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ERRATUM TO "BAIRE SPACES AND VIETORIS HYPERSPACES"

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Abstract. The main purpose of this short note is to correct an error in "Baire spaces and Vietoris hyperspaces", Proc. Amer. Math. Soc. **135** (2007), no. 1, 299–303.

In our article [2], on page 301, the proof of Theorem 2.1 contains a gap that the map $f : X^n \to \mathcal{F}_n(X)$ is open. We are grateful to Dariusz Tywoniuk, who communicated to us with the following arguments: Take $n = 3$ and two non-empty disjoint open subsets $U, V \subset X$. Then,

$$f(U \times V \times V) = \big\{S \in \langle\{U, V\}\rangle \cap \mathcal{F}_3(X) : |S \cap U| = 1\big\}.$$

In case $x \in U$ is a non-isolated point and $y \in V$, then $f(x, y, y) = \{x, y\} \in f(U \times V \times V)$, but any τ_V-neighbourhood of $\{x, y\}$ will contain an $S \in \mathcal{F}_3(X)$, with $|S \cap U| = 2$.

The gap can be overcome using the authors' original arguments to prove Theorem 2.1; see [1]. In fact, the proof that appeared in print was suggested by the referee and was intended to shorten the original one.

The authors' arguments in [1] involve two auxiliary sets as an interface between $\mathcal{F}_n(X)$ and X^n:

$$[X]^n = \{S \in \mathcal{F}_n(X) : |S| = n\}$$

and

$$\mathcal{D}(X^n) = \big\{(x_1, \dots, x_n) \in X^n : |\{x_1, \dots, x_n\}| = n\big\},$$

where $n \geq 1$. Here, X^1 is identified with X; hence $\mathcal{D}(X^1) = X^1 = X$. Also, $\mathcal{D}(X^n)$ will always carry the relative topology from X^n, and, as in [2], all spaces X are assumed to be Hausdorff and infinite.

The following two observations now reduce the proof of Theorem 2.1 to only the sets $[X]^n$ and $\mathcal{D}(X^n)$.

Proposition 2.1. *Let X be a space and $n \geq 1$. Then, $\mathcal{F}_n(X)$ is a Baire space if and only if $[X]^n$ is a Baire space.*

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Proof. The set $[X]^n$ is τ_V-open in $\mathcal{F}_n(X)$ because X is Hausdorff. Hence, $[X]^n$ is a Baire space if $\mathcal{F}_n(X)$ is. Suppose that $[X]^n$ is a Baire space, and take τ_V-open dense subsets $\mathcal{V}_k \subset \mathcal{F}_n(X)$, $k < \omega$. Next, set $\mathcal{G} = \bigcap\{\mathcal{V}_k : k < \omega\}$. Finally, take a finite family $\mathcal{W}$ of non-empty open subsets of X, with $\langle\mathcal{W}\rangle \cap \mathcal{F}_n(X) \neq \varnothing$. To show that $\langle\mathcal{W}\rangle \cap \mathcal{G} \neq \varnothing$, consider the following cases. If each $W \in \mathcal{W}$ consists only of isolated points of X, then $\langle\mathcal{W}\rangle \cap \mathcal{F}_n(X)$ consists only of isolated points of $\mathcal{F}_n(X)$, so $\langle\mathcal{W}\rangle \cap \mathcal{F}_n(X) \subset \mathcal{G}$. If some $W \in \mathcal{W}$ contains a non-isolated point of X, then $\mathcal{H} = \langle\mathcal{W}\rangle \cap [X]^n \neq \varnothing$. However, each $\mathcal{U}_k = \mathcal{V}_k \cap [X]^n$, $k < \omega$, is open and dense in the Baire space $[X]^n$. Hence, $\mathcal{D} = \bigcap\{\mathcal{U}_k : k < \omega\}$ is τ_V-dense in $[X]^n$. This implies that $\varnothing \neq \mathcal{H} \cap \mathcal{D} \subset \langle\mathcal{W}\rangle \cap \mathcal{G}$, which completes the proof. □

Proposition 2.2. *Let X be a space and $n \geq 1$. Then, X^n is a Baire space if and only if $\mathcal{D}(X^n)$ is a Baire space.*

Proof. The set $\mathcal{D}(X^n)$ is open in X^n; hence it is a Baire space if X^n is. To show the converse, suppose that $\mathcal{D}(X^n)$ is Baire. Then, each $\mathcal{D}(X^m)$, $1 \leq m \leq n$, is also a Baire space because the natural projection $P_m : X^n \to X^m$ onto the first m coordinates is an open continuous map, with $P_m(\mathcal{D}(X^n)) = \mathcal{D}(X^m)$. Thus, $\mathcal{D}(X^1) = X^1$ is Baire, and the proof can proceed by induction if $n > 1$. Namely, suppose that X^m is a Baire space for some $m < n$, with $1 \leq m$. To show that X^{m+1} is also a Baire space, let $G = \bigcap\{V_k : k < \omega\}$ for some open dense subsets $V_k \subset X^{m+1}$, $k < \omega$, and let $W = W_1 \times \cdots \times W_{m+1} \subset X^{m+1}$ be a non-empty open subset. Consider the following cases. There exists a $j \in \{1, \ldots, m+1\}$ such that W_j contains an isolated point x. In this case, let $Y = \pi_j^{-1}(x)$ where $\pi_j : X^{m+1} \to X$ is the projection onto the jth-factor of the product X^{m+1}. Then, Y is homeomorphic to X^m, and it is an open subset of X^{m+1} because x is an isolated point of X. Therefore each $U_k = V_k \cap Y$, $k < \omega$, is open and dense in Y, and, by assumption, $D = \bigcap\{U_k : k < \omega\}$ is also dense in Y. So, $\varnothing \neq W \cap D \subset W \cap G$ because $W \cap Y \neq \varnothing$. Consider finally the case when each W_j, $1 \leq j \leq m+1$, consists only of non-isolated points. In this case, $T = W \cap \mathcal{D}(X^{m+1}) \neq \varnothing$, while each $L_k = V_k \cap \mathcal{D}(X^{m+1})$, $k < \omega$, is open and dense in $\mathcal{D}(X^{m+1})$ because $\mathcal{D}(X^{m+1})$ is open in X^{m+1}. Hence, $R = \bigcap\{L_k : k < \omega\}$ is dense in $\mathcal{D}(X^{m+1})$ because $\mathcal{D}(X^{m+1})$ is a Baire space. Thus, $\varnothing \neq T \cap R \subset W \cap G$, which completes the proof. □

Proof of Theorem 2.1. By Propositions 2.1 and 2.2, it suffices to show that $\mathcal{D}(X^n)$ is a Baire space if and only if $[X]^n$ is a Baire space. Consider the map $f : \mathcal{D}(X^n) \to [X]^n$ defined by $f(x_1, \ldots, x_n) = \{x_1, ..., x_n\}$, $(x_1, \ldots, x_n) \in \mathcal{D}(X^n)$. Then, f is a continuous, open and finite-to-one surjection; hence [2, Proposition 2.2] completes the proof. □

References

[1] J. Cao, S. García-Ferreira, and V. Gutev, *Barely Baire spaces and hyperspaces*, University of Auckland, Department of Mathematics Report Series, No. 522, 2004 (`http://www.math.auckland.ac.nz/Research/Reports/view.php?id=522`).

[2] ———, *Baire spaces and Vietoris hyperspaces*, Proc. Amer. Math. Soc. **135** (2007), no. 1, 299–303. MR2280197 (2007i:54059)

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