

CANONICAL COVERS AND DIMENSION OF Z-SETS IN THE HILBERT CUBE

E. CUCHILLO-IBÁÑEZ AND M. A. MORÓN

(Communicated by Alexander N. Dranishnikov)

ABSTRACT. In this paper we characterize the finite dimensionality of a compact Z -set in the Hilbert cube in terms of the existence of a particular canonical cover in its complement.

1. INTRODUCTION

In [1] it was suggested that topological invariants of compact Z -sets of the Hilbert cube, Q , could be described in terms of metric-uniform invariants of their complements. In particular it was asked how the dimension can be described in this context. Later, in [4] it was proved that the topological category for compact Z -sets of the Hilbert cube is isomorphic to the C_0 -coarse geometry category of their complements. The C_0 -coarse structure of a metric space was introduced by Wright [11] (see also [12]). We recommend [10] for the basic definitions of general coarse spaces.

One of the consequences of [4] is that the complements of two compact Z -sets are C_0 -coarsely equivalent if and only if those complements are uniformly homeomorphic (with respect to the metrics induced by that fixed on the Hilbert cube) and it is equivalent to the fact that the two compact Z -sets are homeomorphic.

In view of the equivalence of categories described in [4], it is natural to consider the problem of describing topological invariants of Z -sets in terms of C_0 -coarse invariants of their complements. In this way the asymptotic dimension of such a coarse structure on the complement appears to be an appropriate tool for describing, from the outside, the topological dimension of a Z -set. Indeed in [4] it was proved that the dimension of a Z -set can be interpreted in terms of the asymptotic dimension of the C_0 -coarse structure on its complement. See [10] for the definition of asymptotic dimension of a coarse space.

Instead of the C_0 -asymptotic dimension, in this paper we use the well-known concept of *canonical covers*, used by Dugundji in [5], to characterize the finite-dimensionality of a Z -set or, equivalently, the finiteness of the asymptotic dimension of the C_0 -coarse structure in its complement. We then point out that although the tools used in the definition of canonical cover are topological in nature, in fact this concept is in the core of the C_0 -coarse geometry.

Received by the editors October 12, 2006, and, in revised form, August 20, 2007.

2000 *Mathematics Subject Classification*. Primary 54F45, 52B11; Secondary 54E15.

Key words and phrases. Compact Z -set, canonical cover, nerve of a cover, covering dimension.

The authors were supported by the MEC, MTM2006-0825.

©2008 American Mathematical Society
Reverts to public domain 28 years from publication

First we show how to project to the outside certain cofinal sequences of open covers of the Z -set, hence constructing canonical covers of its complements.

Later, in the usual way, we replace this complement by a countable locally finite polyhedron where a topological copy of the Z -set can be viewed as its end space. This construction is an extension of the fact that 0-dimensional metrizable compacta can be represented as the end space of a locally finite simplicial tree.

The general result we obtain is that a compact Z -set is finite-dimensional if and only if its complement admits a canonical cover of finite order. We also relate the dimension with the order of the canonical cover. Kroonenberg [9] obtained some dual related results. In particular she studied, in terms of the complement, when a finite dimensional closed subset of the Hilbert cube is a Z -set.

For the reader's convenience we recommend [3] for the study of Z -sets; see also [6, 8] for dimension theory and [10] for a global view of coarse geometry concepts quoted, but not used, in this paper.

The authors are very grateful to the referee whose suggestions improved significantly an earlier version of this paper.

2. CANONICAL COVERS AND DIMENSION FROM THE OUTSIDE

Our final intention is to interpret the inequality $\dim X \leq n$ in terms of $Q \setminus X$, where $X \subset Q$ is a compact Z -set in the Hilbert cube Q . Recall that a closed subset A in the Hilbert cube (Q, d) is said to be a Z -set in Q provided that for every $\epsilon > 0$ there exists a continuous map $f : Q \rightarrow Q \setminus A$ such that $d(f(p), p) < \epsilon$ for each $p \in Q$. Moreover, any compact metric space can be embedded in the Hilbert cube as a Z -set.

Definition 2.1. Let X be a topological space and A a closed subset of X . A *canonical cover* of $X \setminus A$ in X , γ , is an open cover of $X \setminus A$ such that:

- i) γ is a locally finite family, and
- ii) for each neighborhood V , in X , of a point $a \in A$ there exists another neighborhood W of a in X , $W \subset V$, such that every element $U \in \gamma$ which meets W is contained in V .

This concept was used by Dugundji [5] to obtain his famous generalization of Tietze's Extension Theorem. As Antonyan noted in [2], the second condition implies that every neighborhood of any boundary point of A in X contains infinitely many open sets in γ . This was a requisite to be a canonical cover in its original definition (see [5]). Other useful results obtained using these techniques can be found in the book [7].

To treat the dimension from the outside, that is from the complement of a Z -set, we need some preliminary results:

Proposition 2.2. *Let X be a compact subset of Q . For each canonical cover γ of $Q \setminus X$ and every open set $G \subset Q$, with $X \subset G$, the family of sets of γ that have empty intersection with G is finite.*

Proof. Assume that there exists a sequence $\{U_n\}_{n \in \mathbb{N}}$ of different elements of γ such that, for every $n \in \mathbb{N}$, $U_n \cap G = \emptyset$. Consider a sequence $\{x_n\}_{n \in \mathbb{N}}$ of points such that $x_n \in U_n$. As this sequence is contained in the compact $Q \setminus G$, the sequence has an accumulation point $z_0 \in Q \setminus G \subset Q \setminus X$. In this way, every neighborhood of z_0 intersects an infinite number of members of the canonical cover γ , which is not possible. \square

Similar arguments as those in the above proposition lead us to the following

Proposition 2.3. *Let X be a compact subset of Q . For each canonical cover γ of $Q \setminus X$ and every open set $G \subset Q$ such that $X \subset G$, the family of sets of γ that are not contained in G is finite.*

Remark 2.4. Note that the results above say that the canonical covers in $Q \setminus X$ are in fact *uniform covers* (see Definition 5.29 in [10]) for the C_0 -coarse structure in $Q \setminus X$.

Another property we will use is

Proposition 2.5. *Let X be a compact subset of Q . Each member of a canonical cover γ of $Q \setminus X$ intersects only a finite family of elements of γ .*

Proof. Let $U_0 \in \gamma$. As γ is a canonical cover of $Q \setminus X$, $\overline{U_0} \cap X = \emptyset$, where $\overline{U_0}$ means the closure of U_0 in Q .

Define $G = Q \setminus \overline{U_0}$. Observe that G is open in Q and $X \subset G$. As every $U \in \gamma$ that intersects U_0 satisfies $U \not\subset G$, from Proposition 2.3 they are a finite collection. \square

To fix the notation we choose the following definition of order.

Definition 2.6. Let X be a set and \mathcal{A} a family of subsets of X . By the *order* of the family \mathcal{A} , $\text{ord } \mathcal{A}$, it means the largest integer n such that there are $n + 1$ members of the family \mathcal{A} which have a non-empty intersection. If no such integer exists, we say that \mathcal{A} has order ∞ .

Proposition 2.7. *Let X be a compact subspace of the Hilbert cube (Q, d) and $\mathcal{U} = \{U_i\}_{i=1}^m$ be a finite open cover of X such that $\text{ord}(\mathcal{U}) \leq n$. For every $\varepsilon > 0$ there exists a family (an ε -envelopment) $\mathcal{W} = \{W_i\}_{i=1}^m$ of open subsets of Q that satisfies:*

- a) $W_i \cap X = U_i$ for each $i \in \{1, \dots, m\}$.
- b) $W_i \subset B(U_i, \varepsilon)$ and, so, $\text{diam}(W_i) \leq \text{diam}(U_i) + 2\varepsilon$ for each $i \in \{1, \dots, m\}$.
- c) For every subset $\Lambda \subset \{1, \dots, m\}$, $\bigcap_{r \in \Lambda} W_r \neq \emptyset$ if and only if $\bigcap_{r \in \Lambda} U_r \neq \emptyset$.
In particular $\text{ord}(\mathcal{W}) = \text{ord}(\mathcal{U})$.

Proof. Choose an index $i \in \{1, \dots, m\}$. If $U_i = X$, take $W_i = B(X, \frac{\varepsilon}{2})$. In the general case, $U_i \neq X$, for every $x \in U_i$ consider

$$\varepsilon_{x,i} = \frac{1}{2} \min\{d(x, X \setminus U_i), \varepsilon\}.$$

Clearly $B(x, \varepsilon_{x,i}) \cap X \subset U_i$. Define

$$W_i = \bigcup_{x \in U_i} B(x, \varepsilon_{x,i}).$$

By its construction,

$$W_i \cap X = U_i \quad \text{and} \quad W_i \subset B(U_i, \frac{\varepsilon}{2}).$$

When $U_{j_1} \cap \dots \cap U_{j_k} \neq \emptyset$, $j_1, \dots, j_k \in \{1, \dots, m\}$, then $W_{j_1} \cap \dots \cap W_{j_k} \neq \emptyset$. Reciprocally, if $W_{j_1} \cap \dots \cap W_{j_k} \neq \emptyset$ and $U_{j_r} \neq X$ for all $r \in \{1, \dots, k\}$ (the other option is reducible to this one), let $z \in W_{j_1} \cap \dots \cap W_{j_k}$. There exist $x_1 \in U_{j_1}, \dots, x_k \in U_{j_k}$ such that

$$z \in B(x_1, \varepsilon_{x_1,j_1}) \cap \dots \cap B(x_k, \varepsilon_{x_k,j_k}).$$

Suppose that $s \in \{1, \dots, k\}$ satisfies

$$\varepsilon_{x_s, j_s} = \min\{\varepsilon_{x_r, j_r} : r \in \{1, \dots, k\}\}.$$

For all $r \in \{1, \dots, k\}$, $r \neq s$, we have

$$d(x_r, x_s) \leq d(x_r, z) + d(z, x_s) < \varepsilon_{x_r, j_r} + \varepsilon_{x_s, j_s} \leq 2\varepsilon_{x_r, j_r}.$$

From the definition of ε_{x_r, j_r} , we have that $x_s \in U_{j_r}$ for every $r \in \{1, \dots, k\}$. In such a way,

$$x_s \in U_{j_1} \cap \dots \cap U_{j_k} \neq \emptyset.$$

Finally, we conclude that

$$\text{ord}(\{W_i\}_{i=1}^m) = \text{ord}(\{U_i\}_{i=1}^m). \quad \square$$

Remark 2.8. Open sets of Q belonging to envelopments (constructed as in the preceding proposition) of different open covers of X have a non-empty intersection if and only if the original members of the open covers of X also have a non-empty intersection.

Proposition 2.9. *Consider the Hilbert cube with a fixed metric (Q, d) . Let $X \subset Q$ be a compact subset with $\dim(X) = n$. Then there is a canonical cover γ of $Q \setminus X$ with $\text{ord}(\gamma) \leq 2n + 1$.*

Proof. Consider a sequence $\{\mathcal{U}_m, m \in \mathbb{N}\}$ of finite open coverings of X with the following properties:

1. $\mathcal{U}_1 = \{X\}$ and $\mathcal{U}_{m+1} \geq \mathcal{U}_m$ for every $m \in \mathbb{N}$. That is, \mathcal{U}_{m+1} is a refinement of \mathcal{U}_m .
2. $\lim_{m \rightarrow \infty} \text{mesh}(\mathcal{U}_m) = 0$, where mesh represents the maximum of the diameters of all elements in \mathcal{U}_m with respect to the inherit metric, and the sequence $\{\text{mesh}(\mathcal{U}_m)\}_{m \in \mathbb{N}}$ is strictly decreasing.
3. $\text{ord}(\mathcal{U}_m) \leq n$ for every $m \in \mathbb{N}$.

For each $m \in \mathbb{N}$, take a $1/m$ -envelopment $\mathcal{W}_m = \{W_{m,j}\}_{j=1}^{k_m}$ of the cover $\mathcal{U}_m = \{U_{m,j}\}_{j=1}^{k_m}$ as described in the above proposition. We can suppose that if $B_m = \bigcup_{j=1}^{k_m} W_{m,j}$, then $\{B_m, m \in \mathbb{N}\}$ is a base of open neighborhoods of X in Q such that $\overline{B_{m+1}} \subset B_m$ for every $m \in \mathbb{N}$ and that $\overline{B_2} \neq Q$. Define

$$\beta_{1,1} = Q \setminus \overline{B_2},$$

$$\beta_m = \{\beta_{m,j} = W_{m-1,j} \setminus \overline{B_{m+1}}, j = 1, \dots, k_{m-1}\} \text{ for } m \geq 2.$$

We will show that the family

$$\gamma = \{\beta_{p,i} : p \in \mathbb{N}, i = 1, \dots, k_p\}$$

is a canonical cover of $Q \setminus X$, where $k_1 = 1$. In fact, it is clear that γ is an open cover of $Q \setminus X$. Since the families β_m are finite sets, $\{B_m, m \in \mathbb{N}\}$ is a base of open neighborhoods of X and, for all m , $\overline{B_{m+1}} \subset B_m$ and $\beta_{m,j} \cap \overline{B_{m+1}} = \emptyset$, $j \in \{1, \dots, k_{m-1}\}$, then γ is a locally finite family. Given $x_0 \in X$ and an open neighborhood of x_0 in Q , A_{x_0} , from the properties of the envelopments \mathcal{W}_m there exists $m_0 \in \mathbb{N}$ which is the minimum of the numbers $m \in \mathbb{N}$ which satisfy the following property: “if $x_0 \in W_{m,j}$, then $W_{m,j} \subset A_{x_0}$ ”. If we choose V_{x_0} equal to any of the $W_{p,j}$, for $p \geq m_0 + 2$, containing x_0 , it is easy to verify that if an element of γ meets V_{x_0} , then it is contained in A_{x_0} .

Finally, $\text{ord}(\gamma) \leq 2n + 1$, because for each p there are at most $n + 1$ elements of β_p with a non-empty intersection and if $\beta_{p,i} \cap \beta_{l,j} \neq \emptyset$, necessarily $|p - l| \leq 1$. \square

Remark 2.10. Note that in the above result there is a gap between the dimension of the Z -set, n , and the order of the constructed canonical cover, $2n + 1$. The gap is just $n + 1$. In the case $n = 0$, this gap must be at least 1. Moreover, using the fact that the family of partitions is cofinal for finite open covers in 0-dimensional compacta, the above construction can be refined to obtain the classical result that those spaces are just the end spaces of locally finite simplicial trees.

Can we always find a canonical cover with gap 1? In a work in progress we are studying this question, but until now we have only been able to prove that the answer to this question is affirmative provided $n = 0, 1$.

In order to obtain a reciprocal result we first note:

Let γ be a canonical cover of $Q \setminus X$, $\mathcal{N}(\gamma)$ the nerve of γ , and consider the space $\mathcal{N}(\gamma) \cup X$ topologized as it is indicated in [5, Theorem 3.1] (see also [7, Proposition II.12.1]). Specifically, a basis of open sets for this topology is given by

- the open subsets of $\mathcal{N}(\gamma)$ endowed with the Whitehead topology and
- the subsets V^* constructed as follows: for each point $x \in X$ and each open neighborhood V of x in Q , the set V^* in $\mathcal{N}(\gamma) \cup X$ consists of the points of $X \cap V$ and those of the stars of every vertex of $\mathcal{N}(\gamma)$ corresponding to a set of the covering γ contained in V .

We denote by

$$\pi : Q \longrightarrow \mathcal{N}(\gamma) \cup X$$

the continuous map built in [5, Theorem 3.1] which is the identity, and a homeomorphism, over X .

In this case we obtain

Proposition 2.11. *The space $\mathcal{N}(\gamma) \cup X$ is compact and metrizable.*

Proof. Let $\mathcal{G} = \{G_\lambda\}_{\lambda \in \Lambda}$ be an open cover of $\mathcal{N}(\gamma) \cup X$.

Since X is compact, there exists a finite number of elements of \mathcal{G} which cover X ,

$$X \subset G_1 \cup \dots \cup G_m,$$

and there exists a finite number of basic open sets V_1^*, \dots, V_n^* such that

$$X \subset V_1^* \cup \dots \cup V_n^* \subset G_1 \cup \dots \cup G_m.$$

As

$$(\mathcal{N}(\gamma) \cup X) \setminus (G_1 \cup \dots \cup G_m) \subset (\mathcal{N}(\gamma) \cup X) \setminus (V_1^* \cup \dots \cup V_n^*) \subset \mathcal{N}(\gamma),$$

it suffices to see that the closed subset

$$(\mathcal{N}(\gamma) \cup X) \setminus (V_1^* \cup \dots \cup V_n^*) = \mathcal{N}(\gamma) \setminus (V_1^* \cup \dots \cup V_n^*)$$

is compact. Following their definition, for each $i \in \{1, \dots, n\}$,

$$V_i^* = \pi(V_i \cap X) \cup \bigcup_{U \in \gamma, U \subset V_i} St(v_U),$$

where V_i is an open set in Q , v_U is the vertex in $\mathcal{N}(\gamma)$ corresponding to $U \in \gamma$ and $St(v_U)$ is the star of v_U in the infinite polyhedron $\mathcal{N}(\gamma)$. Clearly

$$X \subset V_1 \cup \dots \cup V_n.$$

We will show there are a finite number of vertices of $\mathcal{N}(\gamma)$ in $\mathcal{N}(\gamma) \setminus (V_1^* \cup \dots \cup V_n^*)$. In fact, since X is compact and γ is a canonical cover of $Q \setminus X$, there exist W_1, \dots, W_l , open subsets of Q , such that they have a non-empty intersection with

X , they cover X and if $U \in \gamma$ intersects some of them, then U is contained in V_i for certain $i \in \{1, \dots, n\}$. Take $U_0 \in \gamma$ and its associated vertex $v_{U_0} \in \mathcal{N}(\gamma)$. The following are equivalent:

- i) $v_{U_0} \notin V_1^* \cup \dots \cup V_n^*$.
- ii) For all $i \in \{1, \dots, n\}$, $v_{U_0} \notin \bigcup_{U \in \gamma, U \subset V_i} St(v_U)$.
- iii) For every $i \in \{1, \dots, n\}$, $U_0 \not\subset V_i$.

Hence, when $v_{U_0} \notin V_1^* \cup \dots \cup V_n^*$, then

$$U_0 \cap (W_1 \cup \dots \cup W_l) = \emptyset.$$

In this manner, as $W_1 \cup \dots \cup W_l$ is an open set of Q that contains X , by Proposition 2.2 there is a finite collection of elements of the canonical cover γ which have an empty intersection with $W_1 \cup \dots \cup W_l$. Therefore, in $\mathcal{N}(\gamma) \setminus (V_1^* \cup \dots \cup V_n^*)$ there is a finite number of vertices of $\mathcal{N}(\gamma)$.

Now we classify the vertices of $\mathcal{N}(\gamma)$ into two classes:

- a) Vertices that are not in $V_1^* \cup \dots \cup V_n^*$. There are only a finite number of them: v_{U_1}, \dots, v_{U_k} .
- b) Vertices v_U that are in $V_1^* \cup \dots \cup V_n^*$, and, therefore, there exists V_i , $i \in \{1, \dots, n\}$, such that $U \subset V_i$.

Obviously, when all the vertices are considered:

$$\mathcal{N}(\gamma) = \bigcup_{j=1}^k St(v_{U_j}) \cup \bigcup_{i=1}^n \bigcup_{U \in \gamma, U \subset V_i} St(v_U).$$

So

$$\mathcal{N}(\gamma) \setminus (V_1^* \cup \dots \cup V_n^*) = \mathcal{N}(\gamma) \setminus \bigcup_{i=1}^n \bigcup_{U \in \gamma, U \subset V_i} St(v_U) \subset \bigcup_{j=1}^k St(v_{U_j}) \subset \bigcup_{j=1}^k \overline{St}(v_{U_j}),$$

where \overline{St} denotes the closed star.

Finally, due to Proposition 2.5, as each closed star is a finite polyhedron they are compact subspaces. Hence, the closed subset $\mathcal{N}(\gamma) \setminus (V_1^* \cup \dots \cup V_n^*)$ is contained in a finite union of compact sets and it is also compact.

As was pointed out in [5] it is not difficult to verify that the space $\mathcal{N}(\gamma) \cup X$, with the mentioned topology, is a Hausdorff space. Thus, if we show that $\mathcal{N}(\gamma) \cup X$ is second countable then we obtain its metrizability. In fact, a countable base for its topology is the union of the following families of open subsets:

- $\{V_n^*\}_{n \in \mathbb{N}}$, where $\{V_n\}_{n \in \mathbb{N}}$ is the family of all elements in an open countable base of Q meeting X .
- $\{W_{m,k} \mid m, k \in \mathbb{N}\}$, where, for each $m \in \mathbb{N}$, $\{W_{m,k}\}_{k \in \mathbb{N}}$ is a countable open base of the interior (in $\mathcal{N}(\gamma)$) of the finite polyhedron contained in $\mathcal{N}(\gamma)$ determined by the members of the canonical cover that are not contained in $Q \setminus B(X, \frac{1}{m})$. □

Proposition 2.12. *Let X be a compact metric space embedded in the Hilbert cube Q as a Z -set. If there exists a canonical cover of $Q \setminus X$ whose order is less than or equal to m , then $\dim(X) \leq m$.*

Proof. If γ is a canonical cover of $Q \setminus X$ whose order is less than or equal to m , the nerve of γ , $\mathcal{N}(\gamma)$, is an infinite polyhedron whose simplices have dimension less

than or equal to m . Consider the mentioned extended canonical projection

$$\pi : (Q, d) \longrightarrow (\mathcal{N}(\gamma) \cup X, d')$$

where d' is a distance function which defines the topology of $\mathcal{N}(\gamma) \cup X$.

Now we are going to bound the dimension of the compact metric space X via ε -mappings to polyhedra [6, Theorem 1.10.14]. Choose a number $\varepsilon > 0$. As $\pi : Q \rightarrow \mathcal{N}(\gamma) \cup X$ is a uniformly continuous map, take $\delta > 0$ such that if $d(z_1, z_2) < \delta$, $z_1, z_2 \in Q$, then $d'(\pi(z_1), \pi(z_2)) < \varepsilon/2$. Since X is a Z-set, for that $\delta > 0$ there exists a continuous map

$$f_\delta : Q \longrightarrow Q$$

which verifies

$$f_\delta(Q) \cap X = \emptyset$$

and

$$d(x, f_\delta(x)) < \delta \text{ for all } x \in Q.$$

Define the map

$$F_\varepsilon : (X, d') \longrightarrow \mathcal{N}(\gamma) \cup X$$

as

$$F_\varepsilon(t) = (\pi \circ f_\delta \circ (\pi|_X)^{-1})(t).$$

Observe that $F_\varepsilon(X) \subset \mathcal{N}(\gamma)$. Since X is a compact space, F_ε is a continuous map and $\mathcal{N}(\gamma)$ is an infinite polyhedron whose simplices have dimension less than or equal to m , the image of F_ε is a compact space which, necessarily, is contained in a finite subpolyhedron of $\mathcal{N}(\gamma)$. So, the dimension of the image of F_ε is less than or equal to m .

Finally, the diameter of all the fibres of F_ε is less than ε . In fact, as the image of F_ε lies in $\mathcal{N}(\gamma)$, take $y \in \mathcal{N}(\gamma)$ and $x_1, x_2 \in X$ such that $F_\varepsilon(\pi(x_1)) = F_\varepsilon(\pi(x_2)) = y$:

$$\begin{aligned} d'(\pi(x_1), \pi(x_2)) &\leq d'(\pi(x_1), \pi(f_\delta(x_1))) + d'(\pi(f_\delta(x_1)), \pi(x_2)) \\ &= d'(\pi(x_1), \pi(f_\delta(x_1))) + d'(\pi(f_\delta(x_2)), \pi(x_2)) < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon. \end{aligned}$$

Therefore,

$$\dim(X) \leq m.$$

□

Using the results above and the equivalence of categories in [4] we have

Theorem 2.13. *Consider the Hilbert cube with a fixed metric (Q, d) . Let $X \subset Q$ be a compact Z-set. Then the following sentences are equivalent:*

1. X is finite-dimensional.
2. The asymptotic dimension of the C_0 -coarse structure induced by the metric d in $Q \setminus X$ is finite.
3. $Q \setminus X$ admits a finite order canonical cover.

REFERENCES

1. M. Alonso-Morón and A. González Gómez, *The Hausdorff metric and classifications of compacta*, Bull. London Math. Soc. **38** (2006), 314–322. MR2215924 (2006m:54019)
2. S. Antonyan, *A short proof that hyperspaces of Peano continua are absolute retracts*, Proc. Amer. Math. Soc. **129** (2001), 2187–2189. MR1825932 (2002c:54004)
3. T.A. Chapman, *Lectures on Hilbert Cube Manifolds*, Amer. Math. Soc., Providence, RI, 1976. MR0423357 (54:11336)
4. E. Cuchillo-Ibáñez, J. Dydak, A. Koyama and M.A. Morón, *C_0 -coarse geometry of complements of Z -sets in the Hilbert cube*, Transactions A.M.S. (to appear).
5. J. Dugundji, *An extension of Tietze's theorem*, Pacific Journ. Math. **1** (1951), 353–367. MR0044116 (13:373c)
6. R. Engelking, *Theory of Dimensions Finite and Infinite*, Heldermann Verlag, Lemgo, 1995. MR1363947 (97j:54033)
7. S.-T. Hu, *Theory of retracts*, Wayne State University Press, Detroit, MI, 1965. MR0181977 (31:6202)
8. W. Hurewicz and H. Wallman, *Dimension Theory*, Princeton University Press, Princeton, NJ, 1941. MR0006493 (3:312b)
9. N. Kroonenberg, *Characterization of finite-dimensional Z -sets*, Proc. Amer. Math. Soc. **43** (1974), 421–427. MR0334221 (48:12540)
10. J. Roe, *Lectures on Coarse Geometry*, American Mathematical Society, Providence, RI, 2003. MR2007488 (2004g:53050)
11. N. Wright, *C_0 coarse geometry and scalar curvature*, J. Funct. Anal. **197** (2003), 469–488. MR1960422 (2003m:58034)
12. N. Wright, *The coarse Baum-Connes conjecture via C_0 coarse geometry*, J. Funct. Anal. **220** (2005), 265–303. MR2119281 (2006d:58023)

DEPARTAMENTO MATEMÁTICA APLICADA, E.T.S.I. MONTES, UNIVERSIDAD POLITÉCNICA, 28040 MADRID, SPAIN

E-mail address: `eduardo.cuchillo@upm.es`

DEPARTAMENTO GEOMETRÍA Y TOPOLOGÍA, FACULTAD DE CC. MATEMÁTICAS, UNIVERSIDAD COMPLUTENSE, 28040 MADRID, SPAIN

E-mail address: `ma_moron@mat.ucm.es`