y \xrightarrow{j} X \xrightarrow{q} Z \to 0 \equiv F$. We are going to connect the existence of a w^*-continuous selector for j^* with the extension problem for $C(K)$-valued z-linear maps through j. It will be helpful to consider the subspace $ZL(X, \mathbb{R})$ of $Z(X, \mathbb{R})$ of maps in linearized form (with respect to a given fixed Hamel basis). We begin with an easy observation.

Lemma 2. *There is a w^*-continuous selector $\omega : B_{Z(Y,\mathbb{R})} \to \lambda B_{Z(X,\mathbb{R})}$ for j^* if and only if there is a w^*-continuous selection $\omega' : B_{ZL(Y,\mathbb{R})} \to \lambda B_{ZL(X,\mathbb{R})}$ for $j^* : ZL(X,\mathbb{R}) \to ZL(Y,\mathbb{R})$.*

Proof. It is clear that if we have a w^*-selector ω for $j^* : Z(X,\mathbb{R}) \to Z(Y,\mathbb{R})$, then $\omega'(F) = \omega(F) - L_{\omega(F)}$ is a w^*-continuous selection for $j^* : ZL(X,\mathbb{R}) \to ZL(Y,\mathbb{R})$. Conversely, if there exists a w^*-continuous selection ω' for $j^* : ZL(X,\mathbb{R}) \to ZL(Y,\mathbb{R})$ (which we shall also call w^*-selector), then $\omega(F) = \omega'(F - L_F) + L_F l$ (where $l : X \to Y$ is a linear retraction for j) is the new selector we were looking for. $\square$

Theorem 2 (Zippin's lemma for z-linear maps)**.** *Let $0 \to Y \xrightarrow{j} X \xrightarrow{q} Z \to 0 \equiv G$. Every z-linear map $F : Y \curvearrowright C(K)$ extends to a z-linear map $F^\omega : X \curvearrowright C(K)$ with $Z(F^\omega) \leq \lambda Z(F)$ if and only if there is a w^*-selector $\omega : B_{Z(Y,\mathbb{R})} \to \lambda B_{Z(X,\mathbb{R})}$ for j^*.*

Proof. Let $\omega : B_{ZL(Y,\mathbb{R})} \to \lambda B_{ZL(X,\mathbb{R})}$ be a w^*-selector for j^*. Given a z-linear map $F : Y \curvearrowright C(K)$ with $Z(F) \leq 1$, we can define the extension $F^\omega : X \curvearrowright C(K)$ of F through j:

$$F^\omega(x)(k) = \omega(\delta_k F)(x).$$

The map F^ω is well defined since, for each $x \in X$, $F^\omega(x)$ is a continuous function on K. Moreover, F^ω is a z-linear map in linearized form: the z-linearity of F^ω is a consequence of the estimate

$$Z(F^\omega) \leq \sup\{Z(\omega(\delta_k F) : k \in K\} \leq \lambda Z(F).$$

It is finally clear that F^ω extends F.

To obtain the reciprocal, and by virtue of Lemma 1, it is only necessary to show how to extend the map $\Delta_Y : Y \curvearrowright C(\Xi[Y])$. Assume then that $\nabla : X \curvearrowright C(\Xi[Y])$ is a z-linear extension of Δ_Y through j. We can define a map $\omega : B_{ZL(Y,\mathbb{R})} \to Z(\nabla)B_{ZL(X,\mathbb{R})}$ as

$$\omega(F) = \phi_F \nabla.$$

The map ω is clearly well defined and $j^*\omega = id$; moreover, ω is w^*-continuous since the factorization process is w^*-continuous. $\square$

The reader might have been surprised by the fact that a w^*-continuous extension process is not automatically guaranteed. The problem is that the simple method of extension discussed at the beginning of this section, $F \to \widehat{F}$ assigning to $F = B + L$ the extension $\widehat{F} = Bm + Ll$, is not, in general, w^*-continuous; the reason is that from $F = w^* - \lim F_n$ and $F_n = B_n + L_n$ it does not automatically follow that $L = w^* - \lim L_n$. There is a way to try to obtain that: choosing as L an optimal approximation to F, i.e., $\|B\| = \|F - L\| \leq Z(F)$ (see [1]) and since m can be chosen verifying $\|m\| \leq 1 + \varepsilon$ we get the estimate

$$Z(\widehat{F}) = Z(Bm) \leq \|B\|\|m\| \leq Z(F)(1 + \varepsilon).$$

All this means that in order to have a w^*-continuous extension process it would be enough to have a w^*-continuous metric projection $Z(Y,\mathbb{R}) \to Y'$. However, as we have already shown, this means exactly $\mathrm{Ext}(Y, C(K)) = 0$ for all $C(K)$-spaces, which clearly explains why there is an extension in such a case.

References

[1] F. Cabello Sánchez and J.M.F. Castillo, *Duality and twisted sums of Banach spaces,* J. Funct. Anal. 175 (2000) 1–16. MR1774849 (2001i:46011)

[2] F. Cabello Sánchez, J.M.F. Castillo, N. J. Kalton and D. Yost, Twisted sums with $C(K)$-spaces, Trans. Amer. Math. Soc. 355 (2003) 4523–4541. MR1990760 (2004f:46010)

[3] F. Cabello Sánchez, J.M.F. Castillo and F. Sánchez, *Nonlinear metric projections in twisted twilight*, Rev. Real Acad. Cienc. Exactas Fís Nat. (Esp.) 94 (2000) 473–483. MR1865749 (2002j:46004)

[4] J.M.F. Castillo and M. González, *Three-space problems in Banach space theory,* Lecture Notes in Math. 1667, Springer-Verlag, Berlin, 1997. MR1482801 (99a:46034)

[5] W.B. Johnson and M. Zippin, *Extension of operators from weak*-closed subspaces of l_1 into $C(K)$ spaces*, Studia Math. 117 (1995) 43–55. MR1367692 (96k:46025)

[6] N. J. Kalton, *The three-space problem for locally bounded F-spaces*, Compos. Math. 37 (1978) 243–276. MR0511744 (80j:46005)

[7] N. Kalton, *On subspaces of c_0 and extension of operators into $C(K)$-spaces,* Quart. J. Math. Oxford 52 (2001) 313–328. MR1865904 (2002k:46045)

[8] N. J. Kalton and N. T. Peck, *Twisted sums of sequence spaces and the three space problem*, Trans. Amer. Math. Soc. 255 (1979) 1–30. MR0542869 (82g:46021)

[9] N. Kalton, A. Pełczyński, *Kernels of surjections from $\mathcal{L}_1$-spaces with an application to Sidon sets,* Math. Ann. 309 (1997) 135–158. MR1467651 (98h:46013)

[10] M. Zippin, *The embedding of Banach spaces into spaces with structure*, Illinois J. Math. 34 (1990) 586–606. MR1053564 (91f:46017)

Departamento de Matemáticas, Universidad de Extremadura, Avenida de Elvas, 06071 Badajoz, Spain

E-mail address: castillo@unex.es

Departamento de Matemáticas, Universidad de Extremadura, Avenida de Elvas, 06071 Badajoz, Spain

E-mail address: ymoreno@unex.es