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A RIGIDITY THEOREM FOR HOLOMORPHIC GENERATORS ON THE HILBERT BALL

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Abstract. We present a rigidity property of holomorphic generators on the open unit ball $\mathbb{B}$ of a Hilbert space H. Namely, if $f \in \mathrm{Hol}(\mathbb{B}, H)$ is the generator of a one-parameter continuous semigroup $\{F_t\}_{t\geq 0}$ on $\mathbb{B}$ such that for some boundary point $\tau \in \partial\mathbb{B}$, the admissible limit $K\text{-}\lim\limits_{z\to\tau} \frac{f(x)}{\|x-\tau\|^3} = 0$, then f vanishes identically on $\mathbb{B}$.

Let H be a complex Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and induced norm $\|\cdot\|$. If H is finite dimensional, we will identify H with $\mathbb{C}^n$. We denote by $\mathrm{Hol}(D, E)$ the set of all holomorphic mappings on a domain $D \subset H$ which map D into a subset E of H, and put $\mathrm{Hol}(D) := \mathrm{Hol}(D, D)$.

We are concerned with the problem of finding conditions for a mapping $F \in \mathrm{Hol}(D, E)$ to coincide identically with a given holomorphic mapping on D when they behave similarly in a neighborhood of a boundary point $\tau \in \partial D$.

A number of basic results in this direction are due to D. M. Burns and S. G. Krantz [6]. They establish conditions at a boundary point for a holomorphic self-mapping F of the open unit disk $\Delta := \{z \in \mathbb{C} : |z| < 1\}$ to coincide with the identity mapping (see Proposition 1 below). Then they generalize this fact to the n-dimensional case: for holomorphic self-mappings of the open unit ball (see Proposition 3 below) and of strongly pseudoconvex domains in $\mathbb{C}^n$. Further developments of this theme are presented by X. J. Huang in [15], where he obtains similar results for weakly pseudoconvex domains. More recently, L. Baracco, D. Zaitsev and G. Zampieri [3] have proved local boundary rigidity theorems for mappings defined only on one side as germs at a boundary point, and extended their results from boundaries of domains to submanifolds of higher codimension. More higher-dimensional results can be found, for instance, in [2] and [11].

In this paper we present a rigidity theorem for holomorphic generators on the open unit ball $\mathbb{B}$ of a Hilbert space H which generalizes the analogous theorem for the one-dimensional case [8, 17, 7] and properly contains the above-mentioned Burns–Krantz theorem for the open unit ball in $\mathbb{C}^n$.

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We begin by recalling the result of D. M. Burns and S. G. Krantz [6] for holomorphic self-mappings of the open unit disk Δ.

Proposition 1. *Let $F \in \mathrm{Hol}(\Delta)$. If the unrestricted limit*

$$\lim_{z\to 1} \frac{F(z)-z}{(z-\tau)^3} = 0$$

for some $\tau \in \partial\Delta$, then $F \equiv I$ on Δ.

This assertion also holds when the unrestricted limit is replaced with the angular one (see [22] and [5]). Recall that a function $f \in \mathrm{Hol}(\Delta, \mathbb{C})$ has an angular limit $L := \angle \lim_{z\to\tau} f(z)$ at a point $\tau \in \partial\Delta$ if $f(z) \to L$ as $z \to \tau$ in each nontangential approach region

$$\Gamma_k(\tau) := \left\{ z \in \Delta : \frac{|z-\tau|}{1-|z|} < k \right\}, \ k > 1.$$

In this case it is convenient to set $f(\tau) := \angle \lim_{z\to\tau} f(z)$. Moreover, in a similar way, one defines the angular derivative of f at $\tau \in \partial\Delta$ by $f'(\tau) := \angle \lim_{z\to\tau} \frac{f(z)-f(\tau)}{z-\tau}$.

A point $\tau \in \overline{\Delta}$ is a fixed point of $F \in \mathrm{Hol}(\Delta)$ if either $F(\tau) = \tau$, where $\tau \in \Delta$, or $\lim_{r\to 1^-} F(r\tau) = \tau$, where $\tau \in \partial\Delta = \{z : |z| = 1\}$. If F is not an automorphism of Δ with an interior fixed point, then by the classical Schwarz–Pick lemma and the Julia–Wolff–Carathéodory theorem, there is a unique fixed point $\tau \in \overline{\Delta}$ such that for each $z \in \Delta$, $\lim_{n\to\infty} F_n(z) = \tau$, where the n-th iteration F_n of F is defined by $F_1 = F$, $F_n = F \circ F_{n-1}$, $n = 2, 3, \dots$. This point is called the Denjoy–Wolff point of F. Moreover, a boundary fixed point $\tau \in \partial\Delta$ of F is its Denjoy–Wolff point if and only if $F'(\tau) \in (0, 1]$.

A rigidity result for generators of one-parameter continuous semigroups on Δ (see Proposition 2 below) has been proved in [8] and [17]. To formulate it, we first recall the definitions of these notions.

Let $D \subset H$ be a domain in the Hilbert space H. We say that a family $S = \{F_t\}_{t\geq 0} \subset \mathrm{Hol}(D)$ is **a one-parameter continuous semigroup on** D (a semigroup, for short) if

(i) $F_t(F_s(z)) = F_{t+s}(z)$ for all $t, s \geq 0$ and all $z \in D$,

and

(ii) $\lim_{t\to 0^+} F_t(z) = z$ for all $z \in D$.

A semigroup $S = \{F_t\}_{t\geq 0} \subset \mathrm{Hol}(D)$ is said to be generated if for each $z \in D$, there exists the strong limit

$$\lim_{t\to 0^+} \frac{1}{t}[z - F_t(z)] = f(z).$$

In this case the mapping $f : D \mapsto H$ is called the **(infinitesimal) generator** of S.

A well-known representation of generators on Δ is due to E. Berkson and H. Porta [4], namely:

A function $f \in \mathrm{Hol}(\Delta, \mathbb{C})$ is a generator if and only if there are a point $\tau \in \overline{\Delta}$ and a function $p \in \mathrm{Hol}(\Delta, \mathbb{C})$ with $\mathrm{Re}\, p(z) \geq 0$ for all $z \in \Delta$ such that

$$f(z) = (z-\tau)(1-\overline{\tau}z)p(z), \quad z \in \Delta. \tag{1}$$

This point τ is the common Denjoy–Wolff point of the semigroup generated by f.

The following rigidity result for generators has been proved in [8] and [17].

Proposition 2. *Let $g \in \mathrm{Hol}(\Delta, \mathbb{C})$ be the generator of a one-parameter continuous semigroup. Suppose that*

$$\angle \lim_{z \to 1} \frac{g(z)}{|z-1|^3} = 0. \tag{2}$$

Then $g \equiv 0$ in Δ.

Here we take this opportunity to present a completely different proof of this assertion.

Proof. Suppose that g does not vanish identically on Δ. Condition (2) implies that $\tau = 1$ is the Denjoy–Wolff point of the semigroup generated by g (see Lemma 3 in [10]). So, g has no null point in Δ (see Theorem 1 in [10]). Consequently, g can be represented by the Berkson–Porta formula

$$g(z) = -(1-z)^2 p(z), \quad z \in \Delta,$$

where p is a holomorphic function of nonnegative real part which does not vanish in Δ.

Consider the function

$$g_1(z) := \frac{-z}{(1-z)^2} \cdot g(z) = zp(z), \quad z \in \Delta.$$

This function is the holomorphic generator of a semigroup on Δ with its Denjoy–Wolff point at zero.

However, the equality

$$\angle \lim_{z \to 1} \frac{g_1(z)}{z-1} = \angle \lim_{z \to 1} \frac{-z}{(1-z)^3} \cdot g(z) = 0$$

implies that $g_1(1) = 0$ and $g_1'(1) = 0$. Therefore $\tau = 1$, too, is the Denjoy–Wolff point of the semigroup generated by g_1 (again by Lemma 3 in [10]). The contradiction we have reached proves that $g \equiv 0$ on Δ. □

As we have already mentioned above, D. M. Burns and S. G. Krantz generalize their one-dimensional result for holomorphic self-mappings of Δ (Proposition 1) to the open unit ball $\mathbb{B} := \{x \in \mathbb{C}^n : \|x\| < 1\}$, where $\|x\| = \sqrt{|x_1|^2 + |x_2|^2 + ... + |x_n|^2}$.

Proposition 3 (see [6])**.** *Let $\mathbb{B} \subset \mathbb{C}^n$ be the open unit ball. Let $\Phi : \mathbb{B} \to \mathbb{B}$ be a holomorphic mapping of the ball to itself such that*

$$\Phi(x) = \mathbf{1} + (x - \mathbf{1}) + O\left(\|x - \mathbf{1}\|^4\right)$$

as $x \to \mathbf{1}$. (Here $\mathbf{1}$ denotes the distinguished boundary point $\mathbf{1} = (1, 0, ..., 0)$ of the ball.) Then $\Phi(x) = x$ on the ball.

At this juncture, a natural question arises: does the rigidity result for generators (Proposition 2) admit an analogous generalization to the open unit balls of either $\mathbb{C}^n$ or a Hilbert space H? The following theorem gives an affirmative answer to this question. Moreover, we show that it is sufficient to consider the K-limit instead of the unrestricted one in the assumption of the theorem.

Let $\mathbb{B}$ be the open unit ball of the Hilbert space H. For $\alpha > 1$, we denote by

$$D_\alpha(\tau) := \left\{ x \in \mathbb{B} : |1 - \langle x, \tau \rangle| < \frac{\alpha}{2}(1 - \|x\|^2) \right\}$$

the Korányi approach regions at $\tau \in \partial\mathbb{B}$ and say that a mapping $f : \mathbb{B} \mapsto H$ has a K-limit M at τ if it tends to M along every curve ending at τ and lying in a Korányi region $D_\alpha(\tau)$.

Theorem. *Let $f \in \mathrm{Hol}(\mathbb{B}, H)$ be the generator of a one-parameter continuous semigroup on $\mathbb{B}$. If for some $\tau \in \partial\mathbb{B}$, the K-limit*

$$K\text{-}\lim_{x\to\tau} \frac{f(x)}{\|x-\tau\|^3} = 0, \tag{3}$$

then $f \equiv 0$ on $\mathbb{B}$.

Proof. We prove this assertion by reduction to the one-dimensional case. Namely, we consider the restriction of the orthogonal projection of an appropriate modification of the generator f to a one-dimensional disk touching $\mathbb{B}$ at the point $\tau \in \partial\mathbb{B}$. To this end, we fix a point $y \in \mathbb{B}$ and define the mapping

$$M_y(x) := \frac{y - P_y x - sQ_y x}{1 - \langle x, y\rangle}, \quad x \in \mathbb{B},$$

where P_y is the orthogonal projection of H onto the subspace generated by y ($P_0 \equiv 0$ and $P_y x = \dfrac{\langle x, y\rangle}{\|y\|^2} y$ for $y \neq 0$), $Q_y = I - P_y$ and $s = \sqrt{1 - \|y\|^2}$. This mapping is an automorphism of $\mathbb{B}$ satisfying $M_y^{-1} = M_y$ (*cf.* p. 98 in [12] and p. 25 in [20]).

Denote by U_y a unitary operator on $\mathbb{B}$ such that $U_y\tau = M_y\tau$. Then the mapping $m := M_y \circ U_y$ is an automorphism of $\mathbb{B}$ which satisfies $m(\tau) = \tau$ and $m(0) = y$.

Obviously, m is a biholomorphism of $\mathbb{B}$ onto $\mathbb{B}$. Therefore, by Lemma 3.7.1 on p. 30 of [9], the mapping

$$f_m(w) = [m'(w)]^{-1} f(m(w)), \quad w \in \mathbb{B}, \tag{4}$$

is also a holomorphic generator on $\mathbb{B}$.

Substituting

$$[m'(w)]^{-1} = [m^{-1}(x)]'_{x=m(w)} = U_y^* M_y'(m(w))$$

in (4), we have

$$f_m(w) = U_y^* M_y'(m(w)) f(m(w)), \quad w \in \mathbb{B}. \tag{5}$$

Now we define a holomorphic function g on the unit disk Δ of the complex plane $\mathbb{C}$ by

$$g(z) := \langle f_m(z\tau), \tau\rangle, \quad z \in \Delta. \tag{6}$$

This function g is a holomorphic generator on Δ. To see this, note that by the Theorem in [1], the generator f_m satisfies the inequality

$$\mathrm{Re}\langle f_m(x) - (1 - \|x\|^2) f_m(0), x\rangle \geq 0 \quad \text{for all} \quad x \in \mathbb{B}.$$

In particular, for $x = z\tau$, where $z \in \Delta$,

$$\mathrm{Re}(\langle f_m(z\tau), \tau\rangle \overline{z}) \geq (1 - |z|^2)\, \mathrm{Re}(\langle f_m(0), \tau\rangle \overline{z});$$

i.e.,

$$\mathrm{Re}(g(z)\overline{z}) \geq (1 - |z|^2)\, \mathrm{Re}(g(0)\overline{z}) \quad \text{for all} \quad z \in \Delta,$$

and, consequently, by the same theorem (see [1]), g is indeed a holomorphic generator on Δ. (We remark in passing that this also follows from the characterization of generators in terms of their ρ-monotonicity [18, 16].)

We claim that under our assumptions, $g \equiv 0$ on Δ. Indeed,

$$\begin{aligned} g(z) &= \langle U_y^* M_y'(m(z\tau)) f(m(z\tau)), \tau\rangle = \langle M_y'(m(z\tau)) f(m(z\tau)), U_y\tau\rangle \\ &= \langle f(m(z\tau)), [M_y'(m(z\tau))]^* U_y\tau\rangle, \quad z \in \Delta, \end{aligned} \tag{7}$$

and, consequently,

$$\begin{aligned} \frac{g(z)}{|z-1|^3} &= \frac{1}{|z-1|^3}\langle f(m(z\tau)), [M_y'(m(z\tau))]^* U_y\tau\rangle \\ &= \frac{\|m(z\tau)-\tau\|^3}{|z-1|^3}\left\langle \frac{f(m(z\tau))}{\|m(z\tau)-\tau\|^3}, [M_y'(m(z\tau))]^* U_y\tau\right\rangle. \end{aligned} \tag{8}$$

Note that each automorphism h of $\mathbb{B}$ is the restriction to $\mathbb{B}$ of a holomorphic mapping defined either on the larger ball $B(0,R)$ centered at zero of radius $R = \dfrac{1}{\|h^{-1}(0)\|}$ if $h(0) \neq 0$ or on all of H if h fixes the origin. So, M_y and m are, in fact, holomorphic mappings defined either on the open ball $B(0,R)$ of radius $R = \frac{1}{\|y\|} > 1$ if $y \neq 0$ or on H if $y = 0$. Hence the first factor on the right-hand side of equality (8) has a finite limit as $z \to 1$, and so has the second factor of the inner product.

Now we show that the first factor of the last inner product in (8) tends to zero as $z \to 1$ nontangentially in Δ.

For z close enough to 1 in the nontangential approach region

$$\Gamma_k = \left\{ z \in \Delta : \frac{|z-1|}{1-|z|} < k \right\}, \quad k > 1,$$

$m(z\tau)$ belongs to the Korányi region $D_\alpha(\tau)$ whenever $\alpha > k$. Indeed, it can be shown by direct calculations that the function m satisfies the equality

$$\frac{|1-\langle m(z\tau),\tau\rangle|^2}{1-\|m(z\tau)\|^2} = L\,\frac{|1-z|^2}{1-|z|^2}, \quad z \in \Delta,$$

where

$$L := \frac{d}{dz}\langle m(z\tau),\tau\rangle\bigg|_{z=1} = \frac{1-\langle y,\tau\rangle}{1-\langle U_y\tau, y\rangle} = \frac{|1-\langle y,\tau\rangle|^2}{1-\|y\|^2} > 0.$$

Consequently, we have for $z \in \Gamma_k$,

$$\frac{|1-\langle m(z\tau),\tau\rangle|}{1-\|m(z\tau)\|^2} = L\,\frac{|1-z|^2}{1-|z|^2}\cdot\frac{1}{|1-\langle m(z\tau),\tau\rangle|} < L\,k\,\frac{|1-z|}{|1-\langle m(z\tau),\tau\rangle|}.$$

Since $\lim\limits_{z\to 1}\left|\dfrac{1-\langle m(z\tau),\tau\rangle}{1-z}\right| = L$, it follows that if $z \in \Gamma_k$ is close enough to 1, then $m(z\tau)$ is in $D_\alpha(\tau)$ ($\alpha > k$). Hence, by hypothesis (3) of the theorem, $\angle\lim\limits_{z\to 1}\dfrac{f(m(z\tau))}{\|m(z\tau)-\tau\|^3} = 0$.

Therefore equality (8) implies that $\angle\lim\limits_{z\to 1}\dfrac{g(z)}{|z-1|^3} = 0$, and by Proposition 2, $g \equiv 0$ on Δ. So, by (7),

$$\langle f(m(z\tau)), [M_y'(m(z\tau))]^* U_y\tau\rangle = 0 \quad \text{for all} \quad z \in \Delta.$$

In particular, this equality holds for $z = 0$; *i.e.*,

$$\langle f(y), [M_y'(y)]^* U_y\tau\rangle = 0 \quad \text{for each} \quad y \in \mathbb{B}. \tag{9}$$

By direct calculations, one obtains that

$$\begin{aligned} &M_y'(x)h \\ &= \frac{1}{(1-\langle x,y\rangle)^2}\Big[-(1-\langle x,y\rangle)(P_y+sQ_y)h+\langle h,y\rangle(y-P_yx-sQ_yx)\Big]. \end{aligned}$$

Hence,

$$M_y'(y)h = -\frac{1}{1-\|y\|^2}(P_y+sQ_y)h,$$

and equality (9) is equivalent to

$$\langle f(y),(P_y+sQ_y)U_y\tau\rangle = 0.$$

Substituting

$$U_y\tau = M_y\tau = \frac{y-P_y\tau-sQ_y\tau}{1-\langle\tau,y\rangle}$$

in this equality, we obtain

$$\langle f(y), y-\tau+\|y\|^2\tau-\langle\tau,y\rangle y\rangle = 0 \quad \text{for all} \quad y\in\mathbb{B}.$$

Let $y = y_1\tau+\tilde{y}$, where $y_1 = \langle y,\tau\rangle$ and $\langle\tilde{y},\tau\rangle = 0$.

Similarly, $f(y) = f_1(y)\tau+\tilde{f}(y)$ with $f_1(y) = \langle f(y),\tau\rangle$ and $\langle\tilde{f}(y),\tau\rangle = 0$ for all $y\in\mathbb{B}$.

Using this notation, we have

$$\langle f_1(y)\tau, y_1\tau-\tau+\|y\|^2\tau-|y_1|^2\tau\rangle = -\langle\tilde{f}(y),\tilde{y}-\overline{y}_1\tilde{y}\rangle$$

and

$$(1-\overline{y}_1-\|\tilde{y}\|^2)f_1(y) = (1-y_1)\langle\tilde{f}(y),\tilde{y}\rangle.$$

Differentiating this equality with respect to $\overline{y}_1$, we conclude that it can hold only if $f_1(y) = 0$ and

$$\langle\tilde{f}(y),\tilde{y}\rangle = 0 \quad \text{for all} \quad y\in\mathbb{B}. \tag{10}$$

Now let σ be an arbitrary unit vector orthogonal to τ, *i.e.*, $\langle\sigma,\tau\rangle = 0$. Suppose that $\tilde{y} = y_2\sigma+u$, where $y_2 = \langle\tilde{y},\sigma\rangle$ and $\langle u,\sigma\rangle = 0$.

Similarly, $\tilde{f}(y) = f_2(y)\sigma+v(y)$ with $f_2(y) = \langle\tilde{f}(y),\sigma\rangle$ and $\langle v(y),\sigma\rangle = 0$ for all $y\in\mathbb{B}$. Then by (10),

$$f_2(y)\overline{y}_2 = -\langle v(y),u\rangle.$$

Differentiating this equality with respect to $\overline{y}_2$, we obtain $f_2(y) = 0$. Hence, $f\equiv 0$ on $\mathbb{B}$. □

Following L. A. Harris [13], we define the numerical range of each $h\in\mathrm{Hol}(\mathbb{B},H)$ which has a norm continuous extension to $\overline{\mathbb{B}}$ by

$$V(h) := \{\langle h(x),x\rangle : \|x\| = 1\}.$$

For an arbitrary holomorphic mapping $h\in\mathrm{Hol}(\mathbb{B},H)$ and for each $s\in(0,1)$, we define the mapping $h_s : \frac{1}{s}\mathbb{B}\mapsto H$ by

$$h_s := h(sx), \quad \|x\| < \frac{1}{s},$$

and set

$$L(h) := \lim_{s\to 1^-}\sup\mathrm{Re}(V(h_s)).$$

It is known (Theorem 1 in [14]) that the mapping $I - h$ is a generator if and only if $L(h) \leq 1$. So the following corollary is an immediate consequence of our theorem.

Corollary. *Let $h \in \mathrm{Hol}(\mathbb{B}, H)$ with $L(h) \leq 1$. If for some $\tau \in \partial\mathbb{B}$, the K-limit*

$$K\text{-}\lim_{x \to \tau} \frac{h(x) - x}{\|x - \tau\|^3} = 0, \tag{11}$$

then $h \equiv I$ on $\mathbb{B}$.

Since obviously $L(h) \leq 1$ for all self-mappings of $\mathbb{B}$, this corollary properly contains Proposition 3.

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